6.1. Introduction:

Modes of oscillations of the cyclotron and synchrocyclotron oscillators have been studied in earlier chapters using lumped equivalent circuits for the transmission lines coupling the oscillator tube to the dees. But in some forms of the oscillators, the length of the coupling lines may be comparable to the wavelength for the fundamental frequency and replacement by the lumped circuit is not strictly justified. In order to investigate the modes of oscillation that may be introduced by the coupling line, study has been made of an oscillator using only a delay line in the feedback path. Approximate analytical methods have been developed to obtain the different modes of oscillation. The characteristics have also been experimentally studied using a model oscillator. Also, to make the study complete, the characteristics of oscillation in the presence of a forcing signal have been examined. Results obtained from these studies are described in this chapter.

Oscillators using delay lines have been discussed in several papers in recent years. The analysis of such oscillators is complicated as a nonlinear difference differential equation is involved. Graphical methods have been given by Cunningham and Met which allow one to find the stable modes of oscillation. Method of slowly varying parameters have also been used by Perepelyatnik and Yakovlev for the study of such oscillators. Almost all these studies are theoretical and it has been
assumed that the delay line in the feedback path is ideal and either preceded or followed by a lowpass or a bandpass network. Not much experimental results are available on the actual performance of such oscillators.

We shall study in the present chapter the performance of an oscillator using only a delay line in the feedback path. The delay line is non-ideal in the sense that the attenuation introduced by the delay line is decreasing slowly with increase in frequency though the phase velocity is nearly independent of frequency. The delay line is not preceded or followed by any frequency discriminating circuit. Evidently, for such a system, one would expect a large number of possible frequencies of oscillation, harmonically related to each other. In other words, the oscillator is essentially a multimode one. It is therefore of interest to examine if any of these modes is preferred by the oscillator when executing self oscillations. One can also guess intuitively that it should be possible to excite synchronous oscillation near any one of the possible frequencies of self oscillation, though the nature of the resonance characteristics or the amplitude of the forcing signal required for synchronisation at a particular mode are not easy to guess. It is therefore also of interest to study the behaviour of the oscillator in the presence of a forcing signal.

We present in this chapter a first order theory for the self and forced oscillations in the oscillator. Some of the theoretical results have been examined by doing experiments on an oscillator constructed with a 36-section delay line and using either a saturation type, or a cubic polynomial type of nonlinearity. These experimental results are also
discussed. The oscillator and its describing equation are presented in Sec. 6.2. The first order theory of free and forced oscillation are respectively given in Secs. 6.3 and 6.4.

The experimental oscillator and the components forming the oscillator are described in Sec. 6.5. The experimental features of self and forced oscillators are presented in Sec. 6.6. The experimental results obtained are also discussed in the light of the first order theory presented in Secs. 6.3 and 6.4.

6.2. The Oscillator and Its Describing Equation:

The oscillator, shown schematically in Fig. 6.1, is analysed in this chapter. It consists of an amplifier the output of which is fed back to the input through a delay line. The linear gain of the amplifier is $G$. We shall analyse the performance of the oscillator for two types of nonlinearity, namely the cubic type and the saturation type. For the cubic type the relation between the output and the input may be expressed as

$$V_2 = -G(1-kV_1^2)V_1$$  \hfill (6.1)

The relation for the saturation type of nonlinearity is

$$V_2 = -G V_1 \quad \text{for} \quad |G V_1| < V_o$$
$$= V_o \quad \text{for} \quad |G V_1| \geq V_o$$  \hfill (6.2)

The amplifier is assumed to invert the polarity of the voltage. We further assume that the output impedance of the amplifier is matched to the characteristic impedance of the delay line, which is terminated by its
FIG. 6.1

Block Diagram of the Delay Line Oscillator
characteristic impedance. We assume that the line is dispersive and has losses associated with it. The resistance, inductance and capacitance of the line per unit length are respectively $R$, $L$ and $C$. The output voltage of the line $V_3$ due to an input voltage $V_2$ is to be obtained from the line equation

$$\frac{d^2 V}{dx^2} = RC \frac{dV}{dx} + LC \frac{d^2 V}{dx^2} \quad \cdots \quad (6.3)$$

If the line is lossless and non-dispersive $V_3$ may be written as

$$V_3 = V_2(t - \tau) \quad \cdots \quad (6.4)$$

$\tau$ being the delay introduced by the line.

For a lossy dispersive line this relation will be more complicated and we shall represent the transfer function of the line by the following equation

$$V_3 = \tau(V_2) \quad \cdots \quad (6.5)$$

The equations describing the oscillator are then

$$V_1 = fV_3 = f\tau(V_2) \quad \cdots \quad (6.6)$$

and Eq. (6.1). We shall assume that $f$ is small compared to $G$ and the nonlinear distortion introduced by the amplifier is small and seek a solution of Eqs. (6.1) and (6.6) by the method of variation of parameters$^2,3$.

6.3. **Amplitudes of Self-oscillation and Stability Conditions** :

Let us assume that the frequency of oscillation is $\omega$ and the voltage $V_2$ is given by

$$A \sin(\omega t + \theta) \quad \cdots \quad (6.7)$$
where the amplitude of oscillation, $A$ and the phase, $\theta$ are allowed to be functions of time. We are interested in finding the equilibrium values of these quantities.

