PART I.

THEORETICAL STUDIES ON HOT-CARRIER GALVANOMAGNETIC PROPERTIES OF ELEMENTAL SEMICONDUCTORS

CHAPTER II. Theory of Hot-Carrier Galvanomagnetic Characteristics of Elemental Semiconductors.

CHAPTER III. High Field Galvanomagnetic Coefficients for Spherical Constant Energy Surface Model.

CHAPTER IV. Effects of Anisotropy in the Band Structure on the Hot-Carrier Magnetoresistance and Hall Mobility Characteristics.
2.1. Introduction:

The theory of hot-carrier conduction involves the solution of Maxwell-Boltzmann transport equation and it is almost impossible to obtain the solution without some simplifying assumptions. Three different types of approximation have been made in the literature for obtaining the solution. In one method collisions between the carriers and lattice are only considered and the transport equation is solved using the diffusion approximation and retaining the first two terms only. In the second method, inter-carrier collisions are assumed to make the symmetric part of the distribution function Maxwellian with an unknown parameter, the carrier temperature. This is determined from the energy balance condition assuming an expression for the carrier mobility as at low fields. In the third method, the distribution function is assumed to be Maxwellian but displaced in the momentum space. The carrier temperature and the displacement are obtained by using the momentum balance condition together with the energy balance condition. Theoretically these methods are applicable for three different ranges of carrier concentration, but conductivity mobility obtained from the three methods are found to be approximately the same.

Though hot-electron conductivity characteristics have been studied in some detail, the galvanomagnetic coefficients have not been much studied. Hot-carrier Hall mobility was first studied by Sodha and Eastman assuming spherical constant energy surface and acoustic phonon scattering. However, the assumption of spherical fermi-surface and scattering by only acoustic
phonons are not, in general, applicable to the elemental semiconductors at room temperature. Budd \(^{41,42,43}\) analysed the hot-carrier galvanomagnetic characteristics assuming scattering by acoustic phonons only but ellipsoidal constant energy surfaces. On the other hand, Dykman and Tolpygo \(^{47}\), Hattori et al \(^{48}\) and Tsutsumi \(^{39}\) studied the Hall mobility and the magnetoresistance in the warm electron region. The author presents in this and in the following two chapters the theory of hot-carrier conduction in the presence of a magnetic field taking into consideration the complex band structure and all the significant scattering mechanisms with particular reference to n-type germanium.

It should be mentioned that the expressions for Hall coefficient under similar conditions have also been given by Conwell \(^{38}\). But the evaluation of the Hall coefficient from these expressions requires a knowledge of the distribution function of the carriers and the energy dependence of the relaxation time. The theory developed here, however, enables one to evaluate both Hall coefficient and magnetoresistance directly using the known values of primary semiconductors constants. The expressions for the magnetoresistance coefficient as given by Das \(^{45}\) for the assumptions mentioned earlier are too complicated for the discussion of the detailed feature of the magnetoresistance characteristics. Also, the values of magnetoresistance calculated by him are found to differ widely from the experimental results obtained by the present author. On the other hand, the characteristics obtained from the present author's theoretical analysis are in excellent agreement with the experiment.

The method of analysis used by the present author is formulated in a general way in this chapter. The concentration of the carriers is assumed to be low so that the inter-carrier scattering may be neglected and the method used by Yamashita and Watanabe \(^{21}\) for the analysis of the hot-carrier dc
conductivity is applicable. In order to solve the Boltzmann equation, the
distribution function is assumed to consist of a symmetric function depending
only on the energy and a directional part having three components along the
three directions of the chosen right handed rectangular co-ordinate system.
On substituting these functions in the Boltzmann equation and expanding the
collision terms one obtains a pair of differential equations which may be
directly solved. The galvanomagnetic coefficients are then evaluated using
these solutions.

2.2. Distribution Function for Hot-carriers in the Presence of a Magnetic Field:

In the absence of any field, the distribution function of the carriers
in semiconductors is Maxwellian at the lattice temperature. When a field is
applied the distribution function is perturbed, but for low fields the pertur­
bation is in the form of a small directional term added to the Maxwellian
distribution. For high fields the perturbation is more complicated but the
distribution function of the carriers in a particular valley of a many valley
semiconductor may be written as

\[ f = f_o(\varepsilon) + \mathbf{K} \cdot \mathbf{f}_i \quad \ldots (1) \]

where \( f_o \) and \( f_i \) are functions of energy only. \( f_o \) is the symmetric part
of the distribution function, \( \mathbf{K} \cdot \mathbf{f}_i \) is the directional part, and \( \mathbf{K} \) is the
wave vector.

