Chapter 1

Introduction

The nucleus is a unique many-body quantum system in the sense that the number of constituent nucleons is sufficiently large to allow the correlations, while at the same time the number is finite. The strong interaction between the fermions allows the shapes of the nucleonic orbitals to influence the overall shape of the nucleus. Non-spherical nuclei can therefore be generated by a few anisotropic nucleonic configurations in the valence shell. A new dimension in the study of nuclear dynamics has opened up through the extension of nuclear spectroscopic investigations to large angular momenta, where the centrifugal and the Coriolis forces are sufficiently strong to modify the single-particle basis of the nuclear correlations. This development is founded on the exploration of the properties of nuclear energy levels at high spin.

The availability of heavy-ion accelerators and improved detector systems for detection of $\gamma$-rays has made it possible to reach high-spin nuclear states and to study their decay properties in detail. The quantitative information on the yrast spectroscopy of high spin states comes from the analysis of discrete $\gamma$-rays emitted from the evaporation residues in (HI, xpyn) reactions. The measured quantity and extracted physical properties are the following; spin-alignments, moments of inertia, size of pair-correlation, band crossings, etc., from spin assignments of energy levels; band structure, shape, etc., from collective E2 transitions; proton- or neutron-configurations, fingerprint of one-particle motion in rotating potentials, etc., from E1 or M1 moments; deviation of nuclear shape from axial symmetry from E2 transitions with $\Delta J = 1$ and so on.

Experimental investigations carried out over the last several years using heavy ion beams and large detector arrays have enabled us to study the behaviour of nuclear sys-
terns with large number of valence nucleons, spanning a wide range between very low and very high excitation energy and rotational frequency. The study of the evolution of nuclear shapes, changing role of pairing and Coriolis forces, etc., with increasing rotational frequency has enriched our knowledge about the interplay of collective and non-collective modes of excitation in producing the richness in nuclear spectra. Such studies have been extended recently to nuclei near closed shell, especially in Sn and Sb nuclei near $Z = 50$ and in Cu, Zn and Ge nuclei near $Z = 28$ closed shell and very interesting results have been obtained.

The nuclei near closed shell have spherical shape. Their low energy spectra are rather simple. Their excitation modes were well understood in terms of simple models like anharmonic vibrator and particle vibrator model in the case of even-even and odd nuclei, respectively [Bohr and Mottelson (1975)]. However, with increasing energy and angular momentum these nuclei attain a deformed shape and existence of rotational bands with moderate to large deformation had been observed experimentally. Such rotational bands were observed in nuclei near $Z = 50$ shell like tin and antimony. Any explanation of these deformed structures require particle-hole excitations across the $Z = 50$ shell gap, which has the effect of increasing the number of valence particles and holes to the point where deformation and collective rotation become energetically favoured.

Thus the nuclei near closed shell offer an unique opportunity to investigate the different modes of excitations with spherical and deformed shapes present in the same system. In the last two decades, the spectroscopy of these nuclei has enriched our knowledge about understanding the basic features like, development of collectivity, the interplay of different modes of excitations, viz., single particle and collective and the roles played by neutrons occupying unique parity high-j orbital, etc. In recent years the general trend in $\gamma$-ray spectroscopy is to populate high spin states of neutron deficient nuclei near drip lines and to produce and study the properties of heavier $N = Z$ nuclei. The heaviest $N = Z$ nucleus so far produced is the $^{100}\text{Sn}$ [Lewitowicz et al. (1994, 1995), Chartier et al. (1996)]. With the advent of radioactive ion beam facilities, it is now possible to produce exotic nuclei near drip lines and this has certainly opened up entirely new horizons in the field of nuclear structure study.

In this Chapter an attempt has been made to review the experimental and theoretical
efforts made so far to understand the underlying excitation mechanisms present in nuclei near $Z = 50$ and $Z = 28$ closed shell at low and high spin regime.

1.1 Near $Z = 50$ closed shell

The nuclei near $Z = 50$ closed shell, particularly tin and antimony have always served as the testing ground for the different models and methods of nuclear structure [Bohr and Mottelson (1975)]. Due to the shell closure at $Z = 50$ the low-lying excitation modes are expected due to neutrons only and therefore the characterisation of low-lying states in these nuclei would be easy and can be made with the help of simple models. Early experiments (prior to 1976) had determined much of the low-spin, low-energy structures of the nuclei in this mass region and they were found to be characterised by two basic excitation modes, viz., the single particle and the collective vibration about the spherical equilibrium shape. In the case of odd-mass nuclei, the energy spectra are non-collective in nature near low spin and low excitation energy. Reasonable successes have been achieved in explaining the low lying states on the basis of the coupling of an odd-proton or an odd-neutron in the available spherical shell model orbitals, to the phonon excitations of Sn-core [Sen and Sinha (1970), VandenBerghe and Heyde (1971)]. However, with increasing spin and excitation energy, collective bands would develop with moderate to large deformation in these nuclei through particle-hole excitations across the $Z = 50$ major shell [Heyde et al. (1983)].

Within the mean field approach, the combination of $g_{9/2}$ holes and $g_{7/2}$ particles is energetically favoured for a prolate deformation of $\beta_2 \sim 0.2$. It is clear from the Woods-Saxon calculation for the proton single particle levels around $Z = 50$ region [Fig. 1.1] that the upsloping $g_{9/2}$ level with $\Omega=9/2$ crosses the downsloping $g_{7/2}$ with $\Omega=1/2$ at that deformation. In this way the 2p-2h excitation involving these orbitals give rise to $\Delta J = 2$ collective band in the even-even Sn isotopes and the 2p-1h excitation give rise to $\Delta J = 1$ collective band in the odd-mass Sb nuclei. The bands in Sn and Sb nuclei are particularly interesting because with the small number of active particles outside the $Z = N = 50$ closed shell, the total angular momentum for configuration with a few holes in the $\pi g_{9/2}$ subshell is limited to $\sim 30-50h$. When a rotational band with particular configuration is followed to a high spin, eventually the total angular momentum available from the spin alignment
Figure 1.1: Woods-Saxon calculation for the single particle levels around $Z=50$ and $N=60$ as a function of quadrupole deformation for a) Proton and b) Neutron.
of the particles outside the closed shells is exhausted and the band terminates [Ragnarsson et al. (1995), Afanasjev and Ragnarsson (1995)]. As the particles gradually align, the nuclear shape has been predicted to change over many transitions from collective near prolate ($\gamma \approx 0^\circ$) at low spin, to noncollective oblate ($\gamma \approx 60^\circ$) at the band termination. Such terminating bands have been studied to near terminating spins in Sn and Sb isotopes [Wadsworth et al. (1994), Käubler et al. (1995a, 1995b), Janzen et al. (1994), LaFosse et al. (1994)].

From a historical point of view the low lying deformed structures involving particle-hole excitation across the $Z = 50$ shell gap with 1p-2h configuration were first introduced by Bäcklin et al. [Bäcklin et al. (1967)] for odd mass In nuclei. Later systematics of these structures with well developed $\Delta J = 1$ rotational band built on $9/2^+$ state, having $\Delta J = 2$ intraband cross-over transitions were observed in odd-mass $^{113-119}$Sb nuclei [Sheroy et al. (1979)]. The $9/2^+$ bandhead was believed to result from the excitation from the $g_{9/2}$ orbital across the $Z = 50$ shell gap. First evidence of 2p-2h excitation across the $Z = 50$ closed shell in even-even nuclei was observed in ($^3$He,n) reaction studies by Fielding et al. [Fielding et al. (1977)]. In this study an excited $0^+$ state in even Cd and Sn nuclei were strongly populated.

1.1.1 Even-even nuclei

Even-even Sn nuclei are expected to have spherical equilibrium shape. This is confirmed by the small magnitude of the quadrupole moment observed for the $2^+$ state in Sn isotopes [Stelson et al. (1970)]. Potential energy calculations have also indicated the presence of a deep spherical minimum. The low-lying states of spherical nuclei can be treated in terms of two basic excitations, phonon and quasi-particles. In even-even nuclei the lowest excitation mode is the phonon. For odd mass nuclei both of these modes of excitations are low in energy and must be considered along with their interaction.

