Chapter-6

Incompletely Modulated PSK Signal Reception using Modified Balanced Optical Phase-Locked Loop

6.1 Introduction

In the optical domain, coherent detection has several fundamental advantages over direct detection. Coherent receivers, operating in shot-noise-limited conditions, can approach the theoretical sensitivity limit of 9 photons/bit (for a bit-error rate (BER) of $10^{-9}$), while direct detection receivers usually suffer from considerable power penalties referred to the quantum limit of direct detection of 10 photons/bit [1]. Also coherent detection offers the use of angular modulation schemes and subcarrier multiplexing techniques [2]. It has been theoretically shown that, in coherent optical detection, an inherent 3-dB power penalty exists for heterodyne receivers compared to homodyne receivers [3]. For this reason, homodyne reception is the technology of choice for an optical transmission system with maximum sensitivity. Homodyning requires an OPLL, for which several designs are known. The most common are the Balanced loop [4], the Costa’s or Decision-driven loop [5] and the Dither loop [6]. In the balanced loop architecture, part of the transmitted power must be used for unmodulated carrier transmission. At the receiver, the LO locks in quadrature to the residual-carrier, and therefore, in phase to the data signal. This architecture has the advantage of suppressing the excess intensity noise of the lasers used but imposes the most stringent requirements on the laser linewidth, mainly due to the data-to-phase-lock crosstalk. Its implementation, on the other hand, is simpler that of the Decision Driven-OPLL.

In the performance analysis of balanced OPLL given in the famous work of [4], the laser linewidth requirement (for keeping the laser linewidth-induced power penalty within 1 dB) was evaluated by assuming a fixed power penalty induced by imperfect phase recovery (i.e. 0.5 dB). An experimental balanced OPLL was demonstrated using two 1320-nm diode-laser-pumped neodymium-doped yttrium aluminum garnet
(Nd:YAG) lasers at 140 Mb/s and 2 Gb/s [7]. Using that OPLL, receiver sensitivities of 25 photons/bit at 140 Mb/s over 28.6-km of SMF and 332 photons/bit at 2 Gb/s were achieved. In another experiment, a 4 Gb/s pilot-carrier homodyne system using external cavity semiconductor lasers and a balanced PIN/HEMT transimpedance receiver achieved a sensitivity of 72 photons/bit [8]. The data-to-phase-lock crosstalk can be greatly reduced if the phase-error signal is slightly delayed so that the decision circuit output can be subtracted from it before it is input to the PLL filter [9]. Also a balanced OPLL based PSK receiver for long-haul communication was reported and laser linewidth requirement was evaluated [10]. The power penalty induced by imperfect phase recovery in PSK homodyne communication systems with balanced OPLL receivers were exactly evaluated by considering the optimum phase deviation between bit ‘1’ and bit ‘0’ [11]. A new sub-carrier OPLL was analyzed and experimentally characterized [12]. BPSK pilot carrier receiver experimentation was presented and a solution for its intrinsic pattern-dependency performance, based on 8B/10B coding, was proposed. Thus, several reports examine the balanced OPLL based PSK receiver, however, the linewidth of laser was narrow and loop propagation delay was not considered. In these works, there were no general conditions for linewidth requirement with nonzero loop propagation delay. Narrow linewidth laser sources are very expensive, and also they consume more power and are larger in size than DFB lasers. DFB lasers also have the advantage of potential optoelectronic integration. Due to the broad linewidth of the optical sources, there will be large fluctuation of the beat-frequency during tracking. As a consequence the loop may become unlocked occasionally resulting in extra phase jitter to the generated reference carrier. This means that not only the pull-in limit should be as large as possible but also pull-in time (both phase and frequency) should be also as small as possible. Moreover, tracking error should also be small. This, in turn, requires larger value of the loop natural frequency. The phase shift due to the transit time of the error signal round the loop becomes non-negligible and the loop-delay effect becomes significant. Loop-delay is known to add instability to the system and put a restriction on the maximum realizable value of loop natural frequency [13]. Loop-delay also induces a phase-delay at the output of the local oscillator. This can be minimized by advancing the phase of local oscillator
through an external control which can be done by controlling the phase of the VCO through phase modulator [14].

