CHAPTER 7

FUZZY STOCHASTIC MODELS

7.1 Options
In this chapter we present a fuzzy stochastic differential equation model for option pricing. We shall refer to this as Model III and it is a SDE consisting of a drift term, diffusion term and a jump term. Moreover, we assume that the underlying stochastic process is fuzzy.

In Chapter 1, we saw that the strategy of hedging is used to minimize one’s loss rather than making profits. This is generally achieved by investing in two instruments simultaneously. A careful choice of the instruments must be done so that any adverse price movements in one will be override by the other. In practice, it is not possible to hedge perfectly. Options can be used for hedging. We have cited an example for this in Chapter 1.

Many new commodity traders start with option contracts. Unlike futures, option prices are based on the actual price of the underlying asset itself. Moreover, the main attraction with options is that one cannot lose an amount more than the initial investment. In this regard an investment in options trading seems to be safer than in futures.

Let \( S_0 \) be the price of the underlying asset at the beginning of the period of entering into a contract. Suppose we are able to get an estimate of the range of variation in the asset price after a period of \( t \) days is as the interval \([LL, UL]\). Now an investor can choose the most favourable strike price (depending on whether it is put or call) offered by the exchange.

7.2 A fuzzy stochastic differential equation model for option pricing (Model III):

We assume that \( X(t) \) where \( t \in [0,T] \) denotes the percentage change in the price of the underlying asset at time \( t \) and the SDE consists of a drift term, a diffusion term and a jump term.
We assume that $x$ follows the process

$$dx = m \, dt + s \, dz + h \, dP \quad (7.1)$$

Here $dz$ is a Wiener process and $P$ is the jump process. We assume that the jump events is a fuzzy process with an alpha level cut $[h^- , h^+]$. $h^+$ and $h^-$ are respectively the positive and negative parts of the jump events restricted to the domains $[\alpha , \delta]$ such that

$$h = \begin{cases} h^- , & [\alpha , \beta] \\ 0 , & [\beta , \gamma] \\ h^+ , & [\gamma , \delta] \end{cases}$$

Consider equation (7.1) in discrete form for $x \in [\gamma , \delta]$ as

$$\Delta x = m \Delta t + s \Delta z + h \Delta P \quad (7.2)$$

Let $f = f(x, t) + h(x, t)$ be the change in the value of the underlying asset at time $t$ satisfying

$$f = \begin{cases} f + h^- , & [\alpha , \beta] \\ f , & [\beta , \gamma] \\ f + h^+ , & [\gamma , \delta] \end{cases}$$

$f$ represents percentage change in the value of the asset at time $t$ under normal conditions over the range $[\beta , \gamma]$ while $f + h^-$ and $f + h^+$ denote the value in the case of extreme variations occurring in the range $[\alpha , \beta]$ and $[\gamma , \delta]$ respectively.

Ito’s lemma in discrete form is

$$\Delta f = \left( \frac{\partial f}{\partial x} m + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} s^2 \right) \Delta t + s \frac{\partial f}{\partial x} \Delta z + \delta_h f \, \Delta P \quad (7.3)$$

where $\delta_h f = f(x + h, t) - f(x, t)$. 
We follow the same idea as in the derivation of Black Scholes equation and construct a portfolio of value $\pi$ percentage as follows[12]:

-1 : derivative (long).

\[ \frac{\partial f}{\partial x} : \text{shares of an equivalent stock (short)}. \]

Then \[ \pi = - (f + h) + x \frac{\partial f}{\partial x} \]

so that the change in $\pi$ is given by,

\[ \Delta \pi = - \Delta f - \Delta h + \Delta x \frac{\partial f}{\partial x} \]

If the rate of interest is $r$ then at time $t$ the change in the value of the portfolio is $r \pi \Delta t$ so that

\[ - \Delta f - \Delta h + \Delta x \frac{\partial f}{\partial x} = r \pi \Delta t \quad (7.4) \]

$r$ is assumed constant during this period.