The low-lying levels of even-even nuclei are characterised by a $2^+$ state at certain energy above the ground state with a fast $E2$ transition to the ground state. The next excited states $0^+$, $2^+$ and $4^+$ states are at approximately twice the energy of the first $2^+$ state and are connected to first $2^+$ state by $E2$ transitions. The strength of these transitions is of the same order as that of the latter to the ground state. The $M1$ transition from the second
2+ to the first 2+ state and the E2 crossover transition to the ground state are much weaker. These data strongly suggest that the lower states of the even-even nuclei in these nuclei are not properly described as two quasi-particle states or other simple-particle states, but more nearly as quadrupole vibrational states. However, the higher lying states do not show the same characteristics and are classified as neutron excitations. Projected BCS quasi-particle calculation or broken-pair model was successful in explaining the experimental data for different even mass Sn isotopes [Van Gunsteren et al. (1974, 1976)]. Later, this calculation was extended to include proton 1p-1h excitations [Van Poelgeest et al. (1980)] and was reasonably successful in the description of the properties of the isomeric states and higher lying states in even-even Sn isotopes. However, experimental transition strengths were not reproduced in this model for stretched E2 and E3 transitions between excited states. The observed strength of these transitions was always less than ten percent of the 'phonon' transition strengths which were improved with small admixtures of two-broken-pair configuration states [Bonsignori et al. (1985)]. However, both these calculations faced difficulties in reproducing the lowest excited 0+ state which appears only about 0.5 MeV above the first excited 2+ state in all even Sn isotopes.

Fielding et al. [Fielding et al. (1977)] in (3He,n) studies found that the first excited 0+ states in even 108-118Sn nuclei were populated as strongly as that of the ground state via L = 0 two-proton transfer. Later collective band structure based upon this 0+ state upto spin 12+ in even 112-118Sn nuclei was identified in γ-spectroscopic study by Bron et al. [Bron et al. (1979)]. Similarities in the energy systematics with odd-mass In and Sb nuclei clearly indicate that these bands are based on the deformed proton 2p-2h configuration. Later the bands in 112Sn and 114Sn were extended to higher spins [Harada et al. (1988, 1989)] and new bands were found in the lighter Sn isotopes down to 106Sn [Viggars et al. (1987), Wadsworth et al. (1993, 1994), Mäkelä et al. (1995)].

Further evidence for the deformed nature of these states was obtained from the electromagnetic transition probabilities. The B(E2) values in these nuclei had indicated much more collectivity for the 0^+_2, 2^+_2, 4^+_1 and 4^+_2 states than predicted by the neutron quasi-particle calculations. Furthermore, the observed E0 transition rate for 0^+_3 → 0^+_2 transition in 116Sn was of the same magnitude as the rate of the E0 transition from the beta-band heads in deformed nuclei. A nuclear monopole transition is a consequence of a change of
mean charge radius in the nucleus and therefore is a measure of deformation and mixing. The strong E0 transition in the $^{116}$Sn can be generated if the associated $0^+$ states are strongly mixed and involve deformation. Wenes et al. [(Wenes et al. (1981))] had obtained good agreement with the experimental E0 and E2 data in a calculation where proton 2p-2h excitation was coupled to the quadrupole vibration of the even Sn core.

1.1.2 Even-odd or odd-even nuclei

Till 1979, there were no data on the high spin states in odd Sn isotopes although low lying states in these nuclei had been studied earlier. The highest spin states ever assigned was the $11/2^-$, arising out of the $\nu h_{11/2}$ unique parity orbital. Hasimoto et al. [(Hasimoto et al. (1979))] first reported the observation of high spin states in $^{109,111,113,115,117}$Sn studied through the ($\alpha$,xn) reaction on Cd isotopes. The excitation energies and the transition probabilities between members of lower group can be well understood by a particle core coupling model. The group of states at higher energy region was interpreted as arising from three quasiparticle configuration. Phonon coupled states were also observed for $g_{7/2}$ neutron orbital in $^{109}$Sn and $^{117}$Sn, although they were not identified in other isotopes. A search for the positive parity states with $J^\pi \geq 9/2^+$ in $^{111}$Sn was made by Prade et al. [(Prade et al. (1984))] and they extended the information upto $J^\pi \geq 17/2^+$ and excitation energy of 2.5 MeV. A remarkable agreement between experiment and theory based on shell model was achieved for these states. Till 1994, no rotational band, based on one quasiparticle coupled to 2p-2h band of the even-even Sn core have been observed in odd mass Sn nuclei.

In the case of odd-mass Sb nuclei, low-lying structures were described in terms of proton quasiparticle excitation in the available shell model space consisting of $1g_{7/2}, 2d_{5/2}, 3s_{1/2}, 2d_{3/2}$ and $1h_{11/2}$ orbitals. Presence of rotational band, with band head $9/2^+$ below 2 MeV of excitation energy provides information on deformed states that coexist with these spherical states.

Before the advent of multidetector systems no further experimental information was added to the above mentioned data on collectivity in this mass region. Since 1988 such information became available and within a short span of a few years, many new features of collective nuclear rotation have been reported. These experiments have confirmed and
extended the information on the 2p-1h deformed states in Sb isotopes with $\pi g_{9/2}^{-1} \otimes \pi g_{9/2}^{2}$ configuration. Apart from this, several new bands, of both positive and negative parity extending up to very high spin were observed in these nuclei. More interestingly, it was shown [Janzen (1995), Afnasjev and Ragnarsson (1995)] that the collective bands observed in $^{109,111}$Sb would offer a unique opportunity to investigate the gradual transition from collective to non-collective behaviour within a specific configuration.

Presence of both decoupled $\Delta J = 2$ and coupled $\Delta J = 1$ rotational bands extending up to very high spin was reported in odd-mass Sb nuclei e.g. in $^{109}$Sb [Janzen et al. (1994)], $^{111}$Sb [LaFosse et al. (1994)], $^{113}$Sb [Janzen et al. (1993)], $^{115}$Sb [Chakrawarthy and Pillay (1996), Moon et al. (1995)] and $^{117}$Sb [LaFosse et al. (1992)]. However, the spins, parities and the excitation energies of some of these bands could not be fixed with definiteness because transitions linking them to states of known spin and parity were not observed and the uncertainties in the spin values were predicted to be ±2h to ±3h. Cranked Strutinsky calculations done for $^{109}$Sb with a Woods-Saxon potential have predicted that the lowest lying structure should correspond to a proton in the $h_{11/2}$ orbital coupled to the $^{108}$Sn deformed (i.e. proton $2p-2h g_{7/2}^{2} \otimes g_{9/2}^{-2}$) core at a deformation $\beta_2 \sim 0.2$ and this is true for the other isotopes also. The intensity of the gamma transitions in the band based on the above configuration supported this argument in all Sb isotopes. However, two possible configurations were interpreted for other two $\Delta J = 2$ bands in $^{109}$Sb. One explanation involves the deformed $2p-2h$ state, from which one of the protons is promoted to the next available $h_{11/2}$ orbital. Excited band with similar structure were also observed in $^{108}$Sn isotope [Wadsworth et al. (1993)]. The other interpretation regarding this configuration is the neutron p-h structure $h_{11/2} \otimes g_{9/2}^{-1}$ coupled to the yrast band. Similar interpretations were given for the positive parity $\Delta J = 2$ bands in $^{111}$Sb.

A number of unusual features was observed in the high spin regions of these bands. In $^{109}$Sb, it was found that for $h\omega$ (0.8 MeV, strong fluctuations appear in the dynamic moment of inertia, while for $h\omega$ > 0.8 MeV they would smoothly decrease to a very low value finally becoming equal to the values of the three known bands in $^{108}$Sn at high rotational frequencies. A large difference between the dynamical moment of inertia ($\sim 13h^2 MeV^{-1}$) and the kinematic moment of inertia ($\sim 36h^2 MeV^{-1}$) was, therefore noted at high frequencies. These quite intriguing features were considered to be the first observation.
of a smooth progression from a collective band to a non-collective band-terminating state.

In a subsequent study by Janzen et al. [Janzen et al. (1994)] a well-deformed rotational band built on the proton h_{11/2} orbital, to a very high spin (79/2)h up to approximately 21 MeV in excitation and large rotational frequency (\(\hbar \omega \sim 1.0 \text{ MeV}\)), were observed in \(^{113}\text{Sb}\). The measured intrinsic quadrupole moment (Q_0 = 4.4 ± 0.6 eb) for this negative parity band was consistent with an axial prolate deformation of \(\beta_2 \sim 0.32\). These data most probably correspond to the first observation of the h_{11/2} orbital as an "enhanced deformation" intruder configuration. Furthermore, it was noted in this work [Janzen et al. (1994)] that though the quadrupole moment and excitation energy of the \(\pi h_{11/2}\) intruder band were predicted well with a standard mean field theory, a large residual n-p interaction strength is essential to describe the \(\pi h_{11/2}\) band-crossing properties. The band-crossing due to rotational alignment of h_{11/2} neutrons in \(^{113}\text{Sb}\) had been observed to be delayed considerably relative to the bands in the neighbouring even-even nuclei.

