SUMMARY OF THE THESIS

The research work presented in this thesis deals with the study of the geometry of "GCR-Lightlike Submanifolds of Indefinite Almost Hermitian Manifolds".

Recently, there has been a keen interest of showing an interplay between Riemannian and semi-Riemannian geometries. In the process of generalization of submanifold theory from Riemannian manifolds to semi-Riemannian manifolds, lightlike submanifolds arise naturally in the semi-Riemannian category. The study of lightlike submanifolds is interesting, due to the fact that the intersection of normal bundle and tangent bundle is non-empty. Thus, this unique feature makes the study of lightlike submanifolds different from the study of non-degenerate submanifolds and therefore one fails to use results of non-degenerate submanifolds in case of lightlike submanifolds. In the process of development of theory of lightlike submanifolds, the notion of CR-lightlike submanifolds of indefinite Kaehler manifolds was introduced by Duggal and Bejancu [15]. Contrary to classical theory of CR-submanifolds, CR-lightlike submanifolds exclude the complex and totally real submanifolds as subcases. Therefore, Screen Cauchy Riemann (SCR)-lightlike submanifolds of indefinite Kaehler manifolds were introduced by Duggal and Sahin [16]. But there was no inclusion relation between CR and SCR cases. Thus Duggal and Sahin [17] introduced Generalised Cauchy Riemann (GCR)-lightlike submanifolds of indefinite Kaehler manifolds which acts an umbrella of real hypersurfaces, invariant, screen real and CR-lightlike submanifolds.

Since lightlike submanifolds are playing an increasingly important role in quantum theory and string theory, as the action and field equations of particles and strings often do not depend on the inverse metric and are well depend even when the metric becomes degenerate. Moreover, the growing importance of lightlike submanifolds in mathematical physics, in particular their extensive uses in relativity
and very limited information available on the general theory of lightlike submanifolds, motivated us to do work on it. Thus in present thesis, our working spaces are Generalised Cauchy Riemann (GCR)-lightlike submanifolds of indefinite almost Hermitian manifolds. The brief chapter wise description of the work presented in present thesis is given in the following paragraphs.

Chapter 1 deals with the preliminaries. In this chapter a brief resume of the results in the differential geometry and their allied structures have been given. Most of the results presented in this chapter are available in review articles, research papers and books, even then we have collected them to fix up our terminology and to make this thesis self contained.

In Chapter 2, we establish conditions for the integrability of various distributions of GCR-lightlike submanifolds and also find the conditions for each leaf of holomorphic distribution and radical distribution to be totally geodesic.

In Chapter 3, we study totally umbilical GCR-lightlike submanifolds of indefinite Kaehler manifolds and prove that the induced connections \( \nabla \) on \( TM \) and \( \nabla^t \) on \( tr(TM) \) are metric connections. We prove that the sectional curvature of a totally umbilical GCR-lightlike submanifold vanishes. Finally we study normal GCR-lightlike submanifolds and obtain necessary and sufficient conditions for a GCR-lightlike submanifold to be a normal GCR-lightlike submanifold.

Chapter 4 deals with study of geodesic GCR-lightlike submanifolds and GCR-lightlike product of indefinite Kaehler manifolds. We obtain some necessary and sufficient conditions for a GCR-lightlike submanifold to be a GCR-lightlike product.

In Chapter 5, we obtain the expressions for sectional curvature, holomorphic sectional curvature and holomorphic bisectional curvature of a GCR-lightlike submanifold of an indefinite Kaehler manifold. We discuss the boundedness of holo-
morphic sectional curvature of GCR-lightlike submanifolds of an indefinite complex space form. We establish a condition for a GCR-lightlike submanifold of an indefinite complex space form to be null holomorphically flat. We also obtain some characterization theorems for holomorphic sectional and holomorphic bisectional curvature.

In Chapter 6, we obtain the expression of Ricci tensor for a GCR-lightlike submanifold of an indefinite complex space form and discuss the properties of Ricci tensor on totally geodesic GCR-lightlike submanifold of an indefinite complex space form. We also prove that every proper totally umbilical GCR-lightlike submanifold of an indefinite Kaehler manifold is a totally geodesic GCR-lightlike submanifold.

Chapter 7 is devoted to the study of lightlike submanifolds of indefinite nearly Kaehler manifolds. We introduce CR, SCR and GCR-lightlike submanifolds of indefinite nearly Kaehler manifolds and obtain their existence in indefinite nearly Kaehler manifolds of constant holomorphic sectional curvature $c$ and of constant type $\alpha$. We also prove characterization theorems on the existence of totally umbilical and minimal GCR-lightlike submanifolds of indefinite nearly Kaehler manifolds.

In the end of the thesis, a bibliography has been given which by no means is an exhaustive one but lists only those research papers and books which have been referred to in the main text of the thesis.