Using equations (7.2) and (7.3) in (7.4) and simplifying we find that

\[ ( - \frac{\partial f}{\partial t} - r x \frac{\partial f}{\partial x} - \frac{1}{2} \frac{\partial^2 f}{\partial x^2} s^2 ) \Delta t + ( \delta_h f + h \frac{\partial f}{\partial x} ) \Delta P - \Delta h = -r (f + h) \Delta t \]

Now, \[ \frac{\delta hf}{h} = \frac{f(x+h,t) - f(x,t)}{h} \approx \frac{\partial f}{\partial x} . \]

As $\Delta h$ is small, we ignore it and the above equation becomes

\[ \frac{\partial f}{\partial t} + r x \frac{\partial f}{\partial x} + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} s^2 = r (f + h) \text{ when } x \in [\gamma, \delta]. \]

A similar equation can be obtained for $x \in [\alpha, \beta]$. Also \( h = 0 \) in the interval \( [\beta, \gamma] \). Thus
\[
\frac{\partial f}{\partial t} + r x \frac{\partial f}{\partial x} + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} s^2 = r (f + h), \quad (t, x) \in [0,T] \times [a, b] \quad (7.5)
\]

**Solution of equation (7.5) by finite difference method**

We divide the region \([0, T] \times [a, b]\) by introducing mesh points \((t_n, x_i)\), where \(n = 0,1,2,..N-1\) and \(i = 0,1,2,..M-1\).

At each point \((t_n, x_i)\) the partial derivatives in equation (7.5) are replaced by the finite difference approximations,

\[
\frac{\partial f}{\partial x}(t_n, x_i) = \frac{f_{i+1}^n - f_i^n}{\Delta x}, \quad \frac{\partial f}{\partial t}(t_n, x_i) = \frac{f_{i+1}^n - f_i^n}{\Delta t}
\]

and \(\frac{\partial^2 f}{\partial x^2}(t_n, x_i) = \frac{f_{i+1}^n - 2f_i^n + f_{i-1}^n}{\Delta x^2}\).

Substituting in equation 7.5,

\[
\frac{f_{i+1}^n - f_i^n}{\Delta t} + r x_i \left(\frac{f_{i+1}^n - f_i^n}{\Delta x}\right) + \frac{1}{2} s^2 \left(\frac{f_{i+1}^n - 2f_i^n + f_{i-1}^n}{\Delta x^2}\right) = r (f_i^n + h_i^n)
\]

Rearranging the terms,

\[
\frac{1}{2} s^2 \frac{\Delta t}{\Delta x^2} f_{i+1}^n + \left(1 - \frac{r x_i}{\Delta x}\right) \frac{\Delta t}{\Delta x} f_i^n + \left(\frac{r x_i}{\Delta x} + \frac{1}{2} s^2 \frac{\Delta t}{\Delta x^2}\right) f_{i-1}^n = r \Delta t (f_i^n + h_i^n)
\]

where \(f_i^n = 0\) for \(i < 1\) and \(n > N\)

Let \(c = \frac{s^2}{2\Delta x^2}\), \(b = \frac{r x_i}{\Delta x} + \frac{s^2}{2\Delta x^2}\) and \(a = \frac{r x_i}{\Delta x} + \frac{s^2}{\Delta x^2}\). Then \(a = b + c\).

Then the system of equations takes the matrix form

\[
A f^{n+1} = B \quad (7.6)
\]
where A is the tri diagonal matrix of order M x M as obtained in model I. Also

\[
\begin{align*}
I^{n+1} &= \begin{pmatrix} f_0^{n+1} \\ f_1^{n+1} \\ \vdots \\ f_{N-1}^{n+1} \\
\end{pmatrix}, \\
B &= \begin{pmatrix} f_0^n + r\Delta t(f_0^n + h_0^n) \\ f_1^n + r\Delta t(f_1^n + h_1^n) \\ \vdots \\ f_{N-1}^n + r\Delta t(f_{N-1}^n + h_{N-1}^n) \\
\end{pmatrix}
\end{align*}
\]

The convergence of the finite difference scheme follows from the result for model I. The condition for uniqueness of the solution (diagonal dominance) is given by

\[ |1 - a\Delta t| \geq |b + c|\Delta t. \]

The quantity b should be non negative for the matrix A to have real eigen values. Otherwise Δx can be chosen such that b ≥ 0.