The high spin study of \(^{117}\text{Sb}\) [LaFosse et al. (1992)], however, had shown a different picture compared to \(^{109,113}\text{Sb}\)-isotopes. Three \(\Delta J = 2\) decoupled rotational bands up to moderate spin were found to be irregular in nature and were interpreted as resulting from the coupling of proton in g_{7/2}, d_{5/2} and h_{11/2} valence orbitals to the deformed 0_2 (2p-2h) states with \(\pi g_{9/2}^2 \otimes \pi g_{7/2}^2\) configuration of \(^{116}\text{Sn}\). The decay pattern of these three bands showed striking similarities to that of the deformed band in \(^{116}\text{Sn}\). It had also been noted by LaFosse et al. [LaFosse et al. (1992)] on \(^{117}\text{Sb}\) that \(Z = 51\) valence proton did not have any significant shape-driving effects on either the spherical or the deformed cores.

1.1.3 Odd-odd nuclei

The structure study of odd-odd nuclei are restricted both in the fields of theory and experiment due to their complex structure and large level densities. However, they are the important candidates in studying the different mode of excitation viz., the single particle and the collective. Also, since these nuclei contain two odd particles of different nature, they offer an opportunity to investigate the residual interaction between a single neutron and a single proton in a variety of different orbitals with a variety of underlying core structures. The residual interaction between the single neutron and the single proton can be studied best in nuclei that are near doubly closed shell. The nuclei near singly closed
shell and away from closed shell then reveal how the residual interaction is modified by the different core structures.

In another context, the interaction between the unpaired neutron and the unpaired proton plays a major role in the description of the high spin states. It is well known that the backbending phenomenon in even-even nuclei is due to the alignment of a pair of particles along the rotation axis of the nucleus. Moreover, the second backbend is ascribed in many nuclei to the breaking of another pair of particles of different type than the first leaving the nucleus with a structure consisting of a core and two unpaired neutrons and two unpaired protons. So the interaction between one unpaired neutron and one unpaired proton provides fundamental information for describing high spin states. Furthermore, the interesting proposition put forward by Federman and Pittel [Federman and Pittel (1979)] that the interaction between protons and neutrons in spin-orbit partner orbitals is particularly strong and responsible for the onset of deformation in $A = 100$ and $A = 150$ regions and is also important in studying the structure of odd-odd nuclides. However, there are few odd-odd nuclides where the orbitals are spin-orbit partners and where anything over a four member multiplet has been fully characterised.

From a theoretical point of view, one serious difficulty in studying odd-odd nuclides lies in the necessity of establishing the position of all the members of a particular multiplet. The manifestation of the interaction between the two odd nucleons is the splitting of the multiplet of states ranging from $j_n-j_p$ up to $j_n+j_p$. The splitting of these states depends on the strength of the interaction between the two odd particles. A second consequence of the interaction is the motion of the whole multiplets relative to their single particle parents. Thus, to fully characterise the interaction, it is necessary to determine the position of all the members of not just one, but several multiplets. Such characterisations are quite important where the possibility of significant odd-even staggering is present.

The systematic study of the properties of the ground and the isomeric states of spherical odd-odd nuclei resulted in the form of Nordheim rules [Nordheim (1950)] and later of the extended Brennan-Bernstein rules [Brennan and Bernstein (1960)] for ground state spins and parities. These were explained with the use of attractive short-range interactions. Later the energy splitting of two-body multiplets in near-magic odd-odd nuclei were systematized [Schiffer and True (1976)] starting from the semiclassical picture that
the residual interaction between two nucleons with $j_1$ and $j_2$ coupled to $J$, depends largely on their angular overlap and thus on the angle between $j_1$ and $j_2$. An analytic expression for the energy splitting of the proton-neutron multiplet in spherical odd-odd nuclei was derived. This is known as "Parabola rule" [Paar (1979)]. In this model, the major contribution to the residual interaction arises from a quadrupole interaction that produced a parabolic splitting when energy is plotted against $J(J+1)$. The orientation of the parabola, "open up" or "open down", depends upon the relative quasiparticle nature, particle or hole, of the proton and neutron. Moreover, the magnitude of the splitting is proportional to $(u^2 - v^2)$. The spin dipole interaction is linear when plotted against $J(J+1)$ and serves to tilt the parabola. The parabolic rule, however, in practice was not rigidly followed. The large deviation from this rule is required to take into account the configuration mixing of different proton-neutron multiplets and higher multipoles in the residual interaction. Higher multipoles, however, can be taken into account in an analytical way, e.g. by expanding the $\delta$-function force in terms of spherical harmonics. So, detailed understanding of the neutron-proton interaction necessitates calculations using multipole-multipole, surface-delta function, spin-spin-$\delta$ function, tensor and isospin-isospin interactions.

The nuclear structure study of odd-odd nuclei near $Z = 50$ shell and $A = 110$ region (especially, indium and antimony) are interesting in the sense that these nuclei bear a closed proton core with a proton hole (particle) respectively and one would expect relatively easier interpretation of the excitation modes. In a phenomenological approach these nuclei contain even-even Sn core which exhibit very specific features. The low lying states in these nuclei are vibration like. However, collective rotational band based on an excited $0^+$ state has been observed in all the Sn isotopes as a consequence of particle excitation across the $Z = 50$ shell gap. Therefore, one can expect the reflection of these features of the core in the level structure of the odd-odd Sb nuclei. The neutrons in these nuclei generally fill the $g_{7/2}$ and $d_{5/2}$ orbital. However, with increasing mass number, the neutrons begin to occupy the unique parity orbital $h_{11/2}$. These neutrons play a crucial role in producing the high spin states in these nuclei and hence produces an opportunity to study the behaviour of n-p interaction at high spin. Also the residual interaction between the odd proton and the odd neutron is sensitive to the particle and hole character of the interacting particles. So, a systematic study of these nuclei will certainly provide important information on the
role played by the neutron-proton interaction with different underlying cores.

The level properties of the ground state and a few excited levels of odd-odd Sb nuclei are known from $\beta$-decay studies, particle transfer reactions, etc., [Lederer and Shirley (1978)]. The level scheme of odd-odd Sb nuclei resulting from these investigations was characterised in terms of few long living isomeric states. Later studies through $(p,n\gamma)$ reaction and with better detector systems had enriched the knowledge about the level structure of these nuclei up to a few hundred keV [Kamermans et al. (1976), Adachi et al. (1979)]. The level scheme of odd-odd Sb nuclei comprise of some noncollective positive and negative parity states which arise mainly due to the coupling of proton and neutron motion to the vibrational states of the even Sn core. The low-lying states in these nuclei were understood theoretically in terms of proton coupled to neutron quasiparticle states [Van Gunsteren et al. 1976]. An interesting feature observed in the low energy spectra of the odd-odd Sb nuclei, from $^{132}$Sb down to $^{110}$Sb, is the presence of an isomeric $8^-$ state. It was also observed that the excitation energy of this state decreases with increasing mass number. This fact can be linked to the decreasing excitation energy of the $11/2^-$ state with increasing mass number in odd mass Sn nuclei.

The high spin states of odd-odd $^{114,116,118,120}$Sb were studied through heavy ion beam [Van Nes et al. (1982), Vajda et al. (1983)]. In recent years these nuclei have attracted many groups and interest has been shown in the production of high spin states in more and more neutron deficient even Sb nuclei, for example $^{114}$Sb [Paul et al. (1994)], $^{112}$Sb [see Chapter 5, Moon et al. (1997)], $^{110}$Sb [Lane et al. (1997)], $^{108}$Sb [Cederkäll et al. (1995)] and $^{106}$Sb [Seweryniak et al. (1994)]. The common characteristic of the level spectra in all nuclei is the presence of a negative parity coupled $\Delta J = 1$ rotational band with $\Delta J = 2$ crossover transitions. The excitation energy of the bandhead is close to that of the $2p-2h$ bandhead in even-even Sn isotopes and also the $\pi g_{9/2}^{-1}$ bandhead in neighbouring odd-mass Sb nuclei. This suggests a similar origin for all these bands. The most probable configuration of this band in even Sb isotopes is $\pi g_{9/2}^{-1}\pi g_{7/2}^{-1} \otimes \nu h_{11/2}$.

The similarity in energy spacings of the bands in all these isotopes suggests that the underlying core has not changed with the increase in neutron numbers. There are also similarities in energy spacing and electromagnetic properties with the neighbouring odd Sb isotopes indicating decoupled nature of the neutron. Attempts were made to analyze
these bands in the framework of particle-rotation coupling model \cite{VanNes1982, Duffait1982}. Good agreement between experimental and calculated results had been obtained for the high spin states. The lower members of the band however showed some deviation presumably due to the contribution from the spherical configuration. The decays from \( J^\pi = 9^- \) and \( 8^- \) states of the band to the levels originating from the spherical configuration support the above conclusion.