We shall first obtain $V_3$ in terms of $V_2$ from Eq. (6.3). One notes that in the equilibrium condition, since the line is assumed to be matched at both ends, $V$ is given by

$$V = A_0 e^{-\alpha x} \sin(\omega t - \beta x + \theta_0)$$

where $A_0$, $\theta_0$ are the equilibrium amplitude and phase of $V_2$; $\alpha_0$ and $\beta_0$ are respectively the attenuation and phase constant of the line for the oscillation frequency $\omega$. Since we have assumed that the nonlinearity is small, the excess of positive feedback should also be small for a possible equilibrium condition and the rate of variation of $A$ and $\theta$ under non-equilibrium conditions would also be small compared to $\omega$. The solution for $V$ under non-equilibrium conditions may hence be taken to be

$$V = A(t) e^{-\alpha x} \sin \left[ \omega t - \beta x + \theta(t) \right]$$

Differentiating $V$, neglecting the higher derivatives of $A$ and $\theta$ compared to $\dot{A}$ and $\dot{\theta}$ and the derivatives of $\alpha$ and $\beta$ in comparison to $\omega$ and $\zeta$, substituting in Eq. (6.6) and equating the sine and cosine terms on the two sides one obtains

$$\left( \alpha^2 - \beta^2 \right) = -\omega^2 LC - 2\omega LC \dot{\theta} + RC \frac{\dot{A}}{A}$$

$$2 \alpha \beta = \omega RC + RC \dot{\theta} + 2\omega LC \frac{\dot{A}}{A}$$

The equilibrium values of $\alpha$ and $\beta$ are obtained by putting $\dot{A} = \dot{\theta} = 0$ and are given by
\[(\alpha_0^2 - \beta_0^2) = -\omega^2 LC \quad \ldots \quad (6.12)\]

\[2\alpha_0\beta_0 = \omega RC \quad \ldots \quad (6.13)\]

In terms of \(\alpha_0\) and \(\beta_0\), one may write (6.10) and (6.11) as

\[(\alpha^2 - \beta^2) = (\alpha_0^2 - \beta_0^2) \left[ 1 + \frac{\dot{\theta}}{\omega} + \frac{2\alpha_0\beta_0}{\alpha_0^2 - \beta_0^2} \frac{\dot{A}}{\omega A} \right] \quad \ldots \quad (6.14)\]

\[2\alpha\beta = 2\alpha_0\beta_0 \left[ 1 + \frac{\dot{\theta}}{\omega} - \frac{(\alpha_0^2 - \beta_0^2)}{\alpha_0\beta_0} \frac{\dot{A}}{\omega A} \right] \quad \ldots \quad (6.15)\]

Solving (6.14) and (6.15), neglecting higher powers of \(\dot{A}\) and \(\dot{\theta}\)

\[\alpha = \alpha_0 \left[ 1 + \frac{\alpha_1}{\alpha_0} \dot{\theta} + \frac{\beta_1}{\alpha_0 \beta_0} \frac{\dot{A}}{A} \right] \quad \ldots \quad (6.16)\]

\[\beta = \beta_0 \left[ 1 + \frac{\beta_1}{\beta_0} \dot{\theta} - \frac{\alpha_1}{\beta_0 \alpha_0} \frac{\dot{A}}{A} \right] \quad \ldots \quad (6.17)\]

where

\[\alpha_1 = \frac{\alpha_0^3 L}{\omega (\alpha_0^2 + \beta_0^2)} \quad \text{and} \quad \beta_1 = \frac{\beta_0^3 L}{\omega (\alpha_0^2 + \beta_0^2)}\]

Thus, if the length of the line be \(L\), \(V_3\) may be written as

\[V_3 = \alpha e^{-\alpha L} \sin(\omega t - \beta L + \theta) \quad \ldots \quad (6.18)\]

Substituting \(\alpha\) and \(\beta\) from (6.16) and (6.17), neglecting small order terms involving \(\dot{\theta}^2\), \(\dot{A}^2\) etc.
Let us now assume that $V_1$ is given by

$$-B \sin(\omega t + \theta) \quad \ldots \quad (6.20)$$

We shall neglect the harmonics introduced by the nonlinearity. This is justified for small deviation of the total loop gain from unity.

The fundamental frequency output of the nonlinearity may be written as

$$V_2(\omega) = \Psi(B) \sin(\omega t + \theta) \quad \ldots \quad (6.21)$$

Replacing $A$ by $\Psi(B)$, using (6.6) and (6.19), equating the sine and cosine terms and rearranging

$$\left(a_1^2 + \beta_1^2\right) \frac{\Psi(B)}{B} \dot{\theta} = \alpha_1 \frac{\Psi(B)}{B} + \frac{1}{f e^{-\alpha_1}} (\alpha \cos \beta_1 l + \beta_1 \sin \beta_1 l) \quad \ldots \quad (6.22)$$

$$\left(a_1^2 + \beta_1^2\right) \frac{\Psi(B)}{B} \dot{\beta} = \beta_1 \frac{\Psi(B)}{B} + \frac{1}{f e^{-\alpha_1}} (\beta_1 \cos \beta_1 l - \alpha_1 \sin \beta_1 l) \quad \ldots \quad (6.23)$$

The equilibrium conditions are obtained by putting $\dot{\beta} = \dot{\theta} = 0$ and putting this condition in (6.22) and (6.23) one obtains

$$\sin \beta_1 l = 0 \quad \ldots \quad (6.24)$$

$$\frac{\Psi(B)}{B} + \frac{1}{f e^{-\alpha_1}} \cos \beta_1 l = 0 \quad \ldots \quad (6.25)$$
Equation (6.24) gives the frequencies of oscillation and (6.25) gives the amplitudes of oscillation.