In this chapter, a incompletely modulated PSK signal reception system that uses a homodyne modified balanced optical phase-locked loop, which contains all the components of a standard balanced OPLL in conjunction of an additional electro-optic phase modulator in the phase-locking branch, is analyzed taking into account shot noise, white frequency induced phase noise, data-to-phase-lock crosstalk in the presence of loop-delay. The degradation of loop stability due to loop-delay is evaluated in terms of the delay bandwidth product \( f_o \tau \). The loop natural frequency \( f_n \) is optimized for minimum phase-error variance, and the corresponding optimum loop performances are evaluated. Bit error rate is calculated for the proposed receiver in the presence of loop-delay. The imperfect-phase-recovery-induced power penalty is evaluated with the optimum phase deviations in the presence of non-negligible loop propagation delay. The optimum phase deviation is chosen in such a way that, depending on the laser linewidth with the bit rate being the parameter, the required received power to achieve the given BER value (typically \( 10^{-10} \)) is minimized. The maximum permissible laser linewidth requirement is investigated and the lock-in range of the receiver is calculated in the presence of the loop-delay using a simplified analysis.

### 6.2 System overview

The modified balanced optical phase-locked loop based PSK signal reception system is shown in Fig.6.1. This homodyne optical reception system contains all the components of a standard balanced OPLL in conjunction of an additional electro-optic phase modulator [15], [16]. The received and the phase modulated optical signals are combined by a 3-dB directional coupler, and the resulting optical signal is converted in the electrical domain by two balanced photodiodes. The diodes are interconnected so that, the signal difference between their photocurrents, drives the following transimpedance amplifier. The transimpedance amplifier provides impedance matching. The balanced detector front end reduces loss by using both branches of the coupler, and it provides LO
intensity noise suppression [17]. The electrical signal at the amplifier output is then processed by a standard first-order active filter and finally sent to the VCO laser input. The electro-optic phase modulator is then used to modulate the phase of the optical signal at the VCO laser output. When OPLL is locked, the receiver tracks the incoming signal frequency and phase. At the photodiode output, the received signal is shifted to baseband, so optical homodyne receiver operations are obtained. In the following analysis we assume that the optical PLL is locked.

![Block diagram](image)

**Fig. 6.1** A block diagram of a homodyne reception system based on a modified balanced optical phase-locked loop.

### 6.3 Mathematical model of the reception system

We write the electric fields of the incoming signal light and the phase modulated local oscillator light, respectively, as

\[
E_R = \sqrt{P_R} \exp\left(j\phi_R(t)\right) \quad (6.1)
\]

\[
E_{LO} = \sqrt{P_{LO}} \exp\left(j\phi_{LO}(t)\right) \quad (6.2)
\]

where \(P_R\) and \(P_{LO}\) denote the power, and \(\phi_R\) and \(\phi_{LO}\) the phase of the complex envelope. Assuming a balanced photodetector, it is easily shown that the photodetector output is given by

\[
V_0(t) = -K_{PD} \cos\phi_E(t) + n(t). \quad (6.3)
\]
The expressions 
\[ \phi_E(t) = \phi_R(t) - \phi_{LO}(t) \]  \hspace{1cm} (6.4) 
\[ K_{PD} = 2RR_T \sqrt{P_R P_{LO}} \]  \hspace{1cm} (6.5)
are the total phase-error and phase detector gain of the modified OPLL, respectively. Consider a standard first order active loop filter with transfer function \( F(s) = \frac{(1+s\tau_2)}{s\tau_1} \), where \( \tau_1 \) and \( \tau_2 \) are filter time constants. If its output is \( V_f(t) \), then it is related to its input \( V_0(t) \), by the following equation
\[ \tau_1 \frac{dV_f(t)}{dt} = \tau_2 \frac{dV_0(t)}{dt} + V_0(t). \]  \hspace{1cm} (6.6)
The received optical phase can be expressed as
\[ \phi_R(t) = \frac{\pi}{2} + \psi d(t) + \phi_{nr}(t) \]  \hspace{1cm} (6.7)
where \( \psi \) corresponds to the phase deviation between two different bits \( d(t) = 1 \) and \( d(t) = -1 \). The first expression in (6.7), \( \frac{\pi}{2} \), describes the quadrature phase-lock behavior of the PLL, i.e., in the locked condition, a 90° phase difference exists between the two lasers [4].
The phase modulated local oscillator optical signal phase can be written as 
\[ \phi_{LO}(t) = \phi_{VCO}(t) + \phi_{PM}(t) + \phi_{nLO}(t) \]  \hspace{1cm} (6.8)
where \( \phi_{VCO}(t) \) and \( \phi_{PM}(t) \) are given by equations (3.8) and (3.9), respectively.
Substituting (6.7) and (6.8) into (6.5), we obtain the output signal of the receiver
\[ V_0(t) = K_{PD} \sin \psi \cos \phi_E(t) d(t) + K_{PD} \cos \psi \sin \phi_E(t) + n(t) \]
\[ = A_d d(t) + A_{PL} \sin \phi_E(t) + n(t) \]  \hspace{1cm} (6.9)
where \( \phi_E(t) = \phi_{nr}(t) - \phi_{nLO}(t) - \phi_{VCO}(t) - \phi_{PM}(t) \) is the total phase-error.
The expressions
\[ A_d = K_{PD} \sin \psi \cos \phi_E(t) \]  \hspace{1cm} (6.10)
\[ A_{PL} = K_{PD} \cos \psi \]  \hspace{1cm} (6.11)
are the amplitudes of the data and of the phase-error signal, respectively. Now, we assume that the loop remains lock with a very small phase-error, i.e.
\[ \phi_E(t) \ll 1. \]  \hspace{1cm} (6.12)