Total routhian calculations based on a Woods-Saxon potential using cranked Strutinsky shell correction approach were performed by Cederkäll et al. \cite{Cederkall1995} for \(^{108}\text{Sb}\) isotope. The lowest lying deformed minimum that appeared in the total routhian surfaces are associated with the \( \pi g_{9/2}^1 \pi g_{7/2}^2 \otimes \nu h_{11/2} \) configuration. This minimum starts to develop at \( \hbar \omega = 0.18 \text{ MeV} \) and has a calculated quadrupole deformation of \( \beta_2 = 0.16 \). The first alignment in the rotational band in this mass region is due to \( \nu h_{11/2} \) at a frequency of \( \hbar \omega \approx 0.42 \text{ MeV} \). The absence of this crossing indicates that the \( \nu h_{11/2} \) orbital is occupied and hence the first alignment is blocked. The first proton crossing as predicted by this calculation is at \( \hbar \omega \approx 0.71 \text{ MeV} \) which is outside the observed range in \(^{108}\text{Sb}\). At somewhat higher excitation energy this calculation predicts a deformed \( \pi h_{11/2} \otimes \nu h_{11/2} \) structure having \( \beta_2 \approx 0.21 \). No clear signature for the existence of this deformed structure has been observed in any odd-odd Sb isotope.

As the \( h_{11/2} \) is not the lowest energy neutron state in this mass region, it is expected that the positive parity intruder states originating from \( \pi g_{9/2}^{-1} \pi g_{7/2}^1 \otimes \nu g_{7/2} \) and \( \pi g_{9/2}^{-1} \pi g_{7/2}^2 \otimes \nu d_{5/2} \) configurations can be found even lower in energy. The \( 7^+ \) isomeric state in \(^{116}\text{Sb}\) \cite{VanNes1982} and \(^{118}\text{Sb}\) \cite{Vajda1983} are found nearly 200 keV below the \( 7^- \) member of \( \pi g_{9/2}^{-1} \pi g_{7/2}^2 \otimes \nu h_{11/2} \) configuration. On the basis of magnetic moment data \cite{Ionescu-Bujor1988}, its configuration has been assigned as \( \pi g_{9/2}^{-1} \pi g_{7/2}^2 \otimes \nu d_{5/2} \). This state is fed by very weak \( \gamma \)-transitions. The weakness of these transitions and their absence in other investigations indicate that the states built on \( 7^+ \) state are not yrast. Thus they cannot be investigated in heavy-ion reactions as concluded from the results of \cite{Cederkall1995}. Recently, a search for the positive parity intruder states has been made in \(^{116,118}\text{Sb}\) via \( (\alpha,n\gamma) \) reaction \cite{Fayez-Hassan1997a}. Two positive parity \( \Delta J = 1 \) bands in addition to negative parity ones, which are connected to \( 7^+ \) isomeric states in both nuclei have been observed. The experimental results have been investigated
using interacting boson fermion fermion model formalism. The results of this calculation are in accordance with the parabolic rule, predict open and down parabolic splittings for the positive, and open up splittings for the negative parity intruder states, leading to the prediction of $\pi g_9/2 \nu d_5/2 \; 7^+$ and $\pi g_9/2 \nu g_7/2 \; 8^+$ bandheads. The experimentally observed $\Delta J = 1$ nature of the positive parity band is in accordance with the QQ rule which predicts this nature for the band in which holes are coupled to a prolate core.

In a heavy ion reaction study only yrast and near yrast levels are populated. However, to characterise the residual interaction between the odd proton and the odd neutron one would have to have detailed knowledge about the multiplets of different-proton-neutron combination. These non-yrast levels are mostly populated in nuclear reactions using light ions or proton projectiles.

In the last few years the low lying states in odd-odd Sb isotopes were extensively studied using alpha and proton beam as well as by electron spectroscopy, for example in $^{118}\text{Sb}$ [Gulyás et al. (1992)], $^{116}\text{Sb}$ [Gácsi et al. (1991)], $^{114}\text{Sb}$ [Gácsi and Dombrádi (1994)] and $^{112}\text{Sb}$ [Fayez-Hassan et al. (1997b)]. Interacting boson fermion fermion model has been used to understand the low-lying features observed in these nuclei. The features of these calculations are described by coupling the odd neutron and odd proton to the spherical even-even core. Surface delta, spin delta and spin-spin interaction have been used to describe the residual interaction between the proton and the neutron. Most of the features, e.g. energy and electromagnetic properties, have been reproduced well by these calculations. Also, $E \times J(J+1)$ plot of different proton neutron multiplets supports the 'parabolic rule'. However, deviations from the parabolic rule have been observed in higher $^{120,122,124}\text{Sb}$ nuclei presumably due to collective contributions in their wavefunctions [Dombrádi et al. (1993)].

1.2 Near $Z = 28$ closed shell

The spectroscopy near $Z = 28$ closed shell contributed a lot towards the investigation of the structure of nuclei with different shapes viz., prolate, oblate and triaxial and with different modes of excitations involving different degrees of freedom present both at low and high spin regime. Many interesting features like transition of shape from oblate to prolate, octupole collectivity, gamma instability, etc., made this region attractive for
nuclear structure study. The low energy spectra of nuclei like Ni and Cu are rather simple and were well understood in terms of collective vibration about an equilibrium spherical shape and weak coupling of odd-particle to the vibrating even-even core, respectively.

As more and more particles are added to $^{56}\text{Ni}$, the interaction between the core and the particles outside the core produces a deformed equilibrium shape in these nuclei. Existence of fair to large deformation had been observed in Ge-Kr-Sr nuclei. The nuclei whose shape is not properly defined at low spin and are soft to deformation are called transitional nuclei. Their wavefunctions are superpositions of rotational, vibrational and single particle contributions. However, with increasing spin and rotational frequency the nuclei become much stiffer and the wavefunctions are dominated by rotational components.

In recent years the high spin states of Zn [Crowell et al. (1994), See also Chapter 4] and Ge [Hermkens et al. (1992), Chaturvedi et al. (1991)] nuclei were populated through heavy ion reaction to study the behaviour of such transitional nuclei at high spin. These studies are intended to test the theoretical prediction of gamma stability in $^{64}\text{Zn}$ and enhanced octupole correlation in $^{66}\text{Ge}$ nuclei. However, their results seem to contradict the theoretical predictions.

The nuclei in this region are also ideal for testing models based upon some approximations since the results can be compared with the full space shell model calculations. The shell model space available for the neutrons and protons in the nuclei in this mass region consists of $p_{3/2}$, $f_{5/2}$, $p_{1/2}$ and $g_{9/2}$ orbitals. However, inclusion of first three orbitals in the calculation is sufficient for the description of low-lying structure of the nuclei such as Ni and Cu. The effect of $g_{9/2}$ orbital in the low energy region is less pronounced. But as the number of neutrons in these nuclei increases the $g_{9/2}$ orbital plays a major role in producing high spin states. This intruder, on one hand, gives rise to the appearance of gaps in the single particle energies at large deformations and on the other hand, its coupling to the close lying $p_{3/2}$ orbital via the $\Delta L = 3$ interaction produces enhanced octupole correlations. Fig. 1.2 shows the Woods-Saxon single particle level calculated for $Z=28$ and $N=32$ as a function of quadrupole deformation.

The question whether certain nuclei can be octupole unstable has been a subject of much experimental and theoretical interest during the past ten years. Nazarewicz et al. [Nazarewicz (1990), Nazarewicz et al. (1994)] predicted softness toward octupole
Figure 1.2: Woods-Saxon calculation for the single particle levels around $Z=28$ and $N=32$ as a function of quadrupole deformation for a) Neutron and b) Proton.
correlations for nuclei with $N, Z \simeq 34, 56, 88, \text{ and } 134$ from Strutinsky-type potential-energy calculations. However, Cottel [Cottel (1990)] analyzed systematically the behaviour of $3^{-}$ states from the available observed data and identified the $N$ and $Z$ values 40, 64, 88, and 134 for maximum octupole collectivity. The softness to octupole deformation is demonstrated by the presence of low-lying collective $3^{-}$ state in even-even nucleus and a negative parity band connected to the ground band by enhanced $E1$ transitions. From a microscopic point of view, octupole collectivity originates in the interaction between the unique parity orbit in a major shell and the common parity orbitals having both orbital and total angular momentum $3h$ less than that of the unique parity orbit [Lecomte et al. (1982)].

Several attempts were made theoretically to investigate the different modes of excitations observed at low-energy and low spin region in nuclei near $Z = 28$ closed shell. Due to shell closure with $f_{7/2}$ orbital the model space for the protons and neutrons in the fp shell nuclei becomes similar to that of sd shell nuclei and a few calculations based on shell model were tried for the nuclei in the middle of fp shell such as Ni and Cu. The model space used in these calculations consists of $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ orbitals for both type of particles, protons and neutrons, outside $^{56}\text{Ni}$ core. It has been found that the inclusion of $g_{9/2}$ orbital in these calculation does not provide much better results [Rustgi et al. 1971]. These calculation are based either on reaction matrix element or empirically determined interactions.