The distribution function \( f \) satisfies the Boltzmann transport equation
for carriers in the valley under consideration for steady electric field. So
one can write

\[ \left. \frac{\partial f}{\partial t} \right|_{\text{field}} + \left. \frac{\partial f}{\partial t} \right|_{\text{collision}} = 0 \quad \ldots (2) \]
The first and the second term respectively represent the rate of change of the distribution function due to the applied field and due to collisions of the carriers with the different scattering sources.

The first term is given by

$$\frac{\partial f}{\partial t} \bigg|_{\text{Field}} = \left( e / h \right) \left[ \overrightarrow{F} + \frac{1}{h} \left( \nabla_x E \times \overrightarrow{B} \right) \right] \cdot \nabla_x f \quad \ldots \quad (3)$$

where \( e \) is the charge of a current carrier; \( h = h / 2\pi \), \( h \), being Planck's constant; \( \overrightarrow{F} \) is the total electric field in V/m; \( \overrightarrow{B} \), the magnetic field in WB/m^2 and \( E \) is the energy of a carrier in the valley under consideration and can be written as

$$E = \frac{\hbar^2}{2} \left[ \frac{k_1^2}{m_1} + \frac{k_2^2}{m_2} + \frac{k_3^2}{m_3} \right] \quad \ldots \quad (4)$$

where \( k_1, k_2 \) and \( k_3 \) are the components of the wave vector \( \overrightarrow{k} \) and \( m_1, m_2 \) and \( m_3 \) are the effective masses along the directions of the principal axes of the energy ellipsoid of the chosen valley.

The change in the distribution function of the carriers in that valley due to the collisions may, in general, be written as

$$\frac{\partial f}{\partial t} \bigg|_{\text{Collision}} = \frac{\partial f}{\partial t} \bigg|_{\text{Acoustic}} + \frac{\partial f}{\partial t} \bigg|_{\text{Optical}} + \frac{\partial f}{\partial t} \bigg|_{\text{Intervley}} + \frac{\partial f}{\partial t} \bigg|_{\text{Impurity}} + \frac{\partial f}{\partial t} \bigg|_{\text{Inter-carrier}} \quad \ldots \quad (5)$$

where the terms on the right hand side represent the change due to the significant scattering mechanisms in elemental semiconductors. These are respectively the acoustic, optical, intervalley, impurity and inter-carrier scattering.

The contribution of the scattering due to impurity centres may, in general, be neglected unless the temperature is very low and the effect of the
scattering due to inter-carrier collisions may be assumed to be negligible for low carrier concentration.

It has been shown by Herring\textsuperscript{49,50} that even for an anisotropic many valley band structure the inter-valley and optical phonon scattering may be assumed to be isotropic but the acoustic phonon scattering may be anisotropic. In the hot-carrier conduction region, however, the contribution of the acoustic phonon scattering is of such magnitude that one can assume the acoustic phonon scattering also to be isotropic without introducing significant error in the final result. Thus the scattering terms have been assumed to be isotropic in the present analysis.

Then the collision term is given by\textsuperscript{39}

\[
\frac{\delta f}{\delta t} \mid \text{collision} = \sum_i B^i(E) P_i \cos \Theta
\]  \hspace{1cm} \ldots \hspace{0.5cm} (6)

where \( \Theta \) is the angle between the wave vector and the field.

Substituting Eq. (1) in Eq. (2) and equating the zeroeth and the first order terms one obtains

\[
B^0(\omega) = \left( \frac{e}{\kappa} \right) \left[ \vec{F} \cdot \left\{ \frac{2}{3} \frac{W^{\frac{1}{2}}}{dW} \left( \frac{W^{\frac{3}{2}}}{dW} \cdot \vec{F} \right) \right\} \right] \]  \hspace{1cm} \ldots \hspace{0.5cm} (7)

\[
B^1(\omega) = \frac{\epsilon t}{m_e} \left[ \vec{M} \cdot \frac{\vec{F}}{\kappa T} \frac{dF_e}{dW} + \frac{1}{\kappa} \vec{M} \cdot (\vec{G} \times \vec{F}) \right] \]  \hspace{1cm} \ldots \hspace{0.5cm} (8)

where \( \kappa \) is the Boltzmann constant, \( T \) is the lattice temperature, \( \omega \) is the energy of a carrier in the chosen valley scaled in units of thermal energy \( kT \).
and $\mathbf{M}$ is the tensor of the reciprocal effective mass multiplied by the conductivity effective mass, $m_e$, and it can be written as

$$
\mathbf{M} = \begin{vmatrix}
  m_e & m_e & m_e \\
  m_e & m_e & m_e \\
  m_e & m_e & m_e \\
\end{vmatrix}
$$

$m_e/m_{xx}$, $m_e/m_{yy}$ etc. being its components along the directions of the chosen right-handed rectangular co-ordinate system.