Apparently (6.24) indicates that all frequencies at which the total phase shift around the loop is a multiple of $\pi$ is a possible frequency of oscillation. But, physically one would expect that the oscillator may execute oscillations only at those frequencies for which the phase shift introduced by the line is an odd multiple of $\pi$, since the amplifier introduces an additional phase shift of $\pi$. On applying the stability criteria it is found that though (6.24) indicates oscillations also at frequencies for which $\beta\ell$ is an even multiple of $\pi$, these are really unstable singularities in the phase plane. The stability conditions given below bring out this result.

Applying Liapunov's criteria of stability the stability condition is found to be,
\[
\left(1 - \frac{\mathcal{Y}(B)}{B}\mathcal{Y}(B)\right) < 0
\]
the subscript $|_0$ indicates the value at the equilibrium point.

Let us consider the case of a cubical nonlinearity for which
\[
\frac{\mathcal{Y}(B)}{B} = G(1 - \frac{3}{4}kB^2)
\]
Substituting in (6.26) and using (6.25), (6.26) reduces to
\[
\left(-\frac{3}{4}kB^2\right)\left(2 - \frac{3\cos\frac{\beta\ell}{Gf^2e^{-\frac{x}{L}}}^{-1}}{Gf^2e^{-\frac{x}{L}}}\right) < 0
\]
Clearly the above inequality will not be satisfied if \( \cos \beta l \) is positive or \( \beta l \) is an even multiple of \( \pi \). On the other hand, if \( \cos \beta l \) is negative the inequality may be satisfied if \( G f e^{-\alpha l} < \frac{3}{2} \).

Hence the oscillations will have stable amplitudes if \( \beta l \) is an odd multiple of \( \pi \). The same conclusion is also reached when we consider the saturation type of nonlinearity.

We give below the equations giving the amplitudes of stable self oscillations for the two type of nonlinearities:

For cubic nonlinearity,

\[
B^2 = \frac{4}{3k} \left(1 - \frac{1}{G f e^{-2l}}\right) \quad \ldots \quad (6.29)
\]

and for saturation type nonlinearity,

\[
\left[ \sin^{-1} \frac{E_c}{GB} + \frac{E_c}{GB} \left(1 - \frac{E_c^2}{GB^2}\right)^{\frac{1}{2}} \right] = \frac{\pi}{2} \frac{1}{G f e^{-2l}} \quad \ldots \quad (6.30)
\]

where \( E_c \) is the limiting voltage.

6.4. Amplitude of Forced Oscillations:

In the present section we shall study the characteristics of the oscillator in the presence of a forcing signal at the input of the amplifier. Let the frequency of the forcing signal applied to the input of the amplifier be \( C \sin \omega t \). The voltage at the input of the amplifier is now,

\[
f V_3 + C \sin \omega t \quad \ldots \quad (6.31)
\]
where $V_3$ is the voltage fed back at the input of the amplifier from the output of the delay line. In the general case, $V_3$ will include the oscillation at the frequency of the forcing signal and also any free oscillations that may be excited. We shall assume that the oscillator is so adjusted that there is only one frequency of the free oscillation, and this oscillation at the input of the amplifier is\(^\text{35,39}\) 

\[-D \sin(\omega t + \theta) \quad \ldots \quad (6.32)\]

Combining equations (6.31) and (6.32), the total input voltage at the amplifier may be written as \(^\text{39}\) 

\[-B \sin(\omega t + \phi) - D \sin(\omega t + \theta) = fV_3 + C \sin \omega t \quad \ldots \quad (6.33)\]

The output voltage of the nonlinearity may be written as \(^\text{39}\) 

\[\psi_1 \sin(\omega t + \phi) + \psi_2 \sin(\omega t + \theta) \quad \ldots \quad (6.34)\]