A number of such calculations had been performed earlier by several groups e.g. for Ni isotopes [Glaudemans et al. (1972)], Cu isotopes [Phillips and Jackson (1968), Wong (1970), Perazzo (1972), Wang et al. (1975)] and for both Ni and Cu isotopes by Koops and Glaudemans [Koops and Glaudemans (1977)]. Most of the features e.g., energy value, electromagnetic transition probabilities, magnetic moments and spectroscopic factors for one nucleon transfer reaction, etc., were reasonably reproduced by these calculations for the low-lying states in Ni and Cu isotopes. Besides shell model calculations, investigations had been made on the structure of these nuclei using anharmonic vibrator model for even-even nuclei and weak coupling of particle or hole to the vibrating core in the case of odd nuclei [Gomez (1971)]. Though shell model reproduced all the observables for the fp shell nuclei well, the necessity of doing such calculations was to check the validity of
the phenomenological models which could be used to describe nuclei with large number of protons and neutrons outside the core.

For nuclei with still higher $Z$ e.g., Zn, Ga, Ge, etc., it was difficult to do such shell model calculations due to the large configuration space needed. As a result phenomenological models like anharmonic vibrator model and quasiparticle-phonon model [Kisslinger and Kumar (1967)] had been used to understand the low-lying structure of even-even and even-odd Zn isotopes, respectively. These calculations were restricted to the specific properties of the described nucleus. A more general description of the properties of these nuclei, using Hartree-Fock-Bogoliubov calculations, had predicted an oblate shape of the ground states of even zinc isotopes [Chandra and Rustgi (1971), Sharma (1980)]. This calculation had suggested also that the excitation from $\pi f_{7/2}$ are important for Zn isotopes. Later, with the development of techniques to handle the large dimensionality of energy matrix, shell model calculation for the $^{62-68}\text{Zn}$ nuclei was performed in $p_{3/2}, f_{5/2}$ and $p_{1/2}$ model space with an empirical interaction determined from the spectra of the Ni and Cu isotopes [Van Hienen et al. (1976), Sakahura et al. (1979)]. This calculation explained most of the features like the binding energies, low-lying energy spectra and electromagnetic transition probabilities for the lower mass Zn nuclei. The poor agreement for higher Zn isotopes was due to the exclusion of $g_{9/2}$ orbit from the model space. Their inability to explain the occurrence of three close-lying $8^+$ states in even Zn isotopes can also be traced to the same reason. Later, Weeks et al. [Weeks et al. (1981)] showed that by including $g_{9/2}$ in the model space one could explain the forking of $8^+$ states.

Apart from the above microscopic approaches, few calculations, based upon semiphenomenological models, in which two quasi-particles are coupled to a vibrating core, had also been tried [de Vries and Brussard (1978)]. Reasonable agreement had been obtained only for states $J^\pi (6^+)$ in even-even Zn nuclei. They achieved good agreement for the strong E2 transition probabilities, however, weak transitions and quadrupole moments of $2^+$ were not reproduced. The reason for this deviation from vibrational character had been predicted due to presence of an other collective mode in the nucleus. The low and high spin states of Ge and Zn isotopes including negative parity and $\gamma$-bands were investigated in a model in which two quasi-particles were coupled to a triaxial Davydov rotor [Petrovici and Faessler (1983)]. The Nilsson single particle energies for $\beta_2 = 0.1$ and $\gamma = 25^\circ$ were
used in this calculation. The three \( J=8^+ \) states in Ge were identified as the \( 8^+ \) member of the ground-state band, the aligned two-neutron \( (\nu g_{9/2})_{I=8}^2 \) with \( J=R+I \) and \( (\nu g_{9/2})_{I=6}^2 \) with \( J=R+I \) configurations. The \( R \) corresponds to the Rotor angular momentum. The aligned proton \( (\pi g_{9/2})_{I=8}^2 \) with \( J=R+I \) configuration was predicted to lie higher in energy.

A close look at all the above model studies (in 28-50 shell) reveal that though energy spectra could be well reproduced, it was difficult to simultaneously account for the electric quadrupole moment of the \( 2^+ \) state along with the \( B(E2) \) value. The Projected-Hartree-Fock model as well as shell model and Hartree-Fock-Bogoliubov models give in general large quadrupole moment compared to experiment. This suggest the need to include prolate-oblate shape mixing. Potential energy calculation, which incorporates shell correction in the liquid drop formula, for the even Ge isotopes reveals the existence of two minima of the potential energy. This indicates the coexistence of oblate and prolate equilibrium shapes.

In recent years, the high spin states of \( Zn \) and Ge isotopes were explored extensively using heavy ion beams and better detector systems [Crowell et al. (1994), Ge [Hermkens et al. (1992), Chaturvedi et al. (1991)]. These nuclei were predicted to be quit soft with respect to triaxial and octupole deformations [Ennis et al. (1991)] and detailed examination of the data reveals that a larger variety of rotational and vibrational degrees of freedom are present in these nuclei. The high spin states above \( J=8h \) are expected to exhibit a complex interplay between the single particle and the collective excitations which originate due to alignment of neutrons in \( g_{9/2} \) orbital. This phenomenon was observed in \( ^{66,68}Ge \) [Hermkens et al. (1992), Chaturvedi et al. (1991)] and \( ^{64}Zn \) [Crowell et al. (1994)]. The outstanding feature of the level scheme is a combination of three close lying \( 8^+ \) states which are qualitatively explained by an \( 8^+ \) of the ground band and the alignment of proton and neutron \( g_{9/2} \) pairs. Furthermore, one finds a \( \gamma \)-band, several negative-parity bands and a low-lying excited \( 0^+ \) state.

Among the other models, Cranked Strutinsky calculations have proved to be an extremely powerful tool in the interpretation of the high spin structure of open-shell nuclei, for which traditional shell model calculations are not feasible. This calculation predicts that the minimum in the potential energy surface for \( ^{64}Zn \) [Crowell et al. (1994)] would be far more stable with respect to nonaxially symmetric degrees of freedom at high spin than at
low spin, which should result in the appearance of well-defined rotational bands. However, the high spin experimental data appear to support exactly the opposite trend. The rotational band structure is less pronounced at high spins than at low spins. The experimental observables at low spin appear consistent with the collective rotational structure of a modestly deformed stable shape, and calculations that include the effect of shape fluctuations give results which are qualitatively inconsistent with the data. The excellent agreement between the data and the calculations that exclude the effect of shape fluctuations suggests that there is actually a stiff minimum at low spin, and that the very gamma-soft potential energy surface is in fact stiffer. A number of possible reasons for the discrepancy have been suggested, the most likely being that difficulties are encountered in the Strutinsky method in nuclei whose Fermi level lies in a major shell which is truncated by the continuum, leading to a lack of consistency in the treatment of shape polarizing nonnatural-parity states.

1.3 Theoretical models

1.3.1 Introduction

Nucleus is a many body system of interacting fermions. The first model to describe the structure of a nucleus is the shell model proposed by M.G. Mayer and J.H.D. Jensen independently in 1950 [Mayer and Jensen (1955)]. This model in its simplest form assumes the independent motion of a nucleon in a spherically symmetric potential that represents an average interaction with the other nucleons of the nucleus. This simple model can explain the existence of magic numbers of nuclear shell structure and predict spins and parities of low-lying states of closed major shells plus or minus one particle. Two-body residual interaction has been introduced for the description of the nuclei with several valence nucleons. Good results have been obtained using this model for sd shell nuclei. But beyond the sd shell, the configuration space increases rapidly and the calculation becomes formidably large. Fortunately, low-lying structure of nuclei away from closed shell show remarkably simple structure. These types of excitation can be explained only if we assume that nucleons move collectively. The collective behaviour of nuclei can broadly be classified into two groups. The spectra are called vibrational if the interlevel spacing is almost constant,
and rotational if the energy levels obey a $J(J+1)$ rule where $J$ is the nuclear spin. The first phenomenological model able to give a satisfactory description of these features was the collective model of the nucleus proposed by Bohr and Mottelson in 1952 [Bohr and Mottelson, (1975)] called the geometrical model. It assumes the nucleus to have a well-defined surface and shape which can undergo oscillation about a mean position as well as rotation. Usually quadrupole deformation is considered as the most important mode of collective motion. Keeping only the corresponding term in the Hamiltonian, one simply obtains the Hamiltonian of a five dimensional quadrupole harmonic oscillator. This Hamiltonian can be quantised by introducing boson creation and annihilation operators. To the lowest order, it becomes proportional to the boson number operator, thus producing equal separation among the multiplets, features which characterise the vibrational spectra. For the description of nuclear rotation the theory assumes a permanent ellipsoidal shape for the deformed nucleus. Choosing the body-fixed axes to be the principal axes of the ellipsoid, one introduces two new variables $\beta$ and $\gamma$, where $\beta$ is a measure of the total deformation of the nucleus and $\gamma$ is related to its shape, which can be prolate or oblate, axially symmetric ellipsoid or triaxially deformed. Quantising the resultant Hamiltonian, one can express the collective part of the kinetic energy as a sum of $\beta$ and $\gamma$ vibrational energy, which for the special case of the ground state band reproduces $J(J+1)$ rule. The interplay of single-particle and collective degrees of freedom also becomes important. Particle rotor model, particle vibration coupling and various other particle core coupling models try to describe this feature of nuclei. Microscopic description of rotation in the limit of strong deformations is given by the cranked shell model which uses a phenomenological deformed potential or cranked Hartree-Fock-Bogoliubov method.

