4.1. Introduction:

In the previous chapter we have proposed testable designs of EMC networks for detecting stuck-at and bridging faults with universal tests. The EMC expansions are based on fixed polarity of input variables. Pradhan [P-1] pointed out that for certain functions there can be significant saving in logic if mixed polarity of variables is allowed. These networks are called ESP (EXOR sum-of-products) networks. These networks can also be tested by universal tests for stuck-at faults. Pradhan has derived for such networks function independent test set (universal test set) for multiple stuck-at faults [P-1]. The present work in this chapter shows that these ESP realizations of switching functions are also circuits of choice for detecting any arbitrary bridging fault with universal tests. It has been shown in this chapter that the function independent multiple stuck-at fault test set derived by Pradhan is not sufficient for detecting possible bridging faults in the network. However, this insufficiency can be overcome after making certain augmentation of the network which is proposed in this chapter.

4.2. Unate and nonunate ESP networks:

Let $P_1, P_2, ..., P_r$ represent a set of product terms over a set of $n$ variables and their complements, i.e., $x_1, x_2, ..., x_n, \overline{x}_1, \overline{x}_2, ..., \overline{x}_n$.

We shall now give the following relevant definitions from Pradhan [P-1].
Definition 4.1: An expansion \( P_1 \oplus P_2 \oplus \ldots \oplus P_r \) will be referred to as an EXOR sum-of-products (ESP) expansion if both versions of the variables are allowed to appear in the expansion.

Definition 4.2: A two-dimensional AND-EXOR array realizing an ESP expansion with arbitrary sharing of logic between vertical cascades is said to be an ESP network.

Definition 4.3: An ESP network is said to be an unate ESP network if for each variable \( x_i \) two primary input connections are available one for \( x_i \) and the other for \( \overline{x_i} \).

Definition 4.4: An ESP network is said to be a nominate ESP network if the complements of the primary inputs are obtained internally by using inverters.

Definitions 4.5: The order of an ESP expansion \( F \) is the maximum of the number of literals contained in any product term in the expansion and is denoted as \( \text{O}(F) \).

In the actual realization of the logic network, we deviate from Pradhan in that we do not allow sharing of logic between vertical cascades and the AND gates in our realization do not have necessarily the same number of inputs. Also, we are interested in nominate ESP network because it is more practical from the point of view of implementation. Like Pradhan, instead of using NOT gates for obtaining \( \overline{x_i} \) from \( x_i \) we use a control variable \( C \) and \( n \)-two-way EXOR gates to get the complemented variables. One of the two inputs to each of the EXOR gates is \( C \) and the other input is \( x_i \in \{ x_1, x_2, \ldots, x_n \} \). Thus the output of \( i \)-th such EXOR gate is,

\[ x_i^* = x_i \oplus C \quad \text{for} \quad 1 \leq i \leq n \]

where, \( x_i^* = x_i \) for \( C = 0 \) and \( = \overline{x_i} \) for \( C = 1 \).
One such realization is shown in Fig. 4.1 for a function given by, \( F_q = 1 \oplus x_2 \oplus x_3 \oplus \bar{x}_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5 \). Note that \( C_0 \) is the constant term in the expansion which in this example is equal to 1. The corresponding complement free RMO expansion of the function is \( 1 \oplus x_1 \oplus x_2 \oplus x_3 \oplus x_4 \oplus x_5 \oplus x_6 \oplus x_7 \). It is clear from the two expansions that significant saving in logic is obtained in case of EDP realization of the function. Pradhan [P-1] has given the following function independent multiple stuck-at fault detection test set \( T \) in such network. It is given by,

\[
T = T_1 \cup T_2 \cup T_3 ,
\]

where \( T_1 = \begin{bmatrix}
C & x_1 & x_2 & \cdots & x_n \\
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 1 & \cdots & 1 \\
1 & 1 & 0 & \cdots & 0 \\
1 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 0 & 0 & \cdots & 1 \\
0 & 0 & 1 & \cdots & 1 \\
0 & 1 & 0 & \cdots & 1 \\
0 & 1 & 1 & \cdots & 0 \\
0 & 1 & 1 & \cdots & 0 \\
\end{bmatrix}
\]

\( T_2 = \begin{bmatrix}
C & x_1 & x_2 & \cdots & x_n \\
0 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 1 \\
0 & 1 & 1 & \cdots & 1 \\
\end{bmatrix}
\)

with \( C = 0 \)

\( T_3 = \{ I_n | \text{wt}(I_n) \leq j \} \) i.e. set of \( n \)-vectors of weight \( j \) or less, \( j \) being the order of the function realized and weight of the binary \( n \)-vector \( I_n \), \( \text{wt}(I_n) \) is the number of 1's in the binary vector. \( T_3 \) is applied with \( C = 1 \) and \( C_0 \) equal to the value of the constant term in the expansion.

The cardinality of \( T \) is

\[
|T| = |T_1| + |T_2| + |T_3| = (2n+2) + 4 + \sum_{e=0}^{j} \binom{n}{e} = 2n + 6 + \sum_{e=0}^{j} \binom{n}{e}
\]
The test set due to Pradhan for his proposed realization of ESP expansion however, is sufficient to detect all bridging faults in the network considered in this chapter. We are not giving any detail of how the tests in \( T \) detect different stuck faults in the network. It is given in Pradhan's work [P-1].

4.3. Bridging faults in nominate ESP network:

Before going to the detection problem of bridging faults in nominate ESP networks, we first present some auxiliary results in the following subsection.