Using equations (6.19), (6.33) and (6.34) it may be shown that the derivatives of $B$, $D$, $\phi$ and $\theta$ satisfy the following equations \(^\text{39}\) (See Appendix III):

\[
\left(\frac{\partial \psi_1}{\partial B} \frac{\partial \psi_2}{\partial B} - \frac{\partial \psi_1}{\partial D} \frac{\partial \psi_2}{\partial D}\right) B = -\xi_2 \psi_2 \frac{\partial \psi_1}{\partial D} \left[1 + \frac{D (\cos \beta_0 \frac{\partial ^2}{\partial x^2} - \frac{\partial ^2}{\partial x^2} \sin \beta_0 \frac{\partial}{\partial x})}{G_2} \right] + \\
+\eta_2 \psi_1 \frac{\partial \psi_2}{\partial D} \left[1 + \frac{(B + C \cos \phi) (\cos \beta_0 \frac{\partial}{\partial x} - \frac{\partial}{\partial x} \sin \beta_0 \frac{\partial}{\partial x}) + C \sin \phi (\sin \beta_0 \frac{\partial}{\partial x} - \frac{\partial}{\partial x} \cos \beta_0 \frac{\partial}{\partial x})}{G_1} \right] \quad \ldots \quad (6.35)
\]

\[
\left(\frac{\partial \psi_2}{\partial B} \frac{\partial \psi_1}{\partial B} - \frac{\partial \psi_2}{\partial D} \frac{\partial \psi_1}{\partial D}\right) D = -\xi_2 \psi_2 \frac{\partial \psi_1}{\partial B} \left[1 + \frac{D (\cos \beta_0 \frac{\partial ^2}{\partial x^2} - \frac{\partial ^2}{\partial x^2} \sin \beta_0 \frac{\partial}{\partial x})}{G_2} \right] + \\
+\eta_2 \psi_1 \frac{\partial \psi_2}{\partial B} \left[1 + \frac{(B + C \cos \phi) (\cos \beta_0 \frac{\partial}{\partial x} - \frac{\partial}{\partial x} \sin \beta_0 \frac{\partial}{\partial x}) + C \sin \phi (\sin \beta_0 \frac{\partial}{\partial x} - \frac{\partial}{\partial x} \cos \beta_0 \frac{\partial}{\partial x})}{G_1} \right] \quad \ldots \quad (6.36)
\]
where, $\eta_1$, $\eta_2$, $\alpha''_1$, and $\alpha''_2$ are respectively the attenuation and phase/unit length of the line for frequencies $\omega$ and $\omega_0$. $\xi_1$ and $\xi_2$ are respectively the amplitudes of oscillations at the forcing frequency and at the frequency of free oscillation at the output of the nonlinearity. These are in general functions of $B$, $D$ and the relative phase between the two oscillations.

The equilibrium values of $B$ and $D$ may be obtained by solving equations (6.35) through (6.38). The stability of these equilibrium values may also be tested by applying Liapunov's criteria of stability. However, detailed analysis of the stability involves complicated algebraic manipulation and we shall restrict our discussions to two of the simple cases. We shall consider the synchronisation characteristics for
(i) the synchronisation frequency nearly equal to the lowest frequency of oscillation,

(ii) the synchronisation frequency exactly equal to one of the higher frequencies of free oscillation.

6.4.1. Synchronisation characteristics near the lowest frequency of free oscillation: One may assume that the oscillator has been so adjusted that the oscillations at the lowest frequency of free oscillation are the preferred ones. Hence in this case, the synchronisation characteristics may be obtained by confining attention to the condition in which the free oscillations are locked to the forcing signal. In other words, $D$ and $\Psi_0$ may be assumed to be zero. One then obtains from (6.35)

\[
\left[ \frac{f e^{-\alpha l}}{B} \right]^2 + 2 \cos \beta_0 \cdot f e^{-\alpha l} \Psi(B) \frac{1}{B} + \frac{(B^2 - C^2)^{1/2}}{B^2} = 0 \quad \ldots \quad (6.39)
\]

and

\[
\tan \phi = \frac{f e^{-\alpha l} \Psi(B) \sin \beta_0}{B + f e^{-\alpha l} \Psi(B) \cos \beta_0} \quad \ldots \quad (6.40)
\]

To illustrate the variation of $B_n$ for different amplitudes and frequencies of the forcing signal, calculations have been made for a cubical nonlinearity assuming $G_0 = G f e^{-\alpha l} = 1.1$ and the calculated values are presented in Fig. 6.2. It is seen from these plots that for values of $\beta_0$ near $\pi$ and for low amplitudes of the forcing signal there may be three possible amplitudes of oscillation for a fixed value of $\cos \beta_0$. For large
values of $\beta_l$, however, the curves are single valued. To investigate the stability of these oscillations, which are assumed in this analysis to occur at the external signal frequency $\omega$, the following criteria are used.

$$\frac{\partial \dot{B}}{\partial B} |_o + \frac{\partial \dot{\phi}}{\partial \phi} |_o < 0 \quad ... \quad (6.41)$$

$$\frac{\partial \ddot{B}}{\partial B} |_o - \frac{\partial \ddot{\phi}}{\partial \phi} |_o \frac{\partial \dot{B}}{\partial B} |_o + \frac{\partial \dot{\phi}}{\partial \phi} |_o > 0 \quad ... \quad (6.42)$$

The subscript $|_o$ in the above equations give values for $\dot{B} = \dot{\phi} = 0$.