Then equation (6.9) can be linearized using approximation \( \sin x \approx x \)

\[ V_0(t) = A_d d(t) + A_{PL} \phi_E(t) + n(t). \]  \hspace{1cm} (6.13)

Taken together, (6.6), (3.8), (3.9) and (6.13) describes a linearized model of a modified balanced OPLL receiver with delay shown in Fig. 6.2.

\[ H(f) = \frac{K F(j2\pi f) \exp(-j2\pi f \tau)}{j2\pi f + K(1 + j2\pi f \tau_p) F(j2\pi f) \exp(-j2\pi f \tau)}. \]  \hspace{1cm} (6.14)

For modified OPLL based incompletely modulated PSK signal reception system, the closed-loop transfer function will be the same except that \( K = K_{pd} K_{VCO} \) is to be replaced by

\[ K = K_{pd} K_{VCO} \cos \psi. \]  \hspace{1cm} (6.15)
6.4 Function of the phase modulator

In case of incomplete phase modulation, a residual-carrier is available with the transmitted signal, which may contain data noise. The task of the modified balanced OPLL is to demodulate the data while removing as much of the noise as possible. The balanced OPLL achieves this in the following manner. A phase-locked loop is basically an integral control system. The error-signal \( V_0(t) \) is averaged over some length of time and then used to control the phase of the laser VCO while carrying out averaging the loop eliminates noise to a large extent and retains the information signal for the purpose of synchronization. The more faithful the loop is in following the phase variation of the input signal (i.e. lower phase-error \( \phi_e(t) \)) the wider the loop bandwidth and naturally less is the noise rejection. The addition of the electro-optic phase modulator at the output of the laser VCO improves the tracking capability of the loop by causing the phase-error \( \phi_e(t) \) to decrease. As a result the effective bandwidth of the loop can be thought to increase, when the output is taken from the phase modulator. However, if the output is taken from the VCO, the equivalent phase-error \( \gamma(t) \) can be much greater than \( \phi_e(t) \). Consequently the effective bandwidth of the loop becomes much narrower than the case when the output is taken from the phase modulator. This improves the noise squelching property of the modified loop. This explains why the use of the electro-optic phase modulator in the loop can generate much better data signal.

6.5 Stability of the system

In this section, we shall investigate the influence of the loop propagation delay on the stability of the modified OPLL based PSK reception system. The limits on the loop stability can be established using standard Nyquist techniques [18]. The loop stability conditions have been derived in chapter 4 and are given by

\[
\left( \frac{1}{x} \right)^4 \left( 1 + 4x^2\xi^2 \right) \leq \left[ \frac{1}{1 + p^2x^2} \right] \quad (6.16)
\]

\[
d \leq \left( \tan^{-1}(2\xi x) + \tan^{-1}(px) \right)/x \quad (6.17)
\]
where the symbols are as defined in chapter-4 (sub-section 4.4.2). Using equations (6.16), (6.17), and (6.15) we can calculate the limit of loop-delay for stable operation of the modified reception system for a given $p$, a given $\xi$ and a given $\psi$.

