In this thesis, some strongly correlated ladder systems, described by a $t-J$ type of Hamiltonian, have been studied. The ladder systems considered have the same structural feature, namely, two chains coupled by rungs and diagonal links. The different systems studied correspond to the cases of undoped ladder (Chapter 2), ladder doped with a single hole (Chapter 3), ladder doped with two and more than two holes (Chapter 4) and, lastly, ladder doped with a single non-magnetic and a single magnetic impurity (Chapter 5). Many of the results obtained are based on exact, analytic solutions of the eigenvalue problem. In the case of the undoped spin ladder, the exact ground state consists of singlets along the rungs when the rung exchange interaction $J'$ is $\geq \alpha J$ ($\alpha = 2$) where $J$ is the strength of the intra-chain and diagonal exchange interactions. A spin excitation is created on replacing a singlet by a triplet. The excitation has no dynamics and is localized on a rung. The spin gap has a finite value, $J'$. The ground state energy and the excitation spectrum have further been calculated for general values of $J'$, $J$ and $J_D$. $J$ and $J_D$ are now the strengths of the intra-chain and diagonal exchange interactions respectively. In the general case, the spin excitation has a dynamical character. Exact solution of the eigenvalue problem is no longer possible. Approximate analytic theories like strong-coupling expansion, bond-operator based mean-field theory and spin wave theory have been used to obtain the results. In the exactly solvable limit ($J' \geq \alpha J, J = J_D$), the exact ground state energy per site is recovered in all the three methods. The spin wave theory gives a gapless excitation spectrum
which is a wrong result.

In the case of the ladder doped with a single hole, there are two sectors, bonding and antibonding, for hole motion. The exact hole spectrum can be obtained (when \( J = J_D \)) both in the \( N \to \infty \) limit and for finite \( N \), where \( N \) is the number of rungs in the ladder. In the bonding sector, the propagating object (hole-QP) consists of a hole accompanied by a free spin-1/2 and located on the same rung. The hole-QP has a perfect coherent motion in a Bloch-type of propagating state. Evidence for such a propagating mode has also been obtained in the case of two coupled Hubbard chains at half-filling [1]. In the case of the conventional two-chain \( t-J \) ladder model, the hole-QP has an extended size as the hole and free spin-1/2 may be located on different rungs. In the antibonding sector, one rung is occupied by a localized triplet excitation. For real value of \( k \), extended states for the hole-QP are obtained. The states can be symmetric or antisymmetric with respect to reflection around the localized triplet excitation. For complex values of \( k \), one gets bound \( (k \to \pi + ik) \) and antibound \( (k \to ik) \) states of the hole-QP with the localized triplet excitation. The antibound state is obtained only in the symmetric sector. A bound state is always obtained in the symmetric sector. The conditions for the existence of bound and antibound states, in the different parameter regimes, have been derived. For example, for \( J' = 2J \gg t, t' \), there can be three bound states, two in the symmetric and one in the antisymmetric sector. The lowest energy bound state always occurs in the symmetric sector. For \( J', J = 0 \), one finds that the ferromagnetic state is the hole ground state in the parameter regime \( 0 \leq t'/t < 11/12 \). For \( 2.0 > t'/t > 11/12 \), the bound state of the hole-QP with the localized triplet excitation in the antibonding sector, has the lowest energy. When \( t'/t \geq 2.0 \), the ground state is the lowest energy state of the bonding band. For \( J', J \neq 0 \), the existence of a localization to delocalization transition is seen (Fig. 3.10). The localized state is the bound state of the hole-QP with the localized triplet excitation. The delocalized state of the hole corresponds to the bonding band with dispersion relation given by Eq.(3.11) in Chapter 3. Such a transition is unique to our model when the intra-chain and diagonal exchange interactions have the same strength. For unequal strengths, there is no localized triplet excitation and so no localized state for the hole-QP.

In the case of two holes, the free spins combine to form a triplet or a singlet. The states, with two holes on the same rung, also belong to the
singlet sector. The case $J = J_D$ is only considered. In the triplet sector, the eigenvalue problem for two holes is solved exactly and analytically by the Bethe Ansatz. For $J' = 2J$ and $J = 8t$, the Bethe Ansatz equations for $r$ holes are similar to those in the case of $r$ magnons in an isotropic, $S=1/2$, 1D ferromagnetic chain. Apart from a continuum of scattering states, an excitation branch above the continuum is obtained. This corresponds to the antibound state of two hole-QPs. In the singlet sector, the two hole-QPs can scatter against each other giving rise to a continuum of scattering states with appropriate lower and upper boundaries. The two holes can also form bound and antibound states. The bound hole-QPs can propagate along the ladder giving rise to gapless charge excitations. The spin excitation spectrum, however, has a gap. This is the so-called Luther-Emery phase with gapless charge and gapped spin excitations. The most significant result is that the two holes form bound states for all values of $J$, $J'$, $t$, and $t'$ including $J, J' = 0$. The greater the value of $J$, $J'$, the smaller is the spread of the bound state wave function. In the $N \to \infty$ limit, the bound and antibound state solutions are obtained from the cubic equation