On using earlier expressions of $\dot{B}$ and $\dot{\phi}$, the above inequalities reduce to$^5$(See Appendix IV)

$$\left[ 1 + \frac{\cos \beta_l - \frac{\beta_l}{2} \sin \beta_l}{\frac{\psi(B) + \frac{\psi'(B)}{\psi(B)} \beta_l}{\psi(B) \psi'(B)}} \right] < 0 \quad ... \quad (6.42)$$

and

$$\left[ 1 + \frac{\cos \beta_l \left\{ \frac{\psi(B) + \frac{\psi'(B)}{\psi(B)} \beta_l}{\psi(B) \psi'(B)} \right\}}{\frac{\psi(B) + \frac{\psi'(B)}{\psi(B)} \beta_l}{\psi(B) \psi'(B)}} + \frac{1}{\left( \frac{\psi(B)}{\beta_l} \right)^2} \frac{\beta_l}{\psi(B) \psi'(B)} \right] > 0 \quad ... \quad (6.43)$$

The inequalities in (6.42) and (6.43) for a cubical nonlinearity and for $C_o = 1.1$ are also plotted in Fig. 6.2. Eq. (6.42) gives a straight line AB as shown in Fig. 6.2 and the plot of Eq. (6.43) is the curve DCE. It is clear that the amplitude of oscillation outside the zone marked DCE and above line AB are stable, and those inside this zone or below AB are unstable oscillations. The zone of unstable oscillations are shown in Fig. 6.2.

The resonance curves presented in Sec. 6.4.1 are qualitatively similar to those of van der Pol's oscillator$^{36}$. One finds that for large
Theoretical Resonance Curves of the Oscillator near the fundamental frequency.

**FIG. 6.2**

The shaded region indicates the zone of unstable oscillation.
amplitudes of forcing signal the oscillator remains synchronised, i.e. oscillates at the forcing signal so long as the normalised amplitudes of oscillations are larger than 0.5, in the case of a polynomial non-linearity. In the case of a saturation nonlinearity also the limiting value is identical to that of the van der Pol's oscillator37. For low amplitudes of the forcing signal the criterion of synchronisation is given by a more complicated relation; but the shape of the zone of synchronisation is again similar to that of a van der Pol's oscillator.

6.4.2. Synchronisation characteristics for forcing signal frequency equal to a harmonic frequency: In section 6.3 it has been shown that the oscillator under study may execute oscillations at a frequency for which the phase shift introduced by the delay line is an odd multiple of \( \pi \), and the total loop gain is larger than unity. Our discussions up till now has been confined to free oscillations and forced oscillations when the oscillator is adjusted to have only one frequency of stable oscillation. However, in general, even for this adjustment since the phase conditions for synchronisation may be satisfied at frequencies which are near multiples of the lowest frequency of oscillation (referred to here as harmonic frequencies), the oscillator may exhibit locking phenomenon at many frequencies. We shall study in this section the synchronisation characteristics when the forcing signal frequency is adjusted to be equal to one of these harmonic frequencies. In this case it may be intuitively seen that for low amplitudes of the forcing signal the oscillator would exhibit oscillations at the forcing frequency as well as at its lowest natural frequency. One is therefore required to consider the general situation in which oscillations
at both the frequencies are simultaneously present. The detailed
equations for the situation has been already derived and are given by
(6.35) through (6.38). The discussion in this section may therefore be
based on these equations. For this discussion we shall assume that the
nonlinearity is of the cubical type and the term involving the relative
phase between the two oscillations is also neglected. This latter assump-
tion is applicable when the harmonic frequency is higher than the third
multiple of the free oscillation frequency. However, even for the third
multiple it is expected that the qualitative nature of the synchronisation
characteristics will not be significantly altered due to the exclusion of
the above-mentioned term.

One obtains from (6.35) through (6.38) for cubic nonlinearity
the following equations giving the steady state values of \( D, B \) and \( \phi \)
\[
C \sin \phi = 0 \quad \ldots \quad (6.44)
\]
\[
(G_{\psi} - B) = C \cos \phi \quad \ldots \quad (6.45)
\]
\[
G_{\psi} \psi = D \quad \ldots \quad (6.46)
\]

On replacing \( \psi \) and \( \psi_{\ast} \) by the expressions for the cubical nonlinearity
one finds that the equations admit two combinations of steady state values
for \( B \) and \( D \) and also that \( \phi \) may be equal to 0 or \( \pi \). These combinations
of values suitably normalised are given below.

(1) Combination I
\[
D_{n}^{2} = (1 - 2B_{n}^{2}) \quad \ldots \quad (6.47a)
\]
\[
B_{n}^{3} - \frac{1}{3} \left( 2 - \frac{B_{n}^{2}}{D_{n}^{2}} \right) B_{n} + C_{n} = 0 \quad \ldots \quad (6.47b)
\]
(i1) Combination II

\[ D_n = 0 \quad \ldots \quad (6.48a) \]

\[ B_n^{3} - \frac{B_n^{2}}{D_n^{\theta}} \cdot B_n \pm 3C_n = 0 \quad \ldots \quad (6.48b) \]

where,

\[ B_n = \frac{B}{D_n}, \quad D_n = \frac{D}{D_n}, \]

\[ C_n = \frac{4C}{(k_c G_G, D_n^{3})}, \quad D_n = \left[ \frac{4}{3k_c \left( 1 - \frac{1}{G_c} \right)} \right]^{1/2}, \quad B_n = \left[ \frac{4}{3k_c \left( 1 - \frac{1}{G_c} \right)} \right]^{1/2} \]

It should be mentioned that \( D_0 \) and \( B_0 \) represent respectively the amplitude of free oscillation at the fundamental and at the harmonic frequency. For some adjustments of the oscillator it is possible that \( D_0 \) will be positive but \( B_0^2 \) may be negative.