6.6 Performance analysis

In this section, the performance of the modified reception system regarding the phase-error variance, bit error rate and power penalty will be investigated in presence of loop propagation delay.

6.6.1 Calculation of phase-error variance

In this sub-section, we now evaluate the total phase-error variance of the modified OPLL based reception system in presence of loop-delay. Expression (6.13) shows that the output signal of the receiver contains the shot noise $n(t)$, and two signals — the data signal $A_d d(t)$ and the phase-error signal $A_{pl} \sin \phi_e(t)$. Thus, the receiver performance is affected by three major noise interferences: phase noise, shot noise, and cross-talk between the data and phase-lock branches of the receiver. Since the noise processes are statistically independent and model derived in section 6.3 is linear, the total phase-error variance of the system can be expressed as [4], [13]

$$\sigma_k^2 = \sigma_{PN}^2 + \sigma_{SN}^2 + \sigma_{PL}^2$$

$$= \int_{0}^{\infty} S_{PN}(f) |1 - H(f)|^2 df + \left(1/A_{pl}^2\right) \int_{0}^{\infty} S_{SN}(f) |H(f)|^2 df + \left(1/A_{pl}^2\right) \int_{0}^{\infty} S_{D}(f) |H(f)|^2 df$$

(6.18)

The quantities on the right hand side of (6.18), respectively, denote the variances due to laser phase noise, shot noise and data-to-phase-lock cross-talk. $H(f)$ is given by equation (6.14). $S_{PN}(f)$, single-sided PSD of the white frequency induced phase noise is given by (3.10). In (6.18), $S_{SN}(f)$ ($V^2/\text{Hz}$) and $S_{D}(f)$ ($V^2/\text{Hz}$) are the single-sided PSD’s of the shot noise and non-return-to-zero (NRZ) random binary data, respectively, and they are given by [3], [4]

$$S_{SN}(f) = 2qRR_f^2 P_{LO}, \quad 0 < f < \infty$$

(6.19)
where \( T_b \) is the bit duration of NRZ data.

Substituting (3.10), (6.15) into (6.14) and then (6.14), (6.19), (6.20), (6.11) into (6.18), we obtain after some rearrangements

\[
\sigma_E^2 = \Delta \nu \int_{f_n}^{\infty} \left| 1 - H(x) \right|^2 dx + \frac{q f_n \sec^2(\psi)}{2 R P_R} \int_{0}^{\infty} \left| H(x) \right|^2 dx + \frac{2 f_n \tan^2(\psi)}{R_b} \int_{0}^{\infty} \left( \sin \left( \frac{\pi f_n x}{R_b} \right) \right)^2 \left| H(x) \right|^2 dx
\]

(6.21)

where \( \left| 1 - H(x) \right|^2 / x \) and \( \left| H(x) \right|^2 \) are given by the equations (4.33B) and (4.45B), respectively, and \( R_b \) is the bit rate in bit/s.

From the expression for total phase-error variance in (6.21), it can be seen that there exists an optimum loop natural frequency \( f_{n,\text{opt}} \) for which \( \sigma_E^2 \) will be minimum. The optimum value of phase-error standard deviation \( \sigma_{E,\text{min}} \) is determined by optimizing the corresponding expression for the phase-error variance (6.21) per variable parameter \( x \). The optimization can be conveniently carried out by the use of an iterative numerical solution of the noise integral with respect to \( f_n \).

### 6.6.2 Calculation of bit error rate and power penalty

For optical receiver systems, the power penalty is an important measure of the system performance. The power penalty shall be defined as the ratio of the deteriorated SNR to the SNR of an ideal receiver (i.e., phase-error \( \phi_p(t) = 0 \) and phase deviation created by the phase modulator \( \psi = \pi / 2 \)). We evaluate here the power penalty induced by residual-carrier, phase noise and shot noise.
For the analysis of the power penalty, we assume that in the transmitter, the stochastic user signal is applied to an ideal phase modulator. The user signal consists of binary rectangular pulses of duration $T_b$. In the homodyne receiver, the detected base band signal is deteriorated by additive white Gaussian noise. The detected signal is fed into a correlator, followed by a threshold decision circuit. The correlator is built with a matched filter, the noise bandwidth of which amounts to $1/2T_b = R_b/2$.