4.3.1. Some properties of ESP expansions of switching functions:

Lemma 3.1 and lemma 3.2 of the previous chapter help in finding out some important properties of ESP expansions. These are the following:

**Property 1**: In case of ESP expansions another relation \( R_3 \) may exist between two product terms \( P_i \) and \( P_j \) in addition to relations \( R_1 \) and \( R_2 \) of RMC expansions. It is given by

\[
R_3 : P_i \cap P_j = \emptyset \text{ (null)}
\]

**Property 2**: In case of ESP expansions, whenever either of the two relations \( R_1 \) and \( R_2 \) exists between two product terms \( P_i \) and \( P_j \), then it is always possible to find some control literal/literals either in \( P_i \) or in \( P_j \) or in both.

In Fig.4.2 a general model of a nominate ESP network realizing an \( n \)-variable function \( \mathbb{F}_0 \) is shown. The network is considered to be a three level network. The three levels are designated by \( L_0 \), \( L_1 \) and \( L_2 \) as shown in the figure. We now state the fault model we consider in such networks.
4.3.2. Fault model:

1) Only intralevel bridging faults will be dealt with. These are mainly:
   a) Bridging faults involving only two lines in the $L_2$-level. This type of bridging faults also includes bridging between two inputs of a collector row EXOR gate.
   
   b) (i) Intragate bridging faults in $L_1$-level,
   (ii) Intergate bridging faults in $L_1$-level,
   (iii) Some of the lines involved in the bridging are lines that are directly connected to the collector row EXOR gates without passing through any AND gate in $L_1$-level and the rest ones are input lines of some AND gate/gates.
   
   c) Single and multiple bridging faults involving the primary input stem lines or their fanout branch lines in the $L_0$-level. This type of bridging also includes bridging between two inputs of an EXOR gate used to get a complemented variable.

2) All single and multiple stuck-at faults are considered.

3) Feedback bridging faults are not considered.

Without any loss of generality we first assume OR-bridging faults in the network in the following section.

4.3.3. OR-bridging faults in nonunate ESP network:

We consider each intralevel bridging separately in the following way.

4.3.3.1. Bridging fault in $L_2$-level:

We classify the different cases of bridging faults in $L_2$-level in the following subclasses.
Class A: Consider first the case of bridging between two lines h and m such that h and m are both outputs of two AND gates realizing the product terms \( P_h \) and \( P_m \). Now, we know \( P_h \) and \( P_m \) are related to each other by any of the three conditions \( R_1, R_2 \) and \( R_3 \). We explain here what happens if \( P_h \) and \( P_m \) are related by \( R_1 \), or by \( R_2 \), or by \( R_3 \) in the following way.

Case 1: Relation \( R_1 \) holds good: i.e., either \( P_h \supseteq P_m \) or \( P_h \subseteq P_m \).

We choose arbitrarily one of the two conditions, say \( P_m \supseteq P_h \). We give proper values to all the literals in \( P_m \) to make \( P_m = 1 \), i.e. the complemented literals are set at 0 and the uncomplemented ones at 1 with \( C = 1 \) and since \( P_m \supseteq P_h \), so \( P_h \) has at least one control literal, say \( x_1 \) which may be complemented or uncomplemented. If \( x_1 \) appears in uncomplemented form, then it is set at 0 and all other literals in the binary \( n \)-vector \( I_n \) are set at 0. On the other hand, if \( x_1 \) appears in complemented form, it is then set at 1. That is, if \( x_1 = x_1 \) then the test vector sets \( x_1 = 0 \) and if \( x_1 = \bar{x}_1 \), then the test vector sets \( x_1 = 1 \).

In both the cases, we \( (I_n) \leq 0 \) \( (P_o) \), so the test \( I_n \in T_3 \), provided \( C_o \) equals to the value of the constant term in the ESP expansion. Under the application of this test vector \( I_n \), \( P_m = 1 \) and \( P_h = 0 \) in absence of the fault and both \( P_m \) and \( P_h \) becomes 1 in presence of the fault, changing the network output from 0 to 1 or 1 to 0, depending on function realized.

Case 2: Relation \( R_2 \) holds good: i.e. \( P_h \nsubseteq P_m \), \( P_h \not\supseteq P_m \), \( P_h \cap P_m \neq \emptyset \)

In this case also, each of \( P_h \) and \( P_m \) has at least one control literal in it that is not present in the other. Let \( N_h = \) number of literals in \( P_h \) and \( N_m = \) number of literals in \( P_m \). Now, if \( N_h = 0 \) \( (P_o) \) and \( N_m < 0 \) \( (P_o) \), or vice-versa, then we choose \( P_m \), if \( N_m < 0 \) \( (P_o) \) and set all the literals in \( P_m \) to proper values to make \( P_m = 1 \) and the control literal in \( P_h \) is set at 0 if it
appears uncomplemented, otherwise it is set at 1, if it appears in complemented form in \( P_h \) and all other literals in the binary n-vector are set at 0. In either case, \( \text{wt}(I_n) \leq 0 (P_o) \). Hence \( I_n \in T_3 \) under the same constraint as regards \( C \) and \( C_o \) mentioned in the previous paragraph. Next if both \( N_h, N_m \leq 0 (P_o) \), then we choose arbitrarily \( P_h \) or \( P_m \), say \( P_m \) and set \( P_m = 1 \) by proper assignment of values of literals in \( P_m \) and proceed as above.