The condition for stability of the finite difference scheme is \( \Delta t \leq 1/a \), which is the same as for model I.

### 7.3 Application of the model

We apply the option pricing model referred to as model III to data from NSE for options traded on the product INR-US$ exchange (US dollar to Indian rupee exchange rate). All contacts on INR-US$ exchange rate traded at NSE during the period January 2013 to May 2013 and having expiry August 2013 are considered. For this data we calculate the arithmetic mean and standard deviation.

Let us assume that the function f in equation (7.5) is denotes the percentage changes in the daily closing price of the asset at NSE. Further we assume that the underlying stochastic process is fuzzy. So we treat f as a fuzzy quantity and represent it as the trapezoidal fuzzy number \((\alpha, \beta, \gamma, \delta)\). We consider a suitable alpha-cut of the fuzzy number \((\alpha, \beta, \gamma, \delta)\) and denote it by \([LL, UL]\). According to our assumption, the positive jump events occur in the interval \([\gamma, \delta]\). To discuss the strategy for investing in a call option we need to consider only the
interval \([\beta, \delta]\). This is because a call option will be exercised only when the asset price is more than the strike price. Thus a buyer of a call option waits for an increase in the asset price which would fetch more profit. A person who intends to enter into a call option will not be interested in a decrease in the asset price as this would reduce the returns. An idea on the maximum possible change in the asset price after a period of \(T\) days of entering into the contract will benefit such investors. The values we obtain for UL point to the highest possible variations during the period \(T\). Now we illustrate our Model III with an example and explain how we arrive at the quantity UL. Since we are going to estimate only the values of UL, it is sufficient to consider only the positive jump events. Thus we take into account the percentage changes in the asset price in the interval \([\beta, \gamma]\). This is the reason why there is only one jump term in the PDE. The fuzzy number representation of \(f\) allows us to replace the coefficient \(h\) of the jump term by the membership function namely \(\frac{\delta-x}{\delta-\gamma}\). Now we present the algorithm and the Matlab code developed by us to solve the system of equations (7.6).

**Algorithm**

**Step 1:**

Defining the parameters of the model

\(m, s\) - the arithmetic mean and standard deviation of INR –US $ exchange rate during the year 2012.

\(\beta, \gamma\) - the lower and upper limits respectively for the percentage changes under normal circumstances. These quantities are assigned values based on the test data.

\(\delta\) - the maximum permissible change in the case of extreme variations.

\(\text{delx}\) - the step size for \(x\).

\(T\) - the strike time.
Ux0 - the initial value of the variation in the closing price of the underlying asset.

r - the risk-free rate of interest allowed by the Reserve Bank of India.

N - the number of subdivisions of the interval [0, T].

M - the number of subdivisions of the interval [β, δ].

**Step 2:**

Compute the coefficient c of the matrix A and initialise vector UL.

**Step 3:**

Compute the main diagonal entries and the super diagonal entries of the coefficient matrix and form the tri diagonal matrix.

**Step 4:**

Form nested loops to calculate the jump at the different step values for x and t as

i = 1, 2, ..., M and t = 1, 2, 3, ..., N.

**Step 5:**

At the end of each time step compute the new value of the vector UL.

**Step 6:**

Display the value UL(M) at the end of the last time step.