$$e^{2q} - \left( \frac{3J + 3J'}{4t} - \frac{2t'}{T} \right) e^{2q} + \left( \frac{3J}{4t^2} \left( -2t' + \frac{3J'}{4} \right) - 3 \right) e^q - \frac{3J}{4T} = 0$$

(6.1)

with the condition that $e^q > 1$. The two-hole bound state has the dispersion ($\epsilon = E - \frac{3J'}{2} + 2t'$)

$$\epsilon = -4t \cos \frac{k}{2} \cosh q$$

(6.2)

where $k$ is the centre-of-mass momentum wave vector and $q$ the relative momentum wave vector, the value of which is obtained from a solution of Eq.(6.1). More than one bound state is also possible. The conditions for the existence of bound and antibound states have been discussed in detail in Chapter 4. Our ladder model provides an example of a strongly correlated system (no site can be doubly occupied) for which, the binding of two holes propagating in a background of antiferromagnetically interacting spins, can be rigorously demonstrated through an exact, analytic solution of the eigenvalue problem. The wave function of the two-hole bound state is also exactly known and has symmetry of the d-wave type. The binding of holes in a state of d-wave symmetry is a well-known feature of the superconducting state of strongly correlated systems. Binding of two holes is also seen in the conventional two-chain $t-J$ ladder model and occurs in the Luther-Emery phase.
Inclusion of the diagonal links in our model does not change the physical picture much.

In Chapter 5, the problem of a single hole in the presence of a nonmagnetic impurity is studied analytically both in the triplet and singlet sectors. A continuum of scattering states and antibound states are obtained in the triplet sector. In the singlet sector, in addition to the states obtained in the triplet sector, bound states of the hole with the impurity are also found. Consider the case $J' = 2J$. The eigenfunctions are found to be symmetric or antisymmetric about the rung in which the static impurity sits. As the ratio $J/t$ is increased from zero, successive bound states in the symmetric, antisymmetric and symmetric sectors appear. For large $J/t$ values, all the three bound states exist. This result is qualitatively similar to that of Poilblanc et al.\[2\] in the case of finite 2D clusters described by the $t-J$ Hamiltonian. For large $J/t$ values, one finds the co-existence of d-, p- and s-wave bound states which successively appear as $J/t$ is increased from zero. As pointed out by Poilblanc et al, this feature is characteristic of strong correlation. It is well-known that a local impurity introduced into a noninteracting electron system can give rise to bound states\[3\]. In 2D, even an infinitesimal attractive (repulsive) impurity potential can give rise to a bound (an antibound) state. In higher dimensions, this occurs when the strength of the impurity potential exceeds a critical value. The effects of impurity scattering can be described in terms of scattering phase shifts. The phase shift approaches $\pi/2$ for energies near a bound state giving rise to strong resonant scattering\[4\]. The nature of the scattering is determined by the one-body impurity potential. Poilblanc et al. have pointed out that the scatterings caused by an impurity, in weakly-interacting and strongly-interacting hosts, can be different. In the first case, the interaction is spatially located at the perturbed site and the resultant bound states can have only s-wave symmetry. In a many-body AFM ground state, however, AFM correlations are slightly enhanced in the vicinity of a vacancy\[5\] and thus the impurity gives rise to a dynamic finite range potential. Bound states of various symmetries can then occur.

We have compared the binding energy of the hole with the impurity in various symmetry sectors with that of a pair of holes (Fig.5.2). Contrary to what is found by Poilblanc et al. for 2D clusters, we find that the binding energy curve for the hole-impurity bound state (in symmetric sector) is lower than that of a bound pair of holes in almost the whole of the parameter space. We have no explanation for this result. In the case of a single hole in the
presence of a magnetic impurity, the eigenvalue problem is solved numerically for finite-sized ladders. The cases, of both ferromagnetic and antiferromagnetic exchange interaction of the magnetic impurity with neighbouring spins, are considered. Again, the results obtained are qualitatively similar to those obtained by Poilblanc et al. [2] for 2D finite clusters described by the $t - J$ Hamiltonian. Hole-impurity binding is more favourable in the case of ferromagnetic exchange interaction, compared to the case of antiferromagnetic exchange interaction. With increase in the strength of the antiferromagnetic exchange interaction of the impurity with the surrounding spins, the magnitude of the hole-impurity binding energy rapidly decreases to zero. Also, the hole-impurity binding energy is more in the case of a non-magnetic impurity than that in the case of a magnetic impurity.