Now, suppose \( N_h = N_m = 0 (P_o) \). If both \( P_h \) and \( P_m \) have no complemented literal in them, we choose arbitrarily \( P_h \) or \( P_m \) and proceed as above. If on the other hand, at least one of them has some literal in complemented form, then we start with that product term, say \( P_h \) and make \( P_h = 1 \) and the control literal in the other, say \( P_m \) is set at proper value so that \( P_m = 0 \) and proceed as above.

The selection of \( P_m \) instead of \( P_h \) is easily seen to be justifiable. In either case the binary n-vector \( I_n \), which is the test for this bridging fault is included in \( T_3 \).

**Case 3:** Relation \( R_3 \) holds good: i.e., \( P_h \cap P_m = \emptyset \) (null).

We choose any of \( P_h \) and \( P_m \), say \( P_h \). It is set at 1 and all other \( x_k \)'s in \( I_n \) are set at 0 with \( C = 1 \). Under the application of \( I_n \), \( P_h = 1 \), and \( P_m = 0 \) in absence of the fault and both \( P_h = P_m = 1 \) in presence of the fault, making a change in the logic value at the network output.

**Class B:** Here, we consider bridging between two lines \( h \) and \( m \), so that \( f(h) = x_h \), where \( x_h = x_h \) or \( \bar{x}_h \), \( f(m) = P_m \), a product term and \( f(h) \) is the line function at line \( h \). The line \( h \) may be some fanout branch line. Here we consider the following cases.

**Case 1:** \( P_m \) does not contain \( x_h \) or \( \bar{x}_h \).
We set \( f(h) = 1 \), by setting \( x_\wedge = 1 \) if \( f(h) = x_\wedge \) with \( G = 1 \) and \( x_\wedge = 0 \) if \( f(h) = \overline{x}_\wedge \) (\( C = 1 \)) and some control literal \( x_i^* \) in \( P_m \) is set at 1 if it appears complemented in \( P_m \), otherwise \( x_i^* \) is set at 1 and all other literals in \( I_n \) are set at 0. This input vector \( I_n \) detects the presence of the fault as in the earlier cases. Clearly \( I_n \in T_2 \), if we set \( C = 1 \).

**Case 2**: \( x_n \) appears in both \( P_m \) and \( f(h) \) in the same polarity.

We set \( f(x_n^*) = 1 \) and proceed as in the previous case.

**Case 3**: \( x_n \) appears in \( P_m \) and \( f(h) \) in opposite polarity.

We proceed similarly as in Case 1 of Class B.

**Class C**: In this case, we assume \( f(h) = x_n^* \) and \( f(m) = x_m^* \), where \( x_i^* = x_i \) or \( x_i^* = \overline{x_i} \).

We consider the following cases.

**Case 1**: \( x_n^* \) and \( x_m^* \) appear only once in the ESP expansion.

Then we set \( f(h) = 1 \) and \( f(m) = 0 \), or vice-versa and all other literals in the binary \( n \)-vector \( I_n \) are set at 0. Clearly \( I_n \in T_2 \).

**Case 2**: One of the lines of \( h \) and \( m \), or both are fanout branches.

In this case, there is no guarantee that \( T \) detects this bridging fault, because the detection depends on how many times \( x_n^* \) and/or \( x_m^* \) appear in the ESP expansion and in what way they appear in the expansion. We will later show that by adding two extra AND gates, this yet undetectable bridging fault will be detected at either of the extra gate outputs which will be assumed to be observable during testing.

**Class D**: This type of bridging faults deals with the bridging between two inputs of a collector row XOR gate. Let the affected gate be \( G_i \) and its two inputs be \( h \) and \( k \). Tests from the sets \( T_2 \) and \( T_3 \) detect any stuck-type fault.
in such gate \[ P^{-1} \]. In other words, all the four combinations of values on lines h and k can be obtained by the application of some tests in \( T_2 \cup T_3 \). Under one such test vector, if \( h = 0 \) and \( k = 1 \), or vice-versa, the fault is detected at the primary output with a change in logic value.

If one of the inputs, say \( h \), is the control input \( C_o \), then one test vector is always found in \( T_2 \cup T_3 \) that fixes \( C_o \) at 0 and \( k \) at 1. We do not allow any propagation of the effect of the bridging fault through the line \( k \) irrespective of whether \( k \) is a fanout branch or not, because if \( k \) is a fanout branch line then one cannot be sure of the detection of the bridging fault under the application of a test vector \( t \) that fixes \( k = 0 \) and \( C_o = 1 \). Then the detection by \( t \) depends upon the number of fanout branch lines of the corresponding stem line of line \( k \).

4.3.3.2. Bridging fault in \( L_1 \)-level (Intragate and Intergate bridging):

The following different types of bridging faults we consider here.

(i) Intragate bridging:

Here we consider the following cases.

Case 1: None of the involved lines is a fanout branch.

Say, the involved lines are \( h, k, \ldots, l \) and the corresponding line functions are \( f(h) = x_h^*, f(k) = x_k^*, \ldots, f(l) = x_l^* \), where \( x_k^* = x_k \) or \( x_k \). Let us assume that the product term realized by the related AND gate in absence of the fault is \( P \).

Now, if all of \( x_h^*, x_k^*, \ldots, x_l^* \) are uncomplemented, then the test \( t \) sets one of them at 0 and the rest are set at 1 with \( C = 1 \) and all other literals that appear in uncomplemented form in \( P \) are set at 1 and that appear in complemented form are set at 0. The other input variables in the binary
n-vector $t$ are set at 0. Clearly, $t \in T$. In some cases, the test can be a member of $T'$ depending on $?$. Similar conclusion can be drawn if all or some of $x_1^*$, $x_2^*$, ..., $x_n^*$ are in complemented form.