The MATLAB code developed by us is given below:
C-7 Option pricing

```matlab
function f = option(m, s, beta, P3, delta, delx, T, r, Ux0, N, M)
c1 = (s^2)/(2*delx*delx);
for i = 1:M
    UIV(i) = Ux0;
end
M1 = M-1;
for i = 1:M
    for I = 1:M1
        Uc = c1*(UIV(i)^2);
        Ub = (r*UIV(i)/delx)+Uc;
        Ua = Ub+Uc;
        UD(i) = 1-(Ua*(T/N));
        UL(I) = Uc*(T/N);
        UU(I) = Ub*(T/N);
    end
end
UX = diag(UD) + diag(UL, -1) + diag(UU, 1);
UL = inv(UX) * UIV';
Uw = (Ua*(T/N))-1;
for n = 2:N
    UJ = zeros(M, 1);
    Uval = UL(:);
    for j = beta:P3
        for i = 1:M
            UJ(i) = UJ(i) + 0;
        end
    end
    for j = P3:delta
        for i = 1:M
            q = j+i;
            if (q < M)
                jump(i) = UJ(q) - UJ(i);
            else
                jump(i) = 0;
            end
            h = (delta-j)/(delta-P3);
            UJ(i) = UJ(i) + (h*jump(i));
        end
    end
    UL = inv(UX) * (UL + r*(UJ+UL)*(T/N));
end
f = UL(M);
end
```

We compute the values of $Ux_0$ as shown below.

The closing prices as on June 3rd and 4th are 57.49 and 57.1975 respectively. We take
We execute the code C-7 with the command as noted down here. The results obtained are in percentage changes with respect to the settlement price on the first day of the period of 90 days. So we convert them and obtain the likely variations. The estimated asset prices and the spot prices for a period of 90 days starting from June 4th 2013 until the expiry of the contract are compared.

The command is

```
>> option([5.172,2.4738,-3.6,9,0.5,90,6,-0.5111,90,24])
```

We obtain the answer as 4.7564

The initial price of the asset is 57.1975 being the closing price on June 4th 2013. The estimated value of the asset is then computed as follows:

\[ S_{30} = 57.1975 \times (1 + \frac{4.7564}{100}) = Rs \, 59.91804 \]

The comparison of the option value estimated from our model with that obtained from Black Scholes formula as well as the strategy that can be adopted by an investor are explained in the conclusion part.

### 7.2 Fuzzy version of Model I

In this section we discuss the fuzzy version of Model I by assuming that the process \( X(t) \) described there is a fuzzy process.

The fuzzy version of the model is the FSDE having alpha cuts as

\[ [X(t)]^\alpha = [X_0]^\alpha + \left[ \int_0^t f(s, X(s)) \, ds \right]^\alpha + \left[ \int_0^t g(s, X(s)) \, dB(s) \right]^\alpha + \left[ \int_0^t h(s, X(s)) \, dP \right]^\alpha \]
where the functions f, g and h have the same meaning as in model I. Since f and g are estimated from historical data we assume they are deterministic. Only the jump event h is assumed to be a fuzzy quantity. Let us write the alpha cuts of the fuzzy number h as $[h^-, h^+]$. Then the FSDE is equivalent to the set of crisp SDE’s

$$[X(t)]^a = x_0 + \int_0^t f(X(s), s)ds + \int_0^t g(X(s), s)dB(s) + [\int_0^t h^-(X(s), s)dP]^a \quad (7.7)$$

$$[\bar{X}(t)]^a = x_0 + \int_0^t f(X(s), s)ds + \int_0^t g(X(s), s)dB(s) + [\int_0^t h^+(X(s), s)dP]^a \quad (7.8)$$