In this thesis, we have constructed a strongly correlated two-chain ladder model for which several exact, analytical results can be derived for low dopant concentration (one and two holes, one non-magnetic and one magnetic impurity). The associated many body problem is non-trivial as there are $N - x$ ($x =$ dopant concentration ) antiferromagnetically interacting spins in which the dopants are embedded. There are very few exact results known for such many body systems. In the case of our ladder model, the assumption of equal exchange interaction and hopping integral strengths along chains and diagonals, simplifies the eigenvalue problem significantly. One can treat quantum fluctuations and the strong correlation constraint of "no double occupancy of a site" in an exact manner. The spins have no dynamics and so are analogous to Ising spins. Motion of a hole in the spin background can, however, be perfectly coherent without leaving behind strings of wrongly oriented spin pairs. Quantum fluctuations are responsible for the repair of strings. This has been explicitly demonstrated in Chapter 3. The simplification in the eigenvalue problem, however, does not change the physical picture significantly from that of a conventional $t - J$ ladder. For the case of a single hole, the hole-QP is of similar nature in both the cases. For the case of two holes, the Luther-Emery phase is obtained in both the cases. One notable difference, already mentioned, is that the spin excitation spectrum in our model does not have a propagating character in the exactly-solvable limit of equal intra-chain and diagonal exchange interactions. For our ladder model, we have not been able to extend the exact, analytic solutions to the cases of three and more than three holes. We have not calculated dynamic correlation functions and the phase diagram as a function of dopant concentration. Such phase diagrams
are available for the conventional two-chain $t - J$ ladder and for the two-chain Hubbard ladder \[6, 7, 8\]. Recently, some integrable spin ladder models with tunable interaction parameters have been introduced \[6-12\]. The phase diagram of an integrable $t - J$ ladder model has been obtained by Frahm and Kundu \[16\]. The integrable models, in general, contain multi-spin interaction terms (e.g. three- and four-spin interactions) and correlated hole hopping terms. The connection of such models with the two-chain $t - J$ ladder model, as regards physical properties, is not well established as yet.

Lin, Balents and Fisher \[17\] have shown that the weak-$U$ two-chain Hubbard ladder model is integrable and so the excitation spectrum can be determined exactly. One of the possible excitations is a bound pair of holes. Park et al \[8\] have calculated the charge gap and spin gap for the two-chain Hubbard model as a function of the on-site Coulomb interaction and the interchain hopping amplitude using the Density Matrix Renormalization Group method. They have found differences in the weak and strong-$U$ cases. In the limit of strong $U$ (strong correlation), the Hubbard Hamiltonian should map onto the $t - J$ Hamiltonian. An exhaustive study of how physical properties change as one goes from the weak-$U$ to the strong-$U$ limit of the Hubbard model has not been undertaken as yet.

Kagan, Haas and Rice \[18\] have recently derived the phase diagram of the three-chain $t - J$ ladder as a function of hole doping, in the limit where the coupling parameters along the rungs $J'$ and $t'$ are taken to be much larger than those along the chains $J$ and $t$. At large exchange coupling along the rungs, $J'/t' > \frac{3}{\sqrt{2}}$, there is a transition from a Luttinger Liquid phase to a Luther-Emery phase at a critical hole concentration $n_{\text{crit}} \approx 1/3$. In the opposite case, $J'/t' < \frac{3}{\sqrt{2}}$, there is a sequence of three Luttinger Liquid phases as a function of hole doping. It will be of interest to generalise our ladder model to the $n$-chain case and study the phase diagram. Ladders of different geometries, like diagonal \[19\] and zigzag \[20\] ladders have been studied both in the undoped and doped cases. The subject of ladders is vast and rapidly growing. In this thesis, only a part of the work that has been done so far has been reviewed. The work on the $t - t' - J - J'$ ladder model reported in the thesis can be extended to the cases of $n$-chain ladders, a finite number of holes, inclusion of an external magnetic field and new interaction terms, employing a variety of analytical and numerical techniques. Some of these problems will hopefully be addressed in future.
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