An approach, distinct from the geometrical model, is based on algebraic technique. One of the most important aspects in this approach is the use of group theory and dynamical symmetry associated with Lie groups. The first algebraic model in the nuclear structure was the SU(3) model proposed by J.P. Elliot in the year 1958 [Elliot (1958a, 1958b)]. This model is applicable in sd-shell region where SU(3) symmetry is present. In higher shells this model fails due to the breakage of this symmetry as a result of the spin-orbit interaction. A new approach, known as Interacting Boson Model (IBM) was proposed by Arima and Iachello in 1974. In this model an attempt was made to describe
the low-lying collective states in medium and heavy even-even nuclei in terms of bosons. These bosons are supposed to be correlated pairs of fermions outside lying closed shells.

1.3.2 Basic features of different interacting boson models

The basic idea of the interacting boson model is that similar valence nucleons behave pairwise as interacting bosons with angular momentum 0 and 2 called s-bosons and d-bosons, respectively and are sufficient to describe the low-lying structure of the nuclei. The simplest version of this model, IBM-1, makes no distinction between the proton and the neutron bosons. However, they are treated distinctly in the higher version of this model called IBM-2.

In both IBM-1 and IBM-2, the possibility of proton-neutron boson has been ignored since in most medium and heavy mass nuclei protons and neutrons occupy different shell. Two other higher versions of the model, IBM-3 [Elliot and White (1980)] and IBM-4 [Elliot and Evans (1981)], consider bosons made up of proton and neutron. The parent group for IBM-3 is now $U(6) \otimes SU_T(3)$ where $T$ refers to isospin. The IBM-4 group starts with the group $U(36)$ which, in the next step, is decomposed in the group $U(6) \otimes U_{ST}(6)$. Here the suffixes $S$ and $T$ refer to spin and isospin respectively.

The interacting boson model deals with nuclei with an even number of protons and neutrons. However, more than half of the nuclear species have an odd number of protons or neutrons. In these nuclei there is an interplay of both collective (bosonic) and single particle (fermionic) degree of freedom. The extended version of interacting boson model to account the spectra of odd-mass nuclei, in which apart from bosons, there is a fermion outside the even-even core, is called interacting boson fermion model (IBFM).

In odd-odd nuclei, there are two particle, a proton and a neutron outside the even-even core. Since these two particles occupy different shells, we have to treat them as two fermions. So, in an odd-odd nucleus there is a even core and outside this core there are two fermions. The interacting boson model is further extended to describe odd-odd nucleus called interacting boson fermion fermion model (IBFFM). In this model the two fermions are weakly coupled to even-even core. The model also takes care of the residual interaction between the neutron and the proton.
1.3.3 Interacting Boson Model-1 (IBM-1)

The practical advantage of IBM is that it works in a small untruncated model space. The central property of this model is the presence of dynamical symmetry. In particular, for certain values of the parameters, the Hamiltonian can be expressed in terms of the Casimir operators of the subgroups of U(5), O(6) and O(3). In such cases, the eigen values are simple algebraic expressions of the quantum numbers associated with the corresponding subgroups. The Hamiltonian is then said to have a dynamical symmetry associated with a group. These limits are idealised cases of spectra in vibrational, $\gamma$-unstable and rotational nuclei, respectively. These three limits are useful because they provide analytical expression for the excitation energies and transition rates, which can be tested against the experimental results. Another aspect of the group theoretical scheme is the presence of supersymmetry proposed by Iachello [Iachello (1980)]. In this scheme the neighbouring even and odd nuclei are described within the same framework, with the same quantum numbers and parameters. More details about the presence of dynamical symmetries and supersymmetries in $Z = 28$ and 50 region can be found in the work of Gangopadhyay [Gangopadhyay (1987)]. However, very few nuclei show behavior near to one of these three limits. Most nuclei have spectra showing behavior intermediate between these limits. In order to describe them, one has to include in the Hamiltonian terms belonging to more than one limiting symmetries. Dynamical symmetries are then broken, and no analytical solutions can be found. Excitation energy and transition rates have to be calculated numerically.

Hamiltonian

The Basic form of the Hamiltonian in IBM-1 can be written in terms of the interacting bosons as [Iachello and Arima (1987), Bonatsos (1988)]

$$H_{sd} = \epsilon_d[d^d\bar{d}]_0 + \sum_{L=0,2,4,6} \frac{1}{2\sqrt{2L+1}} C_L[(d^d)^L(d\bar{d})_L]_0 + \frac{1}{\sqrt{2}}\tilde{u}_2\{[(d^d)^2(d\bar{d})_2]_0 + h.c.\} + \frac{1}{2}\tilde{v}_0\{[(d^d)^0(ss)_0]_0 + h.c.\} + \tilde{u}_2\{[(d^d)^2(s\bar{s})_2]_0 + \frac{1}{2}\tilde{u}_0[s^t s^t s^t]_0 \} \quad (1.1)$$

where $\epsilon_d$ is the d-boson energy and $C_0$, $C_2$, $C_4$, $\tilde{v}_0$, $\tilde{u}_2$, $\tilde{u}_0$, and $\tilde{u}_2$ are the strength parameters for the boson-boson interaction terms. The creation operator $s^t$ creates a
boson with \( L=0 \) (s-boson) while \( d^\dagger \) creates a boson with \( L=2 \) (d-boson) and projection \( \mu \), respectively.

The appropriate boson commutators are

\[
[\sigma, \sigma^\dagger] = 1, \quad [d_\mu, d^\dagger_\nu] = \delta_{\mu\nu}
\]

with all other commutators zero.

The operator \( d \), defined by \( \tilde{d}_m = (-1)^m d_m \) is a spherical, or irreducible, tensor operator of degree two as is \( d^\dagger \).

The Hamiltonian (1.1) may be diagonalized by using the code PHINT [Scholten (1976)].

In most medium and heavy nuclei, apart from positive parity states, negative parity states occur which are thought to correspond to octupole vibrations around a spherically or quadrupole deformed ground state shape. In the interacting boson model, octupole mode of excitation is taken care of by introducing an \( f \) (\( L^r = 3^- \)) boson of negative parity [Arima and Iachello (1976)]. The total boson Hamiltonian incorporating both quadrupole and octupole mode of excitations takes the form [Barfield et al. (1986)]

\[
H_B = H_{sd} + H_f + V_{sd}f
\]  

(1.2)

where \( H_{sd} \) is the usual s-d boson Hamiltonian (1.1) [Iachello and Arima (1987)], \( H_f \) is the pure f-boson term, which in lowest approximation is taken as

\[
H_f = \epsilon_f n_f
\]  

(1.3)

where \( n_f \) is number of f-bosons and \( V_{sd}f \) is the interaction between quadrupole and octupole degrees of freedom in the simplest form, taken as [Bonatsos (1988)]

\[
V_{sd}f = A(L_{sd} \cdot L_f) + B(Q_{sd} \cdot Q_f) + C : (E_{sd}^\dagger \cdot E_{sd}) : 
\]  

(1.4)

In the eqn. (1.4) \( L_{sd} \) and \( Q_{sd} \) are the usual IBM-1 angular momentum and quadrupole operators defined as,

\[
L_{sd} = \sqrt{10}[d^\dagger \otimes \tilde{d}]_1
\]  

(1.5)

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respectively, while $L_f$ and $Q_f$ are those for the f-bosons

$$L_f = 2\sqrt{7}(f^t \otimes \hat{f})_1$$  \hspace{2cm} (1.7)

and

$$Q_f = -2\sqrt{7}(f^t \otimes \hat{f})_2$$  \hspace{2cm} (1.8)

$E^t_{dq}$ is the exchange operator

$$E^t_{dq} = \sqrt{5}(d^t \otimes \hat{f})_3$$  \hspace{2cm} (1.9)

and $\langle \rangle$ indicates normal ordering of the operators.