We have drawn plots showing the variation of \( B_n \) and \( D_n \) with change in the amplitude of the forcing signal; i.e. the magnitude of \( C_n \) for different values of \( \frac{B_n^{2}}{D_n^{\theta}} \). These are shown in Fig. 6.3. One finds on studying these curves, for a value of \( C_n \) greater than a critical value the magnitude of \( D_n \) is zero; or in other words, for these values of \( C_n \) the oscillator executes oscillation only at the forcing frequency. This critical value of \( C_n \) is given by

\[ C_n = \frac{2}{27} \left( 2 - \frac{B_n^{2}}{D_n^{\theta}} \right)^{3/2} \quad \ldots \quad (6.49) \]

Thus, the amplitude of the forcing signal required for establishing the above-mentioned condition increases with decrease in the value of \( B_0 \). It
is also found that for an ideal delay line for which $B_0$ may be assumed to be equal to $D_0$ the value of the critical normalised amplitude is $2/27$. For values of $C_n$ lower than the critical value the situation is fairly complicated. In this region, both the combinations are possible, and also for each combination, for some values of $C_n$, there may be three possible values of both $B_n$ or $D_n$. Of these different possible combinations of $B_n$ and $D_n$, evidently one combination will be excited; the particular combination which will be excited can only be determined from a rigorous stability analysis taking into consideration the initial values of $B_n$ and $D_n$. However, some general observations may be made from the following simplified arguments.

When the amplitude of the synchronising signal is gradually increased from zero one would expect that combination I will be excited and also the value of $B_n$ will start increasing from zero. It would mean that for this case of all the different possible combinations the oscillator will select the combinations represented by $AB$ and $CD$. The amplitude of free oscillation will gradually decrease following the curve $CD$ as $C_n$ is increased to the critical value; at the same time, the amplitude of oscillation at the forcing frequency will increase following the curve $AB$. It is also evident that for the critical amplitude of the forcing signal the free oscillations will vanish and the amplitude of oscillations at the forcing frequency will jump from $B$ to $E$. For further increase in the value of $C_n$, the curve $EF$ will be followed. If now the amplitude of the forcing signal is reduced the curve $FE$ will be retraced; and from point $E$ onwards the curve $EG$ will be followed till a point is
The amplitudes of the fundamental and forcing frequency oscillation, when the forcing signal frequency is equal to a harmonic frequency, for different amplitudes of the forcing signal.

Characteristics for $B_n^2/D_n^2 = 0.8$
The amplitudes of the fundamental and forcing frequency oscillation, when the forcing signal frequency is equal to a harmonic frequency, for different amplitudes of the forcing signal.
even after the forcing signal is removed (Fig. 6.3a). This phenomenon is of importance for some applications of delay line oscillators in multimode memory systems. We find from the above discussion that a delay line oscillator may be operated as a multimode memory system provided the amplitude of the free oscillations at the harmonic frequencies are larger than 0.707 times the fundamental frequency. It is also evident that the mode of oscillation may be changed by cycling the forcing signal through the critical amplitude.

The discussion in this section has been made with reference to a cubic nonlinearity. However, one may expect that the qualitative features of the synchronisation characteristics e.g. the hysteresis region, multimode property etc. will remain even for saturation type of nonlinearity. But the values of the critical amplitude of the forcing signal and the points of switching may be different.

We have discussed in the previous sections some of the features of a delay line oscillator in free and forced condition on the basis of a first order approximate theory. Some experimental results have been obtained on a model oscillator to examine how far the results of first order approximate theory remain valid under experimental conditions. These results are presented in the following section.

6.5. Experimental Oscillator:

The block diagram of the experimental oscillator is shown in Fig. 6.4. We describe below the individual components in block form:
FIG. 6.4
Block diagram of the Experimental Oscillator
(i) **Amplifier**

A stabilised D.C. amplifier with a bandwidth of 100 Kc/s at unity gain and a short drift time of 1 mv/hour has been used. To provide a unity gain around the loop, the amplifier was required to be adjusted to have a gain of the order of 8. The adjustment was done by a suitable choice of the input and the feedback resistance (500 KΩ). For this the gain, the bandwidth of the amplifier will be 12 Kc/s and drift voltage will be 10 mv/hour.

(ii) **The nonlinear element**

The cubic nonlinearity was realised by biased diode elements. The detailed circuit arrangement and the parameter values are shown in Fig. 6.5a. The input output characteristics of the nonlinearity as obtained experimentally is also shown in the same figure. The ratio of K was chosen to be 4/35 to make the value of B unity for G₀ = 1.1. One finds that if the oscillation amplitude is limited within 19.2 volts at the input of the nonlinear element, the deviation of the cubic nonlinearity is less than 2%. A voltage of 12 volts was considered to be an optimum choice to represent unity on the basis of the drift and saturation characteristics of the D.C. amplifier. The bandwidth of the nonlinear element was found to be about 10 Kc/s. It should be noted that in order that the nonlinear element be described by a cubic polynomial, the adjustment of the total loop gain of the oscillator should be so adjusted that amplitude of the self and forced oscillations lie within 19 volts.