For zero phase-error ($\phi_E(t) = 0$) and complete modulation of the transmitter laser ($\psi = \pi/2$), (6.9) reduces to

$$V_0(t) = 2RR_T \sqrt{P_R P_{\text{LO}}} \, d(t) + n(t).$$

(6.22)

The maximum SNR at the input of the decision circuit amounts to

$$\rho_0^2 = \frac{4R^2R_T^2P_R P_{\text{LO}}}{S_{\text{SNR}}(f) (R_b/2)}.$$  

(6.23)

Using (6.19), (6.23) reduces to

$$\rho_0^2 = \frac{4RP_R}{qR_b}.$$ 

(6.24)

The BER of the ideal receiver can be calculated through [4], [5]

$$BER_0 = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\rho_0^2}{2}} \right).$$ 

(6.25)

where $\text{erfc}(.)$ is the complementary error function.

With a zero phase-error ($\phi_E(t) = 0$) and incomplete modulation of the carrier ($0 < \psi < \pi/2$), the SNR at the input of the decision circuit amounts to

$$\text{SNR}_{\text{residual}} = \rho_{\text{residual}}^2 = \frac{4RP_R \sin^2 \psi}{qR_b} = \rho_0^2 \sin^2 \psi.$$ 

(6.26)

and the corresponding bit error rate is given by

$$BER_{\text{residual}} = \frac{1}{2} \text{erfc} \left( \sqrt{\left[ \rho_0^2 \sin^2 \psi \right]/2} \right).$$ 

(6.27)

The power penalty due to the residual-carrier transmission amounts to

$$\epsilon_{\text{residual}} = \frac{\rho_{\text{residual}}^2}{\rho_0^2} = \sin^2 \psi.$$ 

(6.28)
When the bit ‘1’ and bit ‘0’ are transmitted, the demodulator outputs are given from equation (6.9), respectively, as

\[ V_0(t) = +K_{PD} \sin \psi \cos \phi_E(t) + K_{PD} \cos \psi \sin \phi_E(t) + n(t). \]  
\[ V_0(t) = -K_{PD} \sin \psi \cos \phi_E(t) + K_{PD} \cos \psi \sin \phi_E(t) + n(t). \]  

(6.29)  
(6.30)

So, the discrimination level is \( D = K_{PD} \cos \psi \sin \phi_E. \)

(6.31)

Both phase noise and shot noise are assumed to be White Gaussian noise. The probability density function (PDF) of the shot noise and the phase noise are, respectively, given by

\[ p(n) = \frac{1}{\sqrt{2\pi \sigma_{SN}^2}} \exp \left( -\frac{n^2}{2\sigma_{SN}^2} \right) \]  
\[ p(\phi_E) = \frac{1}{\sqrt{2\pi \sigma_{PE}^2}} \exp \left( -\frac{\phi_E^2}{2\sigma_{PE}^2} \right) \]  

(6.32)  
(6.33)

where \( \sigma_{SN}^2 \) represents the phase-error variance due to the shot noise in rad\(^2\).

When bit ‘1’ is transmitted, the bit error rate is given by

\[ P_{e1} = \text{Pr}(V_0(t) < D) \]  
\[ = \int_{V_0(t) < D} \frac{1}{\sqrt{2\pi \sigma_{SN}^2}} \exp \left( -\frac{n^2}{2\sigma_{SN}^2} \right) \frac{1}{\sqrt{2\pi \sigma_{PE}^2}} \exp \left( -\frac{\phi_E^2}{2\sigma_{PE}^2} \right) dn.d\phi_E \]  

(6.34)

Substituting \( \frac{\phi_E^2}{2\sigma_{PE}^2} = t^2 \), and after few steps of mathematical simplifications, we get

\[ P_{e1} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} 0.5 \text{erfc} \left( \sqrt{2R_{P'}} \sin \psi \right) \cos \left( \sqrt{2\sigma_{PE}^2}t \right) \exp \left( -t^2 \right) dt. \]  

(6.35)

On the other hand, when bit ‘0’ is transmitted, the bit error rate is given by

\[ P_{e0} = \text{Pr}(V_0(t) > D) \]  
\[ = \int_{V_0(t) > D} \frac{1}{\sqrt{2\pi \sigma_{SN}^2}} \exp \left( -\frac{n^2}{2\sigma_{SN}^2} \right) \frac{1}{\sqrt{2\pi \sigma_{PE}^2}} \exp \left( -\frac{\phi_E^2}{2\sigma_{PE}^2} \right) dn.d\phi_E \]  

\[ = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} 0.5 \text{erfc} \left( \sqrt{2R_{P'}} \sin \psi \right) \cos \left( \sqrt{2\sigma_{PE}^2}t \right) \exp \left( -t^2 \right) dt. \]  