**Electromagnetic transition operators**

In order to describe electromagnetic transition rates, one has to specify the transition operators in terms of boson creation and annihilation operators. In the lowest order, it is assumed that the transition operators in this model will contain only one-body terms. For electric quadrupole transitions, the relevant form of the operator is [Iachello and Arima (1987)]

$$T^{E2}_B = \alpha((s^t \dagger d + d^t s)_2 + \chi(d^t \dagger \hat{d})_2].$$  \hspace{2cm} (1.10)

In the case of magnetic dipole transitions the relevant simplest transition operator should be

$$T^{M1}_B = \sqrt{\frac{30}{4\pi}}g_4(d^t \otimes \hat{d})_1$$  \hspace{2cm} (1.11)

However, magnetic dipole transitions in this model are not allowed since the M1 transition operator is proportional to the angular momentum $\hat{L}$ and is diagonal in the IBM model space. Therefore to account for the M1 transition, it is necessary to include higher order terms in the eqn. (1.11).

$$T^{M1}_B = \sqrt{\frac{30}{4\pi}}g_4(d^t \dagger \hat{d})_1 + \eta[T_B(E2) \times \hat{L}]_1$$  \hspace{2cm} (1.12)
where, the $\hat{L}$ is the angular momentum operator given by eqn. (1.5).

Incorporating both quadrupole and octupole mode of excitations in IBM-1 the electromagnetic transition operators should have terms from both the degrees of freedom. The most general form of the electromagnetic quadrupole transition operator is

$$T_{B}^{E2} = e_{sd}Q_{sd} + e_{f}Q_{f}$$  \hspace{1cm} (1.13)

However, since E2 collectivity is carried out by d bosons, one can neglect the second term in eqn. (1.13).

In the case of E3 transitions, the corresponding operator is

$$T_{B}^{E3} = e_{s}(s^{1}\hat{f} + f^{1}s)_{3} + \chi_{3}(d^{1}\hat{f} + f^{1}\hat{d})_{3}.$$ \hspace{1cm} (1.14)

The electromagnetic transition rates can therefore be calculated by evaluating the reduced matrix element of the corresponding operator between the initial and the final state. By definition, the B(EL) and B(ML) values are

$$B(L; J_{i} \rightarrow J_{f}) = \frac{1}{2J_{i} + 1} |(J_{f}||T^{L}_{i}||J_{i})|^{2}.$$ \hspace{1cm} (1.15)

The quadrupole moment can be calculated using the reduced matrix element by

$$Q = \sqrt{\frac{16\pi}{5}} \sqrt{\frac{J(2J - 1)}{(2J + 1)(J + 1)(2J + 3)} \langle J||T^{E2}_{i}||J\rangle}.$$ \hspace{1cm} (1.16)

1.3.4 Interacting Boson Fermion Model (IBFM)

**Hamiltonian**

The most general Hamiltonian in IBFM can be written as a sum of three terms, a boson term, a single particle term and a term which represents interaction between particle and even-even core.

$$H = H_{B} + H_{F} + V_{BF}$$ \hspace{1cm} (1.17)

where $H_{B}$ is the core Hamiltonian as described in IBA-1, $H_{F}$ is the single particle term

$$H_{F} = \sum_{jm} \varepsilon_{j} a_{jm}^{\dagger} a_{jm}$$ \hspace{1cm} (1.18)
and $V_{BF}$ is the boson-fermion interaction term [Scholten (1985)]

$$V_{BF} = \sum_j A_j (d^j \bar{d}^j) \langle a^j \bar{a}^j \rangle_0$$

$$+ \sum_{jj'} r_{jj'} \left[ (s^j \bar{s}^j + d^j \bar{d}^j)_2 + \chi (d^j \bar{d}^j)_2 \langle a_j a^j \bar{a}^j \bar{a}^j \rangle_2 \right]$$

$$+ \sum_{jj''} \Lambda_{jj''} \left[ (d^j \bar{d}^j)_{jj''} (d\bar{a}^j a_{j''})_{jj''} \right]_0$$

(1.19)

where $\epsilon_j$ are the single particle energies and $a_j$ and $\bar{a}_j$ are the fermion creation and destruction operators, respectively

$$\{a_j, a_j^\dagger\} = \delta_{jj'}.$$

The three terms in the eqn. (1.19) represent monopole-monopole, quadrupole-quadrupole and exchange interactions. In the case of more than one j-orbit, the number of interaction strength parameters in (1.19) is too large and one uses the following relations obtained microscopically [Scholten (1980), (1985)]

$$A_j = -\sqrt{5(2j + 1)} A_0$$

(1.20)

$$\Gamma_{jj'} = \sqrt{5} (u_j u_{j'} - v_j v_{j'}) Q_{jj'} \Gamma_0$$

(1.21)

$$\Lambda_{jj''} = -2/EA_0 \beta_{jj''} \beta_{jj'}$$

(1.22)

where

$$Q_{jj'} = \langle j||Y_2||j'\rangle$$

(1.23)

$$\beta_{jj'} = (u_j v_{j'} + v_j u_{j'}) Q_{jj'}$$

(1.24)

Here $v_j^2$ is the occupation probability of the single particle orbit $j$ and $u_j^2 + v_j^2 = 1$.

The above Hamiltonian (1.17) may be diagonalized using the computer code ODDA [Scholten (1982)].

**Electromagnetic transition operators**

The electromagnetic transition operators used to calculate gamma transition probabilities in IBFM should contain both boson and fermion terms.
where $g_o$ and $g_s$ are orbital and spin g-factors for the odd fermion and its single particle values are 1.0 and 5.586 for protons and 0 and -3.384 for neutrons, respectively. However, in practical cases, it has been found that quenching by a factor of 0.6 to the free space spin g-factors value is required to reproduce the experimental results.

The magnetic moments can be calculated using the expression

$$H = \frac{1}{4\pi J} \frac{1}{2} \sum_{j,j'} (a_j^+ a_{j'})_2.$$  

(1.26)

Here, $e_B$ and $e_F$ are effective charge parameters for bosons and fermions, respectively.

The lowest order magnetic dipole transition operator can be written as

$$T^{M_1} = \sqrt{\frac{3}{4\pi}} [g_B / 10 (d^d d_1) - \sum_{j,j'} \frac{1}{J} (j+1)(2j+1) \{ a_j^+ a_{j'}^+ (a_j^+ a_{j'}^+) \}].$$  

(1.27)

where the first term operates only on bosons and the second one only on fermions. In order to calculate $M_1$ transition probabilities in IBFM, since the first term is diagonal in the IBM basis, higher order terms in the boson part of the operator should be considered. However, for magnetic moment calculation, this operator is sufficient because it is mainly governed by the single particle.

In the above eqn. (1.27), $g_B$ is the g-factor for the bosons and $g_{jj'}$ are the g-factors for fermions

$$g_{jj'} = \begin{cases} 0, & l \neq l' \\
[(2j - 1) g_l + g_s]/2j, & j = j' = l + 1/2 \\
[(2j + 3) g_l - g_s]/2j + 1, & j = j' = l - 1/2 \\
(g_l - g_s)(j+1)/(l+1/j(j+1)(2j+1)(2l+1)), & j' = j - 1 
\end{cases}$$  

(1.28)

where $g_l$ and $g_s$ are orbital and spin g-factors for the odd fermion and its single particle values are 1.0 and 5.586 for protons and 0 and -3.384 for neutrons, respectively. However, in practical cases, it has been found that quenching by a factor of 0.6 to the free space spin g-factors value is required to reproduce the experimental results.

The magnetic moments can be calculated using the expression

$$\mu = \sqrt{\frac{4\pi}{3}} \frac{J}{(2J+1)(J+1)} \langle J||T^{M_1}||J \rangle.$$  

(1.29)
Single particle transfer operators

From microscopic viewpoint, the image of the shell model single particle creation operator in the boson-fermion space can be written as [Scholten,1985]

\[ c_j^\dagger = u_j a_j^\dagger - \sum_{j'} \frac{v_{jj'}}{\sqrt{N}} \sqrt{\frac{10}{2j+1}} \beta_{jj'}(K_\beta)^{-1} s^\dagger (d_{jj'}^\dagger) + \frac{v_{jj'}}{\sqrt{N}} (s^\dagger a_j^\dagger) + \sum_{j'} u_{jj'} \sqrt{\frac{10}{2j+1}} \beta_{jj'}(K_\beta)^{-1} (d_{jj'}^\dagger) \]

(1.30)

where

\[ K_\beta^2 = \sum_{jj'} \beta_{jj'}^2 \]

and \( \beta_{jj'} \) is defined by eqn. (1.24).

This eqn. (1.30) is used in the calculation of single particle transfer amplitudes. However, this operator is only a lowest order estimate to the full operator and due to the fact that the relation

\[ c_j^\dagger c_j + c_j^\dagger c_j = 1 \]

is no longer obeyed, a normalization factor \( K_j^2 \) and \( K_j^4 \) has to be introduced in order to restore it.