Now $B^o(W)$ and $B^o'(W)$ for isotropic scattering are given by

$$
B^o(W) = -\frac{1}{\tau_{ae}} \left. \frac{d}{dW} \left( W^2 \left( \frac{df^a}{dW} + \frac{f^a}{f} \right) \right) \right|_{W=W_0}
$$

$$
= \frac{1}{\tau_o} \left. \frac{1}{\sqrt{W}} \left[ \left( W+W_0 \right)^{-1/2} \left\{ \left( \tau_{o+1} \right)^{1/2} f_{o+1}(W) - \tau_{o} f_{o}(W) \right\} \right] \right|_{W=W_0}
$$

$$
+ \left( W-W_0 \right)^{-1/2} \left\{ \tau_{o} f_{o}(W-W_0) - \left( \tau_{o+1} \right)^{1/2} f_{o+1}(W) \right\} \right|_{W=W_0}
$$

... (10)

$$
B^o'(W) = \frac{\dot{f}}{\tau_{om}/\Omega} = \frac{\dot{f}}{\Omega}
$$

... (11)

where $\tau_{ae}$ and $\tau_{om}$ are respectively the energy and momentum relaxation time due to the acoustic phonons; $\tau_o$ and $(\Omega^{-1})\tau_{om}$ are respectively the energy and momentum relaxation time due to the optical phonons; $W_o$ is the ratio of the optical phonon energy, $\hbar\omega_o$ and $\hbar\tau_o$, $\omega_o/\hbar$ is denoted by $\theta_o$, the optical phonon characteristic temperature; $\eta_o$ is the optical phonon concentration and is given by $\eta_o = (\Omega^{-1})^{-1}$ and $\Omega = \tau_{om}/\Omega$. The values of $\tau_{ae}$ and $\tau_{om}$, $\tau_o$ and $\Omega$ are given by

$$
\tau_{ae} = \frac{\pi \rho K^4 \left| \alpha \right|^{1/2} \hbar^2}{2 \sqrt{2} \ m_e^{3/2} (\kappa T)^{1/2} (\Sigma_0)^{1/2}} \ W^{1/2}
$$

... (12)
\[ \tau_{\text{ao}} = \frac{\pi \rho k^4 \left| \alpha \right|^2 c_L^2}{\sqrt{2} m_o^{3/2} (\kappa \gamma)^{3/2}} \omega^{-\frac{1}{2}} \] ... (13)

\[ \tau_0 = \frac{\sqrt{2} \pi \rho k^2 (\kappa \gamma)^{1/2} \left| \alpha \right|^2 \omega^{-\frac{1}{2}}}{m_o^{3/2} D_o^{2}} \] ... (14)

\[ \Omega = 1 + \left( \frac{\tau_{\text{ao}}}{\tau_0} \right) \left[ \frac{\alpha \beta (\lambda \gamma) + 1}{\alpha \beta (\lambda \gamma) - 1} \right] \] ... (15)

where \( \omega_s^2 = \xi_0 \omega_d^2 + \eta_0 \omega^2 + \zeta_0 \omega^2 \) and \( \psi_0^2 = \xi_1 \omega_d^2 + \eta_1 \omega^2 + \zeta_1 \omega^2 \) are given by the deformation potential constant respectively; \( m_o \), the free electron mass; \( \rho \), the density of the material of the semiconductor; \( c_L \), the longitudinal acoustic velocity and \( \xi, \eta, \zeta \).

\[ \begin{vmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{vmatrix} \] ... (16)

Assuming \( m_1 \neq m_2 = m_3 \), \( \xi_0, \eta_0 \) and \( \zeta_0 \) are given by \( 1, (2m_1/m_3)(m_1/m_3 + 2) \) and \( (m_1/m_3)(m_1/m_3 + 2) \) respectively, and \( \xi_1, \eta_1, \zeta_1 \) are 1.31, 1.61 and 1.01 respectively for n-type germanium.