Let the expected percentage variation in $X(t)$ at any time $t$ be a fuzzy quantity having alpha–cut $[\underline{V}, \overline{V}]$. For each fixed value of $\alpha \in [0, 1]$ the application of Kolmogorov backward formula to each equation above will give rise to two PIDE. Now we solve each of these equations in the same way as described in Chapter 5. Applying the method of finite differences we get two systems of linear equations in the form $AU = B_1$ and $A\overline{U} = B_2$ with the coefficient matrix A being the same as that obtained earlier and

$$B_1 = \begin{pmatrix} v_0^n + \Delta t (v_0^n + h_0^{-n}) \\ v_1^n + \Delta t (v_1^n + h_1^{-n}) \\ \vdots \\ v^n + \Delta t (v_{N-1}^n + h_{N-1}^{-n}) \end{pmatrix}, \quad B_2 = \begin{pmatrix} v_0^n + \Delta t (v_0^n + h_0^{+n}) \\ v_1^n + \Delta t (v_1^n + h_1^{+n}) \\ \vdots \\ v^n + \Delta t (v_{N-1}^n + h_{N-1}^{+n}) \end{pmatrix}$$

The solution of these equations namely the vectors $\underline{U}$ and $\overline{U}$ are the interval estimates at different time steps for the percentage variations in the commodity prices. Let us denote the solution as LL and UL respectively.

An algorithm similar to the one developed in Chapter 5 can be used with a minor modification for the terms $J_1$ and $J_2$. All the parameters appearing in C-8 have the same meaning as in C-1 except $Lx_0$ and $Ux_0$. The numbers $Lx_0$ and $Ux_0$ denote the low and high price of the contracts traded on the first day of the period of T days. For each commodity, all the details of date wise trading like the day’s low, day’s high,
number of contracts traded, settlement price etc. are obtained from the websites of the commodity exchanges. The probability density function arising from the jump term is replaced by the fuzzy membership function. We have modified the Matlab code C-1 suitably and get C-8.

C-8- Fuzzy model

```matlab
function f = fuzzyprice(m, s, Alpha, Beta, Gamma, Delta, delx, P1, P2, P3, N1, N2, N3, T, Lx0, Ux0, N, M)
c = (s^2)/(2*delx*delx);
for i=1:M
    LIV(i) = Lx0;  % initial lower and upper values%
    UIV(i) = Ux0;
end
NLV = LIV;
NUV = UIV;
M1 = M-1;
b = (m/delx) + c;
a = b + c
w = (T/N) - (1/(2*a))
for i = 1:M
    D(i) = 1 - a*T/N;
    L(I) = c*T/N;
    U(I) = b*T/N;
end
X = diag(D) + diag(L, -1) + diag(U, 1);
LL = (inv(X) * LIV');
UL = inv(X) * UIV';
lamda = P3 + (P2/(P1-1));
mu = -N3 + (N2/(N1-1));
for n = 2:N
    Lprice = LL(:);  % output (LL)%
    Uprice = UL(:);  % output (UL)%
    low = [Lprice(M)]
    high = [Uprice(M)]
    J1 = zeros(M, 1);
    J2 = zeros(M, 1);
    for k1 = P3:Delta
        for i = 2:M
            l1 = round(k1) + i;
            if (0 < l1 && l1 < M)
                jump1 = UL(l1) - UL(i);
                phiplus = (Delta-k1)/(Delta-Gamma);
                J1(i) = J1(i) + phiplus * jump1;
            else
                J1(i) = 0;
            end
        end
    end
    UL = inv(X) * (UL + lamda*(-J1)'*(T/N)*UL);
end
for k2 = Alpha:N3
```
\begin{verbatim}
for i=2:M
    l2=round(k2)+i;
    if ( 0 < l2 && l2 < M)
        jump2(i)=LL(l2)-LL(i);
        phimin=(k2-Alpha)/(Beta-Alpha);
        J2(i)=J2(i)+phimin*jump2(i);
    else
        J2(i)=0;
    end
end
end

LL=inv(X)*(LL+mu*J2'*(T/N)*LL);
\end{verbatim}

We execute the program with the following commands for two different periods of 30
days and 60 days, with the parameters having been computed from data (B). The
estimated values of UL represent the maxima of the likely changes in the price
variation of the commodity during the specified period starting from 2\textsuperscript{nd}
January 2013. We express the low and high prices as percentages of the settlement price in the
first day of the period T. Now the estimated values of LL and UL are compared with
the actual variations in the low and high prices on each day.