**Case 2:** Some or all of the involved lines are fanout branches.

In such cases no guarantee can be given about the detection of this bridging fault by $T$. The tests may depend on the function realized. The reason is similar to Case 2 of Class C in case of $L_2$-level bridging. However, we will show that the augmentation of the network as mentioned earlier will enable $T$ to detect such undetectable bridging fault.

(ii) **Intergate bridging:**

In case of bridging involving inputs to different AND gates, again $T$ may not be able to detect it. The reason is similar as in the previous case. Here also the augmented network as stated earlier will enable $T$ to detect such undetectable intragate bridging. We will show it later.

**4.3.3.3. Bridging fault in $L_0$-level:**

Tests for the detection of bridging faults, single or multiple involving lines in $L_0$-level may be function dependent. The reason is similar as stated in Case 2 of Class C type bridging. However we will show now that the augmentation of the network, stated earlier will make $T$ sufficient to detect the undetectable bridging faults in $L_0$-level at the observation points. Also bridging involving the two inputs of an EXOR gate used to get complemented variable may not be detectable by $T$ at the primary output. The reason is same as before because in such cases of bridging one of the involved inputs is always $C$ which is a fanout branch line. The other input may also be a fanout branch line.
4.3.3.4. Detection of all single and multiple stuck-at faults:

It has already been shown [P-1] that the test set $T$ detects all single and multiple stuck-at faults at the network primary output of the ESP network.

4.3.3.5. A testable design:

In Fig. 4.3 a testable design for the network is shown. The augmented network contains two more AND gates, $A_1$ and $A_2$. The respective output points $O_1$ and $O_2$ are observed during testing. In general, the output functions at $O_1$ and $O_2$ are respectively $(x_1 x_2 \ldots x_n)$ and $(x_1 x_2 \ldots x_n)$. Let us now show that the different cases of bridging faults that remain undetected in the unsupplemented network by the application of tests in $T$ will be detected either at the observation point $O_1$ or $O_2$ by the same test set $T$.

Consider first any bridging fault $f_b$, single or multiple in the $L_0$-level. To find a test, one of the involved lines is set at 0 by setting the corresponding input variable at 0 and all other input variables are set at 1. The values of $C$ and $C_0$ may be anything. In absence of $f_b$, the logic value at $O_2$ is 0. Due to the bridging (we have assumed OR-type) the line set at 0 previously will have now a logic value of 1, thus changing the logic value of $O_2$ from 0 to 1. Clearly if we set $C = 0$, then $T_1$ includes the test. Hence $T$ includes it.

Now let bridging occur between the two input lines of an EXOR gate used to get a complemented variable. Clearly one of the input lines of the gate is $C$. If an input vector $t$ is applied with $C = 0$ and all $x_i$'s = 1 in the binary $n$-vector $t$, then the output at $O_1$ in absence of the bridging fault is 1 and in its presence becomes 0 because output of each such gate becomes 0 due to the bridging. Thus $t$ detects the fault. Here $t \in T_1$.

It is clear that if such bridging occurs in more than one EXOR gate simultaneously then also the above test $t$ detects the bridging fault at $O_1$. 

Next, we consider different intragate and intergate bridging faults in \( L_1 \)-level undetectable by \( T \). Let the involved lines be \( (h, k, \ldots, l) \) and the corresponding line functions be \( f(h) = x_h, f(k) = x_k, \ldots, f(l) = x_l \) respectively, i.e. no line function is a complemented variable. Then as in the previous case, test can be found similarly, that is included in \( T_1 \) and hence in \( T \).

Now, let the involved lines \( (h, k, \ldots, l) \) be such that all of them have complemented variables as their line functions. Then an input vector \( t \) that sets one of the lines, say \( h \) at 0 and all other input lines to gate at 1 is a test. This can be done very easily by setting \( x_h = 0 \), where \( f(h) = x_h \) and all other \( x_i \)'s in the binary \( n \)-vector set at 1 with \( c = 0 \). The fault is detected at 0 with a change in value from 0 to 1. Clearly \( t \in T_1 \) and hence \( t \in T \).

Next, let the involved lines be \( h \) and \( k \) such that \( f(h) = x_h \) and \( f(k) = \bar{x}_h \). Then we set \( x_h = 1, c = 1 \) and all other \( x_i \)'s = 0. Under the application of this input vector, the output at 0 is 0 in absence of the bridging fault and it becomes 1 in presence of the bridging fault. This input vector also is included in \( T_1 \).

There is another case in which the involved lines \( (h, k, \ldots, r, s, \ldots, l) \) are such that some of them, say \( (h, k, \ldots, r) \) have their line functions as \( \bar{x}_h, \bar{x}_k, \ldots, \bar{x}_r \) and the rest, i.e. \( (s, \ldots, l) \) have \( x_s, \ldots, x_l \), but no two lines are involved in the bridging of which the line functions are complementary to each other. Then a test vector belonging to \( T_1 \) is able to detect it at 0. It sets one of \( x_s, \ldots, x_l \) at 0 and all other variables in the binary \( n \)-vector at 1, with \( c = 0 \). The fault is detected at 0 with a change in value from 0 to 1.

Now, we suppose that at least two lines \( h \) and \( s \) with complementary functions \( f(h) \) and \( f(s) \) are involved in the bridging in the last case. Let...
f(h) = x_h and f(s) = x_h. Then the test vector 't' sets f(h) = 0 by making \( x_h = 0, \quad C = 0 \) and all other \( x_j \)'s in the binary n-vector at 1. The fault is detected either at 0_1 or at 0_2 with a change in value from 0 to 1.