Now, for high electric fields the energy of the carrier may be assumed to be much higher than \( \omega_0 \) and \( B^\circ(\omega) \) may be simplified to

\[ B^\circ(\omega) = - \frac{1}{\tau_a} \frac{d}{d\omega} \left[ \omega^2 \left( \frac{d\omega}{d\omega} + \zeta_0 \right) \right] \]

\[ - \frac{1}{\tau_0} \frac{1}{\varepsilon \omega_{\text{co}}} \frac{\omega_0}{\omega} \left[ \omega_0 \left( \frac{\omega_0}{\omega^2} \right) + \frac{d\omega}{d\omega} + \zeta_0 \right] \] ... (17)
Substituting (11) in (8) and solving one obtains

\[
\frac{\dot{f}}{f_i} = \left( \frac{\varepsilon T}{m_e} \right) K T \ln \left[ \frac{\bar{F} + (\varepsilon T/m_e)(\bar{M}, \bar{F}) \times \bar{B} + (\varepsilon T/m_e)^2 |\bar{M}|(\bar{F}, \bar{B})(\bar{M}^{-1}, \bar{B})}{1 + (\varepsilon T/m_e)^2 |\bar{M}|(\bar{M}^{-1}, \bar{B}) \cdot \bar{B}} \right] \frac{df_o}{dW} \quad \cdots \quad (18)
\]

where \( \bar{M}^{-1} \) and \( |\bar{M}| \) are the inverse and determinant of \( \bar{M} \).

Substituting (18) and (17) in (7) one obtains

\[
\frac{d}{dW} \left[ W^2 \left( \frac{d^2 f_o}{dW^2} + \frac{df_o}{dW} \right) + \frac{T e}{c T} \frac{W^2 (e^{W_o} - 1)}{e^{W_o} - 1} \left( W \frac{d^2 f_o}{dW^2} + \frac{df_o}{dW} \right) + \frac{T e \cdot W_o}{c T} \left( W \frac{df_o}{dW} + f_o \right) \right] = \frac{2}{3} \frac{T e \cdot c \cdot e^2}{(K T) m_c} \left( \bar{F}, \bar{M}, \bar{F} \right) = \beta
\]

Since \( T e / c T \) is independent of \( W \) one may put

\[
\frac{T e}{c T} \frac{W^2 (e^{W_o} - 1)}{e^{W_o} - 1} = \beta
\]

\[
\left( \frac{T e}{c T} \right) W_o = \gamma
\]

So, we get

\[
\frac{d}{dW} \left[ W^2 \left( \frac{d^2 f_o}{dW^2} + \frac{df_o}{dW} \right) + \beta \left( W \frac{d^2 f_o}{dW^2} + \frac{df_o}{dW} \right) + \gamma \left( W \frac{df_o}{dW} + f_o \right) \right]
\]

\[
= - W^{-\frac{1}{2}} \frac{d}{dW} \left[ W^{3/2} \beta \left\{ \frac{1 + (\varepsilon T/m_e)^2 |\bar{M}|(\bar{F}, \bar{B}) \cdot \bar{B}}{1 + (\varepsilon T/m_e)^2 |\bar{M}|(\bar{M}^{-1}, \bar{B}) \cdot \bar{B}} \right\} \frac{df_o}{dW} \right] \quad \cdots \quad (20)
\]
Neglecting $W$ compared to $p$ and solving one gets

$$f_0 = N_0 \exp \left[ - \left( W + r \right) \left\{ q + \frac{\left( 1 + \frac{e \xi}{m_e} \right)^2 |\vec{\mathbf{H}}| (\vec{\mathbf{r}} \cdot \vec{\mathbf{B}}) - |\vec{\mathbf{M}}| (\vec{\mathbf{M}}^{-1} \cdot \vec{\mathbf{B}}) \right\} \right]^{-1} \right] \ldots \ (21)$$

where $N_0$ is the normalisation constant given by

$$\frac{N}{n} = N_0 \int_0^\pi \int_0^{2\pi} \int_0^{\infty} f_0 K^2 \sin \theta \ d\theta \ d\phi \ dK$$

$$= 4 \pi N_0 \int_0^{\infty} f_0 K^2 \ dK$$

$N/n$ being the concentration of the carriers in the valley (assumed to be $n$ in number) under consideration, $N$ being the total carrier density.

In obtaining Eqs. (18) and (21) it has been assumed that (a) the scattering is isotropic and there is equipartition of energy of the phonons, (b) the energy of a carrier is higher than the optical phonon energy and (c) the field is high enough to make $W/p$ negligible compared to unity.