(6.36)

When bit ‘1’ and bit ‘0’ have equal probability, the total bit error rate is given by
\[
\text{BER} = \frac{(P_{e0} + P_{e1})}{2} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} 0.5 \text{erfc} \left( \sqrt{\frac{2RP}{qR_b}} \sin \psi \right) \cos \left( \sqrt{2\sigma_{\psi}} t \right) \exp \left( -t^2 \right) dt. \tag{6.37}
\]

This expression is very similar to that obtained previously \[19\] with \( \psi = \pi/2 \).

Using the optimum value of phase-error standard deviation, the bit error rate of the receiver can be written as
\[
\text{BER} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} 0.5 \text{erfc} \left( \sqrt{\frac{2RP}{qR_b}} \sin \psi \right) \cos \left( \sqrt{2\sigma_{\psi,\text{opt}}} t \right) \exp \left( -t^2 \right) dt. \tag{6.38}
\]

From (6.25) and (6.38), the received power that gives the same BER value \(10^{-10}\) is defined as \( P_{R0} \) and \( P_{R1} \), respectively for the same phase deviation \( \psi \). Then, the power penalty due to the imperfect phase recovery is derived as \[3\]
\[
\varepsilon_{\text{imperfect}} = 10 \log_{10} \left( \frac{P_{R0}}{P_{R1}} \right). \tag{6.39}
\]

### 6.7 Pull-in behavior

In optical coherent transmission systems, the most severe problem is the large fluctuation of the beat frequency of the optical sources. The PLL pull-in limit should be as large as possible. To consider the pull-in behavior, we neglect all the components including phase noise and we assume that the loop is operating in the unlocked mode. Under this condition, the output of the phase detector comprises of a dc voltage and alternating voltage component. But due to the presence of the loop filter, the output of the filter consists of a dc component and only the fundamental of the ac voltage. In this chapter, so far, we have considered only first-order active loop filter. The DC gain of this filter is infinite, so the saturation of electrical circuit output is the dominant problem. The results obtained above can, however, be applied to practical systems which use first-order passive lag-lead loop filters \[20\]. Therefore, we focus on the first-order passive lag-lead loop filters in this section.
Let us assume that the incoming signal is $A \sin(\omega t)$ and the free-running laser VCO output is $2\cos(\omega t)$. It can be shown that the governing phase equation of the homodyne OPLL is

$$\frac{d\phi_e(t)}{dt} = \Omega_0 - AKF(s)(1 + s\tau_p)\sin(\phi_e(t))$$

(6.40)

where $\Omega_0 = \omega - \omega_0$ is the open-loop frequency error and $AK$ is the open-loop gain of the OPLL. Now, when loop is beating, the output of the VCO can assumed as

$$2\cos[(\omega_0 + \Delta\Omega)t - M\cos(\Omega t)]$$

where $\Omega = \Omega_0 - \Delta\Omega$ is the closed-loop frequency error, $\Delta\Omega$ is the frequency shift of the VCO, and $M$ is the input-output index error.

The phase detector output is given by

$$V_\phi(t) = A \sin(\Omega t + M \cos(\Omega t)).$$

(6.41)

If we assume the equivalent filter to be of the form

$$F_1(s) = (1 + s\tau_p)\left(\frac{1 + s\tau_4}{1 + s\tau_3}\right)\exp(-s\tau),$$

then the output of the filter is given by

$$V_f(t) = F_1(s)V_\phi(t)$$

$$= \sqrt{1 + \Omega^2 \tau_p^2}F_0V_\phi(t)\exp(-j\delta)$$

(6.42)

where $F_0$ is the asymptotic gain of the filter, $\delta$ is the phase angle introduced due to loop-delay and the phase modulator, and is given by

$$\delta = \Omega \tau - \tan^{-1}(\Omega \tau_p).$$

(6.43)

Eq. (6.41) further simplifies to

$$V_\phi(t) = AJ_1(M) + AJ_0(M)\sin(\Omega t)$$

(6.44)

where $J_1(.)$ is the Bessel function of the first kind of order 1.