The saturation type nonlinearity was also similarly realised by using biased diodes. The circuit arrangement of the saturation type of
Input-Output Characteristics and the experimental Circuit arrangement of the Cubical nonlinearity

FIG. 6.5(a)
nonlinearity is shown in Fig. 6.5b. The amplitude of oscillations on the output of the saturation nonlinearity was limited to ± 1.5 volts, and the gain of the nonlinear element was adjusted to be equal to 1.5.

The nonlinear element is followed by a cathode follower, the output impedance of which was adjustable between 150 Ω to 400 Ω. This was arranged by putting a potentiometer in series with the output of the cathode follower.

(iii) The delay line

The delay line consisted of 36 sections. The inductance in each section had a value of 14 mH and was constructed by using ferrite cores. The capacitance of each section was 0.22 μF, provided by polyester capacitors. The delay line was terminated by its characteristic impedance in parallel with a 10 turn helipot potentiometer of 10 kΩ. The output of the delay line could be adjusted by this potentiometer to provide different values of f.

The attenuation and phase shift characteristic of the delay line is shown in Fig. 6.6. One finds from this characteristic that a phase shift of π is introduced at a frequency of 220 c/s at which the attenuation factor is 0.74. It may also be noted that the attenuation factor at 700 c/s where the phase shift is 3π is 0.705. It is possible to adjust the loop gain such that it is greater than unity at 220 c/s but less than unity for higher frequencies at which the phase shift is an odd multiple of π. This feature enable one to adjust the oscillator such that, only one frequency of oscillation is possible. We describe below the results
FIG. 6.6

Variation of the attenuation factor and phase shift of the delay line with frequency.
obtained on the free and forced oscillations in the oscillator constructed with the components described above.

6.6. Experimental Results and Comparison with Theory:

6.6.1. Experiments on self-oscillations: Self oscillations were obtained experimentally by adjusting the potentiometer at the output of the delay line. Oscillations were found to be first excited for a setting of this potentiometer equal to 0.845. The amplitude of free oscillation which occurred only at the fundamental frequency (220 c/s) were measured with the help of a calibrated wave analyser. The amplitudes of the harmonics were also measured with the same wave analyser. The data for the cubic nonlinearity and for the saturation nonlinearity are respectively presented in Figs. 6.7a and 6.7b. One of the significant features which emerge from these curves is that the percentage harmonic for the saturation type of nonlinearity is nearly double of the percentage harmonic of the cubical nonlinearity for the same setting of the loop gain. One also finds that, for low settings of the feedback ratio the amplitude of the harmonics is very very small fractions of the fundamental amplitude. The characteristics for the cubic nonlinearity show another significant feature: namely that for a setting of feedback ratio larger than 0.88, the amplitude of the 3rd harmonic increases rather rapidly.

It is possible to compare the above experimental results with those obtained from the first order theory given earlier particularly for the cubic nonlinearity. The theoretically calculated values for the amplitude of the fundamental are indicated by crosses in Fig. 6.7a. It
FIG. 6.7(a)

Characteristics of free Oscillation for

a Cubical nonlinearity
should be pointed out that the theoretically calculated values are very sensitive to the values of $G_e^{-1}$. To avoid any error which may arise out of this the value of $G_e^{-1}$ was really obtained from the ratio of the amplitudes of the fundamental for two low settings of the feedback potentiometer. It is found that there is rather close agreement between the theoretically calculated and the experimental values of $D$. The critical feedback factor (0.845) is also found to agree with that expected from the measured value of attenuation at 220 c/s of the line, the gain of the D.C. amplifier and the transmission factors arising from the cathode follower and the nonlinear element. The fact that the amplitude of the 3rd harmonic increases rapidly for values of feedback factor larger than 0.88 also agrees with theory; since for this value of the feedback factor it is found from the attenuation characteristics of the delay line that the loop gain for the 3rd harmonic is unity. The agreement found between the theoretically calculated values and the experimental values for the saturation type of nonlinearity is not, however, as close as for the cubic type of nonlinearity. Apparently, this is due to the presence of the large amount of harmonics. This however, could not be checked theoretically, as calculations of the transfer function for the saturation type of nonlinearity when a significant amount of harmonics are present is somewhat complicated.

6.6.2. Experiments on forced oscillations: The synchronisation characteristics near the fundamental frequency could not be obtained with much accuracy since very small variation of the forcing frequency was required to obtain points on the synchronisation characteristics. However, the qualitative features of the characteristics were found to agree with
those given in Fig. 6.3. The criteria, that oscillations remain synchronised only if the amplitude of oscillations at the forcing frequency is larger than $\sqrt{0.707}$ times the amplitude of the free oscillations, was found to be satisfied. It was also observed that for smaller amplitude of the forcing signal, oscillator remains synchronised if the amplitude of oscillations is higher than the above value and is required to be higher as the amplitude of the forcing signal is decreased. However, reliable data could not be obtained on these characteristics for comparison with theory.