It may be noted here that intervalley scattering has been neglected in the present theory. The intervalley scattering affects the results in two ways. Firstly the momentum and the energy relaxation times are altered. For not too large fields the change in the energy relaxation time is not very significant in $n$-type germanium. The modification of the momentum relaxation time may be expressed as an alteration in $\Omega$ as given by

$$\Omega = 1 + \left( \frac{\tau_{\Omega\omega}}{\tau_0} \right) \left[ \frac{\exp (\theta_0 / \tau) + 1}{\exp (\theta_0 / \tau) - 1} + \frac{3 \left( \frac{D_e}{D_0} \right)^2 (\theta_0 / \exp (\theta_0 / \tau) + 1)}{\theta_0 / \exp (\theta_0 / \tau) - 1} \right] \ldots \ (23)$$
where $D_i$ and $\Theta_i$ are the intervalley phonon deformation potential constant and intervalley phonon characteristic temperature, $\frac{\hbar \omega_i}{K}$. The second effect of intervalley scattering is repopulation of the carriers among the different valleys. In the absence of the magnetic field one may calculate the changed concentration for near-saturation region using the following equation

$$n/N = \left(\frac{r}{R}\right)^{\eta_2} \sum \left(\frac{r}{R}\right)^{\eta_2}$$

... (24)

where $\eta_i = \eta + q_i$ and $\sum$ denotes the summation over all the valleys. For lower fields, the carrier population may be obtained from the analysis of Paige. The repopulation of the carriers in the presence of the magnetic field is rather difficult to determine. However, an estimate of the effect may be obtained assuming $n/N$ is primarily determined by the effective temperature in the different valleys. This has been discussed in details in section (4.4.1).

2.3. Magnetoresistance and Hall mobility:

In a many valley semiconductor with ellipsoidal constant energy surfaces the general expression for the current density $\vec{J}_v$ due to a particular valley in any direction of the chosen right-handed rectangular co-ordinate system may be written as

$$\vec{J}_v = \left(e/\hbar\right) \int f(\nabla_{\vec{r}} \vec{E}) \, d\vec{r}$$

... (25)

To find the total current density $\vec{J}$ in any direction one has to sum the contributions over all the valleys.

The electric field \(\vec{E}\) in the sample consists of the applied field $E_x$, (the electric field is assumed to be applied in the $x$-direction and the
magnetic field in the \( z \)-direction) the Hall field and the Sasaki field (the field produced in the plane perpendicular to \( x \)-direction due to Sasaki effect). The latter two fields may be represented by a field \( F_y \) in the \( y \)-direction and a field \( F_z \) in the \( z \)-direction. It should be noted that \( F_y \) is the resultant field produced by both Hall field and Sasaki effect, whereas \( F_z \) is due to Sasaki effect only. The current density, \( J_x \) in the direction of the applied field is obtained from Eq. (25) by substituting the expression for \( f \) and eliminating \( F_y \) and \( F_z \). \( F_y \) and \( F_z \) are eliminated by using the condition that the currents in \( y \)- and \( z \)-directions are separately equal to zero. The magnetoresistance \( R_m \) may then be obtained from the relation

\[
R_m = \frac{J_x - J_{x,B}}{J_{x,B}} \quad \ldots \quad (26)
\]

where \( J_x \) and \( J_{x,B} \) denote the current densities in the direction of the applied field respectively in the absence and the presence of the magnetic field.

The Hall mobility is given by

\[
\mu_H = \frac{F_y}{F_x \psi_x} \quad \ldots \quad (27)
\]

when it is assumed that the Sasaki field is negligible in comparison to the Hall field.

2.4. Discussion:

The general theory of hot-carrier conduction in the presence of a steady magnetic field has been presented in this chapter. But in order to assess the relative contributions of the acoustic phonon scattering, optical phonon scattering, intervalley scattering and the effect of the many valley band structure, exact formulation of the expressions for the galvanomagnetic...
constants has been made in the following two chapters for the following assumptions:

(a) The constant energy surface is spherical and only acoustic phonon scattering occurs;

(b) The constant energy surface is spherical and both acoustic and optical phonon scattering occurs;

(c) The constant energy surfaces are ellipsoidal and only acoustic phonon scattering occurs and

(d) the constant energy surfaces are ellipsoidal and the scattering is by acoustic, optical and intervalley phonons.

It is, of course, true that any agreement between theory and experiments for n-type germanium or silicon at a temperature of 300°K could be expected from the assumption (d). However the result obtained from (b) gives an indication of the expected characteristics for p-type germanium and silicon and for some particular directions of n-type germanium and silicon. The results obtained from (a) and (c) may be of some importance in considering the characteristics for low temperatures.