The comparison for 30 days can be seen in the graphical presentations 7.1a to 7.1b
(trial 1). The graph 8.1c and 8.1d indicate the comparisons of the actual variations
with the low and high price respectively for 60 days (trial 2).

**Trial 1**

\begin{verbatim}
>>Ufuzzyprice(0.032,0.161,-6,-0.5,1,6,1,-0.7138,0.563,3.46,0.3038,0.312,-3.46,30,-
0.134,0.14,30,12)
\end{verbatim}

**Trial 2**

\begin{verbatim}
>>Ufuzzyprice(0.032,0.227,-6,-0.5,1,6,2.5,-0.7138,0.563,3.46,0.3038,0.312,-3.46,60,-
0.134,0.14,30,10)
\end{verbatim}
Fig 7.1a
Comparison of daily values of “low” and fuzzy model values for gold futures contracts in the beginning of 2013 for 30 days.

Fig 7.1b
Comparison of daily values of “high” and fuzzy model values for gold futures contracts in the beginning of 2013 for 30 days.
Fig 7.1c
Comparison of daily values of “high” and fuzzy model values for gold futures contracts in the beginning of 2013 for 60 days.

Fig 7.1d
Comparison of daily values of “high” and fuzzy model values for gold futures contracts in the beginning of 2013 for 60 days.
7.3 Possible application of Model I to measure rainfall intensity

We discuss a possible application of our model I to determine the intensity of daily monsoon rainfall during a short period of time. In chapter 3 we have given an overview of SDE models in different fields. The extent of application of SDE models inspired us to make an attempt to apply Model I or II to an uncertain situation in a field different from financial engineering. We make an effort to apply model I to predict the rainfall intensity during the monsoon, employing historical rainfall data to determine the parameters of the model. Although this effort seems to be out of focus from the topic there is every possibility that the prediction of daily rainfall intensity can be used to construct a weather index. Developing weather indices would be crucial to trading of weather derivatives if introduced in trading exchanges in India.

The problem of determining the rainfall trend is challenging. Many models of different types like stochastic models, statistical models and meteorological models have been developed by researchers all over the world.

In rainfall process, scaling of space is classified as follows:

(i) Synoptic perturbation – it is a meteorological disturbance controlled by large scale atmospheric circulation (25000 sq km to 2,50,000 sq km).

(ii) Large mesoscale – aggregates of small mesoscale structures controlled by the synoptic perturbation (2000 to 5000 sq km).

(iii) Small mesoscale – aggregates of convective cells controlled by large mesoscale perturbations (100 to 500 sq km).

(iv) Microscale – convective cells with short duration and high intensity, during storms (3 to 10 sq km).

A major contribution in space-time rainfall modelling is the theoretical contribution of Gupta and Waymire[35]. They employed empirical observations of tropical cyclonic storms and formulated a physically based stochastic representation of the rainfall field in space and time. Their formulation was based on the empirical evidence that precipitation patterns have definable characteristics and behaviour. We give a short description of the model.
7.3.1 The Waymire – Gupta Model

Waymire and Gupta developed a multi dimensional precipitation model to represent mesoscale precipitation[35]. This model uses many observed features of rainfall at that scale and represents rainfall in a hierarchical approach with rain cells embedded in rain bands. The occurrences of rain cells and rain bands are assumed to be random, i.i.d in space-time co ordinates with a common probability density function. They also gave a modified form of the model which is described below.