The above discussion takes care of all intragate and intergate bridging faults in \( L_1 \)-level as well as the third type of bridging faults considered in the fault model in this level. We do not explain the case of bridging faults undetected by T as stated in case 2 of Class C type bridging in \( L_2 \)-level, because it has already been considered implicitly while we have explained the cases with intragate and intergate bridging.

We state the result so far obtained the detection of OR-bridging in the following theorem.

**Theorem 4.1** A nominate ESP network realizing a switching function \( F \) of \( n \)-variables can be so augmented by adding two extra AND gates that the universal test set \( T \) of cardinality \( 2n + 6 + \sum_{e=0}^{j} (\binom{n}{e}) \) is sufficient to detect the different intralevel OR-bridging faults and all single and multiple stuck-at faults in the network.

Note that any bridging fault single or multiple involving the input lines of the testing AND gates is always detectable either at 0_1 or at 0_2 by the application of tests in T. It is clear from the previous discussion of testable design. Also all single stuck-at faults of the added AND gates are detected at the observable outputs 0_1 and 0_2 by the same test set T.

**4.3.4. AND-bridging faults in nominate ESP network:**

We assume that the network shown in Fig.4.2 has been realized using positive logic. In this network T is insufficient to detect all bridging faults of our fault model at network output point. The reason is same as explained in
the case of OR-bridging. We will now show that if we augment the network by adding two extra OR-gates in the same way as shown in Fig. 4.3 where two extra AND gates have been added in case of OR-bridging and if we assume that the two outputs of the added gates are observed during testing, then the test set $T$ together with some more function independent tests will be sufficient for the detection of any yet undetectable bridging fault of the fault model at either of the observable output points $0'_1$ and $0'_2$.

The testable design is shown in Fig. 4.4. The extra OR-gates are OR$_1$ and OR$_2$ respectively with output points $0'_1$ and $0'_2$. In general the output function at $0'_1$ is ($\bar{x}_1 + x_2 + \ldots + \bar{x}_n$) and at $0'_2$ is ($x_1 + x_2 + \ldots + x_n$).

We will now see that together with the test set $T$, the following two function independent test sets $T_4$ and $T_5$, each of cardinality $n$ are required. They are given by

$$T_4 = \begin{bmatrix} C & x_1 & x_2 & \ldots & x_n \\ 1 & 0 & 1 & \ldots & 1 \\ 1 & 1 & 0 & \ldots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \ldots & 0 \end{bmatrix} \quad \text{and} \quad T_5 = \begin{bmatrix} C & x_1 & x_2 & \ldots & x_n \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}$$

We now consider the different intralevel bridging faults and show their detectability by tests in $T^* = (T \cup T_4 \cup T_5)$ in the augmented network.

Any intralevel bridging fault in $L_0$-level is always detected at the output $0'_2$ like the case of OR-bridging and any bridging involving the two inputs of an EXOR gate used to obtain a complemented variable is always detected at $0'_1$ by the test $t$ belonging to the set $T$, which sets $C = 1$ and all $x_i's = 0$. Under the application of this test, the logic value at the control line $C$ of the affected gate will be zero and this effect of the bridging fault will propagate
to the control inputs of all other such gates. As a result the output at $0_1'$ changes from 1 to 0, thereby ensuring the presence of the fault. Problems arise when we consider intragate and intergate bridging in the $L_1$-level because such bridging faults may involve lines that are fanout branches and in these cases tests for their detection at the network output may be function dependent. We now show the necessity of the two test sets $T_4$ and $T_5$ together with the set $T$ to detect these faults at the observation points $0_1'$ and $0_2'$. However, $T_4$ and $T_5$ are not able to detect these faults at the network output if these faults require function dependent tests for detection at the network output.

Let us consider now bridging faults in the $L_1$-level.

Let the involved lines be $(h, k, ..., l)$ and the corresponding line functions are $x_h, x_k, ..., x_l$. These lines may be fanout branches. There exists a test belonging to $T_1$ as in the case of OR-bridging that detects the fault at $0_2'$. Next, let the involved lines $(h, k, ..., l)$ be such that the corresponding line functions are $x_h, \bar{x}_k, ..., \bar{x}_l$. To test the fault, it is required to set one of the lines of $(h, k, ..., l)$ at 1 and all other input lines of the OR gate at 0. Suppose the line $h$ is set at 1. Then either of the following two tests is necessary, i.e. $C = 1, x_h = 0$ and all $x_i's = 1$ for $1 \leq i \leq n, i \neq h$ or $C = 0, x_h = 1$ and all $x_i's = 0$ for $1 \leq i \leq n, i \neq h$. The first one belongs to the set $T_4$ while the second one to set $T_5$. Thus we see that the set $T$ fails to detect the fault even at the observation points $0_1'$ and $0_2'$.