In deriving the above equation all the higher order terms are removed due to low pass filtering. From equations (6.42) and (6.44), we have

$$V_f(t) = AKF_0\sqrt{1 + \Omega^2 \tau_p^2}J_1(M) + J_0(M)\sin(\Omega t)\exp(-j\delta).$$

(6.45)

From the above equation, it can be easily shown that
For small $M$, $J_1(M) \approx \frac{M}{2}$ and the dc output of the phase detector, which is responsible for pull-in phenomenon in PLL, is given by

$$V_{\phi, dc} = \frac{F_0 \left( AK \right)^2 \sqrt{1 + \Omega^2 \tau^2}}{2\Omega} \cos \left( \Omega \tau - \tan^{-1} (\Omega \tau_p) \right).$$

(6.47)

The beat-error frequency is given by

$$\Omega = \Omega_0 - \Delta \Omega.$$

(6.48)

We approximate $\cos(\Omega \tau - \tan^{-1} (\Omega \tau_p))$ as

$$1 - \frac{\left( \Omega \tau \right)^2 m_i}{\sqrt{1 + \left( \Omega \tau_p \right)^2}},$$

where $m_i = \left( \frac{1 - \tau_p}{\tau} \right)$.

Substituting (6.47) into (6.48) and after few steps of mathematical simplifications, we get

$$\Omega_0 = \frac{F_0 \left( AK \right)^2}{\Omega} \left[ 1 - m_i \Omega^2 \tau^2 \right].$$

(6.49)

To find the pull-in range, we take the derivative of $\Omega$ with respect to $\Omega_0$ and equate that to $\infty$. After few steps of mathematical simplifications, we get

$$\frac{\Omega_0}{AK} = \sqrt{2F_0} \left[ 1 - 0.5m_i F_0 \left( AK \right)^2 \right].$$

(6.50)

### 6.8 Results and discussion

Without loss of generality, the damping ratio $\xi = 1$ and the detectors responsivity $R = 1$ A/W are used in all cases considered here. The values of $q$, $R$, and $BER$ are set to be $1.6 \times 10^{-9}$ C, 10 Gbps and $10^{-10}$, respectively in all calculations. Using equations (6.15), (6.16) and (6.17), the values of the maximum allowable normalized delay $d$ are plotted in Fig. 6.3 as a function of normalized phase control parameter $p$ for different value of phase deviations between the mark and space-state bits. The stable region lies below the curve in Fig. 6.3. As seen in the figure, the maximum allowable normalized loop-delay gradually increases as the normalized phase control parameter $p$ increases. This means that for same value of loop natural frequency, the loop can accommodate...
larger value of the loop delay without hampering the stable operating condition. This figure also shows that the normalized delay \( d \) reaches a maximum at a particular value of \( p \). This optimum \( p \) for maximum normalized delay \( d \) shifts right as the value of phase deviation \( \psi \) between mark and space-state bits increases. The maximum normalized allowable loop delay \( d \) is high when the phase deviation \( \psi \) is large. As indicated in Fig. 6.3, the maximum normalized delay \( d \) (at the optimum operating point) for \( \psi = 10^0 \) is about 0.81 at the optimum phase control parameter \( p = 0.26 \). For a conventional Balanced OPLL (\( p = 0 \) and \( \psi = 0 \)), the maximum allowable normalized delay is about 0.65. This result tallies with the result (\( d < 0.736 \)) given in Grant and Michie [13]. The slight difference results from the different uses of \( \xi = 1 \) and \( \xi = 0.707 \).

Using equation (6.21), the standard deviation of phase-error \( \sigma_E \) is plotted by changing the loop natural frequency \( f_n \) for selected values of loop-delay \( \tau \) in Fig.6.4. The received signal power \( P_r \) is set to -53 dBm and \( \psi = 10^0 \), \( K_{pm} = 15\text{rad/V} \), \( K_{vco} = 5 \) GHz/V, \( \Delta U = 1 \) MHz. It is observed in Fig. 6.4 that for low values of loop natural frequency, the effect of loop-delay on the phase-error standard deviation (PESD) is negligible and is dominated by the phase noise. For large loop natural frequency, the PESD is large and are dominated by the shot noise and data to phase-lock cross-talk noise. It is also observed that reliable phases-lock (\( \sigma_E \leq 10^0 \)) [4] is only possible if loop-delay is smaller than 3 ns. This figure shows that PESD reaches a minimum at a particular value of \( f_n \). This optimum loop natural frequency \( f_{n,\text{opt}} \) for minimum PESD decreases as the loop-delay \( \tau \) increases. For the maximum allowable loop-delay \( \tau = 3 \) ns, the optimum loop natural frequency \( f_{n,\text{opt}} \) is about 269 MHz and the corresponding normalized loop-delay \( d = 0.243 \), normalized phase control parameter \( p = 0.243 \). This is well within the stability boundary as shown in Fig. 6.3, and loop gain margin of 20 dB (using equations (6.16) and (6.17)) from the critical value ensure the system stability.