The synchronisation characteristics when the forcing signal frequency is equal to the harmonic frequency was however investigated in some details. The characteristics obtained for the 3rd and 5th harmonic for the two nonlinearities are shown in Figs. 6.8 and 6.9. One finds that these curves agree closely with the theoretically expected curves given earlier in Fig. 6.3. As expected, for low amplitudes of the forcing signal, oscillations at the forcing frequency are excited together with the free oscillations. As the amplitude of the forcing signal is increased this state continues till a critical state is reached at which only oscillations at the forcing frequency exists. For further increase in amplitude the synchronised conditions continues. When, however, the amplitude of the forcing signal is decreased the synchronised condition continue for amplitudes lower than the critical value and switches back to the condition at which free oscillations exists with the forced oscillation where the normalised amplitude of the forced signal frequency is very near $\sqrt{0.707}$. Thus, the general shape of the characteristics are in agreement with the theory. The hysteresis region is also found to increase
Experimental Synchronisation Characteristics when the forcing signal frequency is equal to the 3rd. harmonic frequency for a cubical type of nonlinearity.

--- Oscillation at the forcing frequency

--- Oscillation at the fundamental frequency
with increase in regeneration, in agreement with theory. Similar features are found to be exhibited for the saturation type of non-linearity.

We should however, mention that the attenuation characteristics of the delay line and available gain of the amplifier was such that the feedback factor could not be adjusted to make \( B_0^2/\mu_0^2 > 0.5 \). Due to this the multimode property could not be studied. Using the delay line described earlier. The multimode property was studied using the oscillator described below.

6.6.3. Experiments on the multimode property of the delayline oscillator: In the earlier sections have been described experiments which were performed on a model oscillator specifically designed with nonlinearities simulating the theoretical model. The operation frequencies were also chosen to be low enough for harmonic analysis using the equipments available in the laboratory. However, as mentioned earlier, the attenuation characteristic of the delay line was of such nature that the oscillator executed stable oscillations only at the fundamental frequency. In order to investigate the possibility of stable oscillations at other frequencies a delay line oscillator as shown in Fig. 6.10 was constructed. The delay line consisted of 36 sections, wound continuously on a 2\( \frac{3}{4} \)" former using 26 gauge enamelled copper wire. No significant variation in attenuation could be detected upto the frequency of 1.5 Mc/s. The cut-off frequency of the line was 10 Mc/s and a phase shift of \( \pi \) is introduced at a frequency of 180 Kc/s. As is evident from the figure, the feedback may be adjusted by the decade box arrangement terminating the transmission line. The characteristics of oscillations observed on this oscillator are summarised below.
FIG. 6.10

Experimental Circuit arrangement of the Delay line Oscillator
(a) Continuous steady state oscillations at 180 Kc/s are excited for a feedback factor greater than 0.202.

(b) Oscillations could be switched from the lowest frequency mode to the 3rd harmonic or 5th harmonic frequencies by signals of amplitudes as shown in Fig. 6.11 for different feedback factors. A larger amplitude is usually required for switching for larger feedback factors.

(c) Stable oscillations at 180 Kc/s and also at 540 Kc/s, are possible when the feedback factor is increased above 0.212. The required amplitude for switching from 540 Kc/s to 180 Kc/s are also shown in Fig. 6.11.

(d) Stable oscillations at 900 Kc/s in addition to those of 180 Kc/s and 540 Kc/s are possible when the feedback factor is increased beyond 0.313.

It is thus evident that the delay line oscillator may be used as a multimode oscillator and switching from one mode to another is possible by applying an external signal of proper amplitude and frequency. The number of such possible modes may be increased by increasing the feedback factor. For the experimental oscillator, adjustments could be made only for three possible modes. It is expected that the number of modes may be further increased by using a delay line of a larger number of sections and of smaller attenuation. It should also be pointed out that the multimode property of these oscillators are exhibited as oscillations mainly at the fundamental, third harmonic, fifth harmonic or higher harmonic oscillations but, oscillations at the fundamental are also associated with
Experimental Synchronisation Characteristics of the oscillator for different feedback ratio.

Curve A corresponds to switching from fundamental to 3rd harmonic frequency.

Curve B corresponds to switching from 3rd harmonic to fundamental frequency.
large amount of harmonics. In fact, when adjustments are so made that oscillations at 900 Kc/s, 540 Kc/s or at 180 Kc/s were stable, the wave shape of oscillations at 180 Kc/s were more near rectangular than sinusoidal. This may lead to some extent limit the applicability of these oscillators as a multimode system, each mode of which are required to contain oscillation of only one frequency.

6.7. Conclusions:

A first order approximate theory of a delay line oscillator using a non-ideal delay line has been developed. The characteristics of free oscillation and forced oscillation has been studied on the basis of this first order theory. One of the important results which emerges from the theory of forced oscillations is that the characteristics will show jump phenomena and hysteresis. Also a delay line oscillator has multiple stable modes and may be switched from one state to another by a forcing signal, by cycling the amplitude of the forcing signal through a critical value. Experiments performed on a model oscillator show features in agreement with theory.