Now, let only two lines $h$ and $k$ be bridged such that $f(h) = \bar{x}_h$ and $f(k) = x_k$, then the test sets $T$ and $T_4$ are seen to fail to detect the fault $f(h/k)$ if the following network structure is assumed. Let all the variables $(x_1, ..., x_n)$, where $n > 2$, appear in both polarity in the ESP expansion of a function. Then in the testable design the OR gate is an $n$-input gate. To detect the fault, it is
necessary to make either $\overline{x}_h = 1$ and $x_k = 0$, or vice-versa. But only this assignment of logic values to the lines $h$ and $k$ is not sufficient for an input vector to be a test. We now explain this. Say, $\overline{x}_h = 1$ and $x_k = 0$ under some input vector. Due to the bridging $f(h)$ becomes zero, i.e. the effect of the fault propagates to the output of $OR_1$ gate and not to that of $OR_2$ gate. So it is seen that under such assignment of logic values the fault can only be detected at the output of $OR_1$ gate if and only if all other inputs to this gate are desensitized, i.e. are set at zero by this input vector. But no such input combination exists either in $T$ or in $T'_4$. Similar is the case if $\overline{x}_h = 0$ and $x_k = 1$ under some input combination belonging to $(T \cup T'_4)$. Here the fault will be detected at $0'_2$ because the faulty logic value of line $k$ will be zero, i.e. the effect of the fault propagates to the output $0'_2$ and not to $0'_1$. The test should be such that it sets $\overline{x}_h = 0$ with $C = 0$ to make $\overline{x}_h = 0$. Otherwise if it sets $x_h = 1$ with $C = 1$ to make $\overline{x}_h = 0$, then as $x_h$ appears as the line function of some input line (not involved in the bridging) of the $n$-input $OR_2$ gate, this assignment always sets the logic value of output line of $OR_2$ gate at 1 irrespective of whether the fault is present or not, meaning thereby it jeopardises the detection. However, the function independent test set $T'_5$ is seen to be sufficient to detect such faults. Consider an input vector $t$ from $T'_5$, in which $C = 0$, $x_h = 0$, $x_k = 1$ and all other $x_i$'s in the binary $n$-vector = 0. Under the application of $t$, the fault free output at $0'_2$ is 1 and the faulty output becomes zero. Thus $t$ is a test.

Let us now consider another case of bridging in which the involved lines $(h, k, \ldots, r, s, \ldots, l)$ are such that some of them, say $(h, k, \ldots, r)$, have their line functions as $\overline{x}_h$, $\overline{x}_k$, $\ldots$, $\overline{x}_r$ and the rest, i.e. $(s, \ldots, l)$ have $x_s$, $\ldots$, $x_l$ and also no two lines are involved in the bridging of which the line functions are complementary to each other. In such case of bridging also $T$ may
fail to detect even in the augmented network. Reason behind this failure of $T$ is clear from the previous discussion. However, tests from $T_5$ detect this type of bridging faults. If the fault is to be detected at the output $0'_1$ then a test from the set $T_5$ is chosen that sets one of the lines $(h, k, \ldots, r)$, say $h$, at 1 by making $x^*_h = 1$ with $C = 0$ and all other $x^*_i$'s $= 0$ for $1 \leq i \leq n$, $i \neq h$. The fault free output at $0'_1$ will be 1 and the faulty output will be zero. On the other hand to detect the fault at the output $0'_2$ a test from the set $T_5$ is chosen, which sets one of the lines $(s, \ldots, l)$, say $s$, at 1 by making $x^*_s = 1$ and all other $x^*_i$'s $= 0$ for $1 \leq i \leq n$, $i \neq s$ with $C = 0$. The logic value at $0'_2$ will change from 1 to 0, thereby ensuring the presence of the fault. Also if we consider that in the above mentioned bridging at least two lines are involved with opposite functional values then also $T$ may fail under the same reason discussed in the last paragraph. However, $T$ is able to detect this fault in a similar way as in the case of the bridging fault $(h/k)$ where $f(h) = \overline{x^*_h}$ and $f(k) = x^*_h$.

The above discussion takes care of all intragate and intergate bridging faults in $L_1$-level as well as the third type of bridging faults considered in the fault model in this level. Also it takes care of any bridging involving two lines (one or both may be fanout branches) that are inputs to two collector row EXOR gates with functional values $x^*_1$ and $x^*_3$, where $x^*_k = x^*_k$ or $\overline{x^*_k}$. The other cases of bridging faults in the $L_2$-level considered in our fault model can be shown similarly as in the case of OR-bridging to be detectable by $T$ at the network output.

We state the result so far obtained in case of AND-bridging faults in the following theorem.
Theorem 4.2: A nominate ESP network realizing a switching function $P_q$ of $n$-variables can be so augmented by adding two extra OR-gates that the function independent test set $T^*$ of cardinality $4n + 6 + \sum_{i=0}^{d} \binom{r}{i}$ is sufficient to detect the different intralevel AND-bridging faults and all single and multiple stuck-at faults in the network.

Corollary 4.1: Any AND-bridging fault in the $L_1$-level, that does not change the fault free function at network primary output 0 is always detected at one of the observable outputs $0_1$ and $0_2$ by a test $t^*$, where $t \in T^*$, provided all of the lines involved in the bridging fault have not the same line function.

Proof: It has been shown in theorem 4.2 that any bridging fault in the $L_1$-level irrespective of whether its effect changes the fault free function at network primary output is always detectable at some of the observable output points in the augmented network, provided of course all of the involved lines have not the same line function. Hence the corollary follows. Q.E.D.

Here note that any bridging fault involving the inputs of the testing OR-gates as well as all single stuck-at faults of these gates are always detectable by $T^*$ at some of the observation points. This is clear from the previous discussion of testable design.