In Fig. 6.5, minimum phase-error standard deviation is plotted as a function of loop-delay for different phase modulator sensitivities. As seen in the figure, the minimum
gradually increases as the loop-delay increases. The phase modulator control is much more pronounced in the presence of high loop-delay condition, and the considerable reduction of PESD can be achieved with phase modulator control.

Using equations (6.21) and (6.38), the receiver sensitivity (defined as the signal powers for $BER = 10^{-10}$) is plotted as a function of phase deviation $\psi$ for various laser linewidths in Fig. 6.6. The phase modulator sensitivity $K_{pm}$ is set to 15 rad/V, and loop-delay $\tau = 2$ ns. It can be seen from the figure that there exists an optimum value of $\psi$ for each laser linewidth. The optimum $\psi$ decreases as the laser linewidth increases.

Fig. 6.7 shows the optimum phase deviation $\psi_{opt}$ as a function of laser linewidths, for different values of loop-delay at fixed phase modulator sensitivity $K_{pm} = 15$ rad/V. The optimum $\psi$ decreases very rapidly as linewidth increases and slope of decrement becomes steeper for larger loop-delay.

Using equation (6.28), the values of residual-carrier penalty are plotted in Fig. 6.8 as a function of laser linewidth for different values of loop-delay and phase modulator sensitivities. The power penalty due to residual-carrier increases as the laser linewidth increases. Consider linewidth as 3 MHz, the residual-carrier power penalty is decreased for 1 ns loop-delay from 7.6 dB to 6.4 dB and for 1.5 ns loop-delay from 9.0 dB to 8.0 dB with $K_{pm} = 20$ rad/V. Thus, improvement in residual-carrier penalty for 1 ns and 1.5 ns loop-delay is 1.0 dB and 1.2 dB, respectively, at the same linewidth with additional phase control.

Fig. 6.9 shows imperfect phase recovery induced power penalty as a function of laser linewidth for several values of loop-delay. The power penalty due to imperfect phase recovery increases very rapidly as linewidth increases and the slope of increment becomes steeper for higher value of loop-delay. In Fig. 6.10, the normalized beat linewidth $\Delta \nu / R_b$ is plotted as a function of normalized loop-delay time $\tau R_b$ for two different phase modulator sensitivities. As seen in the figure, the normalized beat line-width
decreases as the normalized loop-delay increases. As phase modulator sensitivity increases, the normalized linewidth also increases for a particular value of normalized loop-delay. This is in corroboration with Norimatsu and Iwashita [21].

The power penalty due to imperfect phase recovery under the non-negligible loop-delay time $\tau$ is investigated. The loop-delay $\tau$ with fixing the linewidth $\Delta\nu$ and beat line-width $\Delta\nu$ with fixing the loop-delay $\tau$ is changed in the vicinity of $\tau R_b = 10$ or $\Delta\nu / R_b = 2 \times 10^{-3}$ for two different phase modulator sensitivities in Fig. 6.10. This result is shown in Fig. 6.11.

When the loop-delay time $\tau$ is non-negligible, the required beat linewidth $\Delta\nu$ is not proportional to the bit rate but limited by the condition of

$$\Delta\nu \leq 2.1 \times 10^{-3} / \tau$$

(6.51)

This condition gives 1-dB penalty due to imperfect phase recovery with phase modulator sensitivity $K_{PM} = 20$ rad/V. It is also indicated by this figure that the proposed reception system with phase modulator sensitivity $15$ rad/V requires the laser linewidth as

$$\Delta\nu \leq 1.8 \times 10^{-3} / \tau$$

(6.52)

for keeping the imperfect phase recovery induced power penalty within 1 dB. To be more clear, for the modified balanced OPLL based PSK reception system operating