Two types of single particle transfer reactions are possible in IBFM. The first one between an even-even nucleus with \( N \) bosons and an odd-even nucleus with \( N \) bosons and one fermion and vice versa. The second is between an even-even nucleus with \( N+1 \) bosons and an odd-even nucleus with \( N \) bosons and one fermion and vice versa.

The single nucleon transfer operator which conserves the number of bosons can be written as

\[ P_j^\dagger = \frac{1}{K_j^2} \left[ u_j s_j^\dagger - \sum_{j'} \frac{v_{jj'}}{\sqrt{N}} \sqrt{\frac{10}{2j+1}} \frac{N_p}{N} \beta_{jj'}(K_\beta)^{-1} s^\dagger (d_{jj'}^\dagger) \right] \]

(1.31)

and its Hermitian conjugate. For transfer reaction involving change of boson number by one, the transfer operator is given by

\[ Q_j^\dagger = \frac{1}{K_j^2} \left[ v_{jj'} s_j^\dagger a_j - \sum_{j'} \frac{v_{jj'}}{\sqrt{N}} \sqrt{\frac{10}{2j+1}} \frac{N_p}{N} \beta_{jj'}(K_\beta)^{-1} (d_{jj'}^\dagger) \right] \]

(1.32)
and its Hermitian conjugate. Here $N$ is the number of bosons.

The quantity $N_p$ is given by

$$N_p = \sum_j \frac{1}{2} (2j + 1) v^2 \rho_j$$  

(1.33)

The normalization factors are determined by the relation

$$\sum_{\text{odd}} <\text{odd}|P_j^2|\text{even}>^2 = (2j + 1) u_j^2$$  

(1.34)

$$\sum_{\text{odd}} <\text{even}|Q_j^2|\text{odd}>^2 = (2j + 1) v_j^2$$  

(1.35)

Using the matrix element of above transfer operators one can compute the intensities of transfer reaction often expressed in terms of spectroscopic factors by

$$S(J_i \rightarrow J_f) = \frac{1}{2J_i + 1} |\langle J_f||O^1||J_i\rangle|^2$$  

(1.36)

where $O$ may be $P$ or $Q$ defined by eqns. (1.31) and (1.32).

### 1.3.5 Interacting Boson Fermion Fermion Model (IBFFM)

In the case of odd-odd nuclei, there remains two fermions outside the even-even core. The structure of odd-odd nuclei can be described by an unpaired proton and an unpaired neutron coupled to a even-even boson core.

**Hamiltonian**

The Hamiltonian for the odd-odd nuclei therefore should contain a purely boson term, two terms for the boson-fermion interaction, two terms representing purely fermionic contribution and a term to account for the residual interaction between the two odd-fermions [Timár et al. (1994)].

$$H = H_B + H_{B\pi} + H_{B\nu} + H_{\pi} + H_\nu + H_{\pi\nu}$$  

(1.37)

where $\pi$ ($\nu$) refers to proton (neutron) and B refers to the even-even boson core. Here, $H_B$ describes the even-even core nucleus in IBM-1 given by eqn. (1.1), $H_{B\pi}$ and $H_{B\nu}$ describe the corresponding odd-even and even-odd nuclei, respectively in IBFM by eqn. (1.17), $H_\pi$ and $H_\nu$ are the single particle Hamiltonian describe by eqn. (1.18) and $H_{\pi\nu}$ account for the residual interaction between the odd-proton and the odd-neutron.
Electromagnetic transition operators

For the electromagnetic properties, the one body electromagnetic transition operator is taken to be of the form [Bucurescu et al. (1985)].

\[ T^\lambda = T^\lambda_B + T^\lambda_F \]  

(1.38)

where the first term operates only on the boson part and the last term operates on the fermion part of the wave function. The expressions for the \( T^\lambda_B \) and \( T^\lambda_F \), \( \lambda = E2 \) or M1 are given in Sections 1.3.3 and 1.3.4.

1.3.6 Interacting Boson Model-2

In IBM-1 no distinction has been made between proton and neutron bosons. However, realistic description of the nuclei can only be obtained by treating protons and neutrons as different particles. This distinction is made in IBM-2. Therefore, in this model there are four bosons viz. \( s_\pi, d_\pi, s_\nu \), and \( d_\nu \). In medium and heavy nuclei, since neutron and protons occupy different major shells, no neutron-proton bosons are introduced. Obviously in this model, the size of the configuration space will be large compared to that in IBM-1.

Hamiltonian

The Hamiltonian in IBM-2 is [Arima and Iachello (1984)],

\[ H = \epsilon_d (d_\pi^+ d_\pi + d_\nu^+ d_\nu) + V_{\pi\pi} + V_{\nu\nu} + \kappa (Q_\pi \cdot Q_\nu) + M_{\pi\nu}, \]  

(1.39)

where \( V_{\rho\rho} \) represents the interaction between like bosons. Here \( \rho \) refers to \( \pi \) (proton) or \( \nu \) (neutron) bosons.

where

\[ V_{\rho\rho} = \sum_{L=0,2,4} \frac{1}{2} C_{L\rho} \sqrt{2L + 1} [ (d_{\rho}^d d_{\rho}^d) L (d_{\rho}^d d_{\rho}^d) L] \]  

(1.40)

The expression \( Q_\pi \cdot Q_\nu \), where

\[ Q_\rho = (s_\rho^d d_\rho^d + d_\rho^d s_\rho) s_\rho + \chi_\rho (d_\rho^d d_\rho^d), \]  

(1.41)

stands for the quadrupole-quadrupole interaction between the neutron and proton bosons.
\[ M_{\pi \nu} = - \sum_{k=1,3} 2\xi_k (d^+_k d^l_k), (\bar{d}_k \bar{d}_k)_k + \xi_2 (d^+_l s^l - s^l_l d^l)_2, (\bar{d}_\pi \bar{s}_\nu - s_\pi \bar{d}_\nu) \] (1.42)

is the Majorana force operator which controls the separation of F-spin symmetric states from the mixed symmetry states.

IBM-2 Hamiltonian (1.39) may be diagonalized by the computer code NPBOS [Otsuka (1977)].

**Electromagnetic transition operators**

The electromagnetic transition operator in IBM-2 therefore have a more general form than in IBM-1. For electric quadrupole transition, the operator is given by [Arima and Iachello (1984)]

\[ T_B^{E2} = e_\pi Q_\pi + e_\nu Q_\nu \] (1.43)

where \( Q_\pi \) and \( Q_\nu \) are the quadrupole operators for proton and neutron, respectively. They are defined in eqn. (1.41). \( e_\pi \) and \( e_\nu \) are the effective charges for proton and neutron bosons.

The relevant operator for the M1 transitions is

\[ T_B^{M1} = g_\pi L_\pi + g_\nu L_\nu \] (1.44)

where \( L_\pi \) and \( L_\nu \) are the angular momentum operators for proton and neutron bosons

\[ L_\rho = \sqrt{10}(d^l_\rho \bar{d}_\rho) \] (1.45)

Here, \( g_\pi \) and \( g_\nu \) are the \( g \)-factor for proton and neutron bosons, respectively. It is worthwhile to note that while M1 transitions in IBM-1 are forbidden with the lowest order M1 transition operator, this is not the case in IBM-2 if \( g_\pi \neq g_\nu \) [Bonatsos (1988)].

**1.3.7 Configuration Mixing in The IBM**

The low lying spectra of medium-to-heavy nuclei show regular features which can be explained remarkably well by the collective models like interacting boson model. However,
the experimental data indicate the coexistence of two quite different structures in the same energy region. Therefore, the collective models must be extended in order to describe all the low-lying states. Phenomenologically, these two structures are weakly coupled through the pairing interaction.

In order to describe the low-lying structure of the nucleus, both configurations are calculated separately in IBM-2 and are then admixed by the mixing operator [Duval and Barrett (1982)]

\[ H_{\text{mix}} = \alpha (s_\pi^+ s_\pi^+ + s_\sigma s_\sigma) + \beta (d_\pi^+ d_\pi^+ + \tilde{d}_\pi \tilde{d}_\pi) \]

where \( \alpha \) and \( \beta \) are adjustable strength parameters.

The mixed wave functions are used to calculate \( B(E2) \) values of observed transitions and quadrupole moments. The \( E2 \) transition operator is given by

\[ T^{E2} = e_0 (e_{\sigma 0} Q_{\sigma 0} + e_{\sigma 0} Q_{\sigma 0}) + e_2 (e_{\pi 2} Q_{\pi 2} + e_{\pi 2} Q_{\pi 2}) \]

\( e_j \) and \( e_{\mu j} (j = 0, 2) \) are adjustable parameters. The suffixes 0 and 2 refer to the normal and the intruder configurations respectively. \( Q_\pi \) and \( Q_\sigma \) are the quadrupole operator for the bosons and are given by eqn. (1.41).
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