As a consequence of corollary 4.1 questions may arise about the necessity of detecting undetectable bridging faults because in presence of such faults the network operates properly. But the presence of such undetectable bridging faults may invalidate the valid tests for detecting some detectable stuck-at faults at network primary output $[K-1]$ when simultaneously bridging and stuck-at faults occur. We give an example of such a situation. Consider the network shown in Fig.4.5. The fault free function at network primary output 0 is:

$$F_0 = x_1 x_2 x_3 \oplus \overline{x_1} \overline{x_2} x_3 \oplus \overline{x_1} \overline{x_2} \overline{x_4} \oplus \overline{x_3} x_4$$
We consider a bridging fault $f_b = \bar{h}/k$ as shown by the dotted line in the figure. It is seen that $f_b$ does not change the fault free function $F_0$. Now we assume that in presence of $f_b$, the line 1 has been stuck-at-1 (i.e. $l^1$). Due to the simultaneous occurrence of the bridging and the stuck-at fault, the faulty function at network primary output 0 remains same. Hence in presence of $f_b$, the otherwise detectable stuck-at fault $l^1$ has been masked. However this problem is totally circumvented since $f_b$ is detected at the observation point $O_1'$. 

4.4. Conditions for undetectability of bridging faults among primary input lines:

We now give the necessary and sufficient conditions for a bridging fault involving some primary input lines in a nominate ESP network to be undetectable by any test at the network primary output. This is illustrated in the following theorems.

**Theorem 4.5a**: Let $h$ and $m$ be two primary input lines connected to the literals $x_h$ and $x_m$ respectively in a nominate ESP network $N$ realizing a Boolean function $F_0$.

The ESP expansion of $F_0$ can always be factored as:

$$F_0 = A \oplus x_h x_m B \oplus x_h \overline{x}_m C \oplus \overline{x}_h \overline{x}_m D \oplus x_h x_m E$$

where the Boolean functions $A$, $B$, $C$, $D$ and $E$ are all independent of $x_h$ and $x_m$.

Then a bridging fault $f_b = \bar{h}/m$ is undetectable at the network primary output by any test if and only if

$$B = C = D$$

**Proof**: [Only if]: We have the fault free function at network primary output as:

$$F_0 = A \oplus x_h x_m B \oplus x_h \overline{x}_m C \oplus \overline{x}_h \overline{x}_m D \oplus x_h x_m E$$
We now consider the bridging fault $f_b$ as mentioned in the theorem.

Let

$$F_b \xrightarrow{f_b} F_f,$$

where the faulty function $F_f$ at network primary output is:

$$F_f = A \oplus \overline{x_h} \overline{x_m} D \oplus x_h x_m E$$

$$= A \oplus (\overline{x_h} \oplus \overline{x_m} \oplus \overline{x_h} \overline{x_m}) D \oplus x_h x_m E$$

For $f_b$ to be undetectable at the network primary output,

$$F_o = F_f$$

i.e. $A \oplus \overline{x_h} \overline{x_m} B \oplus x_h \overline{x_m} C \oplus \overline{x_h} \overline{x_m} D \oplus x_h x_m E$

$$= A \oplus (\overline{x_h} \oplus \overline{x_m} \oplus \overline{x_h} \overline{x_m}) D \oplus x_h x_m E$$

or, $\overline{x_h} x_m B \oplus x_h \overline{x_m} C = (\overline{x_h} x_m \oplus x_h \overline{x_m}) D$

or, $\overline{x_h} x_m (B \oplus D) = x_h \overline{x_m} (C \oplus D)$

... (1)

Since the functions $B$, $C$, and $D$ are all independent of $x_h$ and $x_m$, condition (1) is true if and only if

$$B \oplus D = C \oplus D = 0$$

which implies $B = C = D$, which is the necessary condition for $f_b$ to be undetectable at network primary output.

[If]:

Let the fault free function at network primary output be

$$F_o = A \oplus \overline{x_h} x_m B \oplus x_h \overline{x_m} B \oplus \overline{x_h} \overline{x_m} B \oplus x_h x_m E$$

where the Boolean functions $A$, $B$ and $E$ are independent of $x_h$ and $x_m$. 


Let the bridging fault be $f_b = (h/m)$ where the line functions of the primary input lines $h$ and $m$ are $x_h$ and $x_m$ respectively. Let

\[
F_f = \overline{A \oplus x_h \overline{x_m} B \oplus x_h x_m E}
\]

where the faulty function $F_f$ at network primary output is:

\[
= A \oplus (\overline{x_h} + \overline{x_m}) B \oplus x_h x_m E
= A \oplus (\overline{x_h} \oplus \overline{x_m} \oplus \overline{x_h} \overline{x_m}) B \oplus x_h x_m E
= A \oplus (\overline{x_h} \oplus \overline{x_m}) B \oplus \overline{x_h} \overline{x_m} B \oplus \overline{x_h} x_m E
= A \oplus \overline{x_h} x_m B \oplus x_h \overline{x_m} B \oplus \overline{x_h} \overline{x_m} B \oplus x_h x_m E
= F_f
\]

Hence $f_b$ is undetectable by any test at network primary output. This proves the sufficiency part of the theorem. \( \Box \).

**Theorem 4.3b**: Let $h$ and $m$ be two primary input lines connected to the literals $x_h$ and $x_m$ respectively in a nominate ESP network $N$ realizing a Boolean function $F_o$. The ESP expansion of $F_o$ can always be factored as:

\[
F_o = A \oplus \overline{x_h} x_m B \oplus x_h \overline{x_m} C \oplus \overline{x_h} \overline{x_m} D \oplus x_h x_m E,
\]

where the Boolean functions $A$, $B$, $C$, $D$ and $E$ are all independent of $x_h$ and $x_m$. Then a bridging fault $f_b = + (h/m)$ is undetectable at the network primary output by any test if and only if

\[
B = C = E
\]

**Proof**: [Only if]: We have the fault free function at network primary output as:

\[
F_o = A \oplus \overline{x_h} x_m B \oplus x_h \overline{x_m} C \oplus \overline{x_h} \overline{x_m} D \oplus x_h x_m E
\]
We now consider the bridging fault $f_b^*$ as mentioned in the theorem.