If \( \psi(t,x) \) is the ground level rainfall intensity in a region \( R \) then \( \psi \) satisfies the SDE

\[
\frac{\partial \psi}{\partial t} + \bar{v} \cdot \nabla \psi + \alpha \psi = Z(t, \bar{v}) ,
\]

where \( \bar{v} \) is a uniform and steady drift velocity vector and

\[
Z(t,x) = \int_R g_2(x-y)X(t,y)dy.
\]

They assumed the functions \( g_1 \) and \( g_2 \) in the form

\[
g_1(t) = \exp(-\alpha t), \quad t \geq 0 \quad \text{and} \quad g_2(t) = \exp(-t^2/2D^2),
\]

where \( \alpha \) is the mean life time of the rain cells and \( D^2 \) is the spatial extent of the rain cells. The function \( \psi \) is of the form

\[
\Psi(t,x) = \int_{-\infty}^{t} g_1(t-s)Z(s,x-\bar{v}(t-s))ds.
\]

7.3.2 Convective rain cells:

According to meteorologists, rain cells are intense rainfall structures with spatial dimensions of about 5–10 km that appear to be embedded in regions of more widespread rainfall. A rain cell may be a localized region where precipitation is formed as a result of transport of moisture due to convection.
Convection is a circulation occurring in a fluid at a non-uniform temperature owing to variations in density with temperature and action of gravity. There is transfer of heat by this circulation. Convection plays a dominant role in the tropics particularly for the development and maintenance of Indian Summer Monsoon (ISM). It affects the tropical circulation through the release of latent heat and vertical transport of the heat. A convection cell can be noticed in the formation of clouds. Tropical rain cells consist of convective clouds or thunderstorms (also called cumulonimbus clouds) which are the source of heavy rainfall and lightning in the equatorial regions[21]. The average life of rain cells have been observed by researchers through images recorded via satellites. We have used one such observed value of rain cell life from a research article by Tsutomu Takahashi[33].

7.3.3 Indian Summer Monsoon Rainfall (ISMR)

Several researchers have studied the developments in the cloud systems over the Bay of Bengal, Indian Ocean and the Arabian Sea and have made observations on the onset and advance of the Indian summer monsoon (south west monsoon). The research articles have been compiled and published as two volumes of Monsoon Monograph by the Indian Meteorological Department (IMD)[17].

Some of the studies published in the Monsoon Monographs point out that Indian rainfall shows peaks in a two to three year period range[17]. Years of heavy rainfall tend to be followed by years of diminished rainfall. Many studies have found an increase in extreme rainfall events and in the recent past, several rainfall models have been developed. (Nichols, Rajeevan & Sen Roy et al.)[18], [25],[26].

We shall now attempt to apply our Model I to estimate the variations in daily rainfall intensity after the onset of Indian summer monsoon. Negative jump events are likely to occur only in the case of scanty rains. Hence we can assume that during a given period of time (after the onset of monsoon) either only positive jumps or only negative jumps occur. Let us make a supposition that only positive jump events occur.
7.3.4 Application of Model I to estimate rainfall intensity

All the notations in this application of Model I have the same meaning as in the code C-1. Since there are no negative jumps the sum $J_1$ is equal to zero.

We now illustrate the use of model I with the drift coefficient as the mean life of a tropical rain cell and the diffusion coefficient as equal to one. To obtain the other parameters of the model, we consider the data sets comprising of the departure from normal of all Indian summer monsoon rainfall (June – September) amount for the period 1960-2012. The data is available in the website of IMD. We sort out those values classified as very high rainfall years and test the goodness of fit for these data. We find that the two parameter Weibull distribution is a good fit. The parameters of this distribution are obtained as explained in Chapter 6. Analysis of rain cell life has been conducted by Tsutomu Takahashi and their conclusion is that the mean cell life of tropical convective rain cells may be a maximum of 20 minutes[33]. This is denoted by $\alpha$ in the Matlab code.

The probability density function of a two parameter Weibull distribution is given by:

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\frac{x}{\beta}\right)^{\alpha}, \ x \geq 0$$

where $\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter.