Let

$$F_b^* = F_f^*$$

where the faulty function $F_f^*$ at network primary output is:

$$F_f^* = A \oplus (x_h + x_m) D \oplus (x_h + x_m) E$$

$$= A \oplus \overline{x_h} \overline{x_m} D \oplus (x_h \oplus x_m \oplus x_h x_m) E$$

$$= A \oplus \overline{x_h} \overline{x_m} D \oplus x_h \overline{x_m} E \oplus \overline{x_h} x_m E \oplus x_h x_m E$$

For $f_b^*$ to be undetectable at network primary output,

$$F_o = F_f^*$$

i.e. $A \oplus \overline{x_h} x_m B \oplus x_h \overline{x_m} C \oplus \overline{x_h} \overline{x_m} D \oplus x_h x_m E$

$$= A \oplus \overline{x_h} \overline{x_m} D \oplus x_h \overline{x_m} E \oplus \overline{x_h} x_m E \oplus x_h x_m E$$

or, $\overline{x_h} x_m B \oplus x_h \overline{x_m} C = x_h \overline{x_m} E \oplus \overline{x_h} x_m E$

or, $\overline{x_h} x_m (B \oplus E) = x_h \overline{x_m} (C \oplus E)$

... (1)

Since the functions $B$, $C$ and $E$ are all independent of $x_h$ and $x_m$, condition (1) is true if and only if

$$(B \oplus E) = (C \oplus E) = 0$$

which implies $B = C = E$.

This is the necessary condition for $f_b^*$ to be undetectable at network primary output by any test.

[IF] : Let the fault free function at network primary output be

$$F_o = A \oplus \overline{x_h} x_m B \oplus x_h \overline{x_m} B \oplus \overline{x_h} \overline{x_m} D \oplus x_h x_m B.$$
Let the bridging fault be $f_b = *(h/m)$, where the line functions of the primary input lines $h$ and $m$ are $x_h$ and $x_m$ respectively.

Let

$$F_o \xrightarrow{f} F_f,$$

where the faulty function is:

$$F_f = A \oplus \left(x_h + x_m\right) D \oplus \left(x_h + x_m\right) B$$

$$= A \oplus x_h x_m D \oplus x_h x_m B$$

$$= A \oplus (x_h \oplus x_m) B \oplus x_h x_m D \oplus x_h x_m B$$

$$= A \oplus x_h x_m D \oplus x_h x_m B$$

Hence $f_b$ is undetectable at network primary output by any test. This proves the sufficiency part of the theorem. Q.E.D.

4.5. Detection of feedback bridging faults:

Although we have not included the occurrence of any feedback bridging fault in our fault model, still the testable designs proposed are capable of detecting a large number of such bridging faults. We are giving an example of one such fault that is detectable by $T$. Consider the situation shown in Fig.4.6.

Here part of the network is shown. Let us now consider a bridging fault $f_b = +(h/k)$ such that $f(h) = \overline{x_h}$ and $f(k) = x_h$ in the fault free condition.

Now, if we set $x_h = 0$ and $0 = 1$, $f(h)$ becomes 1 and $f(k)$ is 0 in absence of $f_b$. But in its presence, both $f(h) = 1$ and $f(k) = 1$, thereby meaning that the effect of the bridging fault propagates to the vertical input of the EXOR gate shown and in turn changes the value of $f(h)$ i.e. $f(h)$ becomes 0. This is purely a feedback bridging fault. But quite interestingly it is seen that if we set $x_h = 1$ and
C = 1, \( f(h) \) becomes 0 and \( f(k) = 1 \) in absence of \( f_b \) and in presence of \( f_b \), the value of \( f(k) \) does not change, thereby ensuring that the value at the vertical input to the EXOR gate shown remains constant at 1. We have actually stopped the propagation of the effect of the bridging fault through the fanout branch line \( k \) to the corresponding stem line. Thus due to the bridging fault, \( f(h) \) becomes 1, changing the value at the observation point \( O_1 \) if we set all other \( x_i \)’s = 0. Evidently this input vector belongs to the set \( T \).

In a similar way, we can show that a large number of feedback bridging faults can be detected by the function independent tests we have considered.

4.6. Conclusion:

In the present work, ESP network has been shown to be the circuit of choice because of the following points in its favour. These are mainly, (1) for certain functions logic cost becomes less compared to that in RMC networks, (2) methods for minimal realization exist [E-4], [E-6] and (3) such network possesses universal test set for stuck-at faults.

In this chapter we have not considered interlevel bridging faults. The reasons behind this consideration are same as those mentioned in case of RMC networks in the previous chapter.
Fig. 4.1. Network realizing $F_0 = 1 \oplus x_2 \oplus x_3 \oplus x_1 \bar{x}_2 \bar{x}_3 \oplus \bar{x}_4 \bar{x}_5$

Fig. 4.2. Nominate ESP network
Fig. 4.4. Testable realization in case of AND-bridging

Fig. 4.5. Testable realization in case of OR-bridging
Fig. 4.5. Simultaneous occurrence of bridging \((h/k)\) and stuck-at \((l^1)\) faults

\[
f_b = \overline{x_1} (h') \overline{x_2} (k) \overline{x_3} \overline{x_4}
\]

\(C = 1\)

\(C_0 = 0\)

\(O_1, O_2\)

\(F_o\)

Fig. 4.6. A typical feedback bridging fault \(f_b = + (h/k)\)