We use the Matlab code developed for model I with suitable changes. The modified code is C-8 with the parameters defined as:

alpha – the mean life of tropical rain cells.
[a1, d1] - the range chosen by us for daily intensity (in % departure from normal) based upon past data.
P1x, P2x – the estimated parameters of the Weibull distribution.
gammax - extreme rainfall intensity as defined by IMD.
The other quantities in the input statement have the usual meaning.
C-9 Extreme events as Weibull distribution

function f =dep(alpha,a1,d1,gammax,delx,Plx,P2x,T,x0,N,M)
K11plus=round((gammax/delx)+0.5);
K21plus=round((d1/delx)-0.5);
c1=1/(2*delx*delx);
delt=T/N;
M1=M-1;
for i=1:M
    IV(i)=x0;%initial value x0=change in the first week of season%
end
NV=IV(:,);
for n=2:N
    J1=zeros(M,1);
    dep=NV(:,);
    f=dep;
b1=(exp(-(1/alpha)*(n-1)*delt)/delx)+c1;
a=b1+c1;
w=(T/N)-(2/a); % stability condition%
    for i=1:M
        for I=1:M1
            D(i)=1-(a*n*T/N);
            L(I)=c1*n*T/N;
            U(I)=b1*n*T/N;
        end
    end
    A=diag(D)+diag(L,-1)+diag(U,1);
    NV= inv(A)*IV';
    lamdax=gamma(round(1/Plx)+1);%meanofWeibulldist,gamma function%
    for i=2:M
        for kx=K11plus:K21plus
            lx=i+kx-2;
            qx=gammax+kx;
            if ( 0 < lx && lx < M ) %phix is weibull density%
                phix=(Plx/P2x)*((qx/P2x)^(Plx-1))*exp(-(qx/P2x)^Plx);
                J1(i)=J1(i)+phix*(NV(lx)-NV(i));
            end
        end
    end
    NV=inv(A)*(NV+(T/N)*lamdax*(-J1')*NV);
end

Now we execute the code with the command given below and the output represents
the estimated percentage departure of the daily rainfall intensity after the onset of
monsoon 2014. The numerical values obtained at different time steps are converted
from percentage changes to actual values and compared graphically with the actual
weekly rainfall during the period June 2014 to July 2014.

>> deponedim(20,20,320,160,20,5.768,33.6,90,89.9,18,15)
Table 7.1
Comparison of the observed rainfall and estimated values from model I

<table>
<thead>
<tr>
<th>observed % departure</th>
<th>Observed (in mm)</th>
<th>Estimated (in mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>91.3384</td>
<td>125.4355</td>
</tr>
<tr>
<td>2.8</td>
<td>92.4172</td>
<td>80.9992</td>
</tr>
<tr>
<td>3.3</td>
<td>92.8667</td>
<td>91.8126</td>
</tr>
<tr>
<td>-5.1</td>
<td>85.3151</td>
<td>89.5196</td>
</tr>
<tr>
<td>0</td>
<td>89.9</td>
<td>89.973</td>
</tr>
<tr>
<td>17</td>
<td>105.183</td>
<td>89.8837</td>
</tr>
<tr>
<td>17.3</td>
<td>105.4527</td>
<td>89.9126</td>
</tr>
<tr>
<td>-0.6</td>
<td>89.3606</td>
<td>89.8655</td>
</tr>
<tr>
<td>-3.8</td>
<td>86.4838</td>
<td>90.0114</td>
</tr>
<tr>
<td>26.6</td>
<td>113.8134</td>
<td>89.542</td>
</tr>
<tr>
<td>-29.5</td>
<td>63.3795</td>
<td>91.0252</td>
</tr>
<tr>
<td>-30.3</td>
<td>62.6603</td>
<td>86.4743</td>
</tr>
<tr>
<td>-36.9</td>
<td>56.7269</td>
<td>99.8475</td>
</tr>
<tr>
<td>-25.3</td>
<td>67.1553</td>
<td>63.1799</td>
</tr>
<tr>
<td>17.6</td>
<td>105.7224</td>
<td>151.4697</td>
</tr>
</tbody>
</table>

Fig 7.1e
Graphical comparison of the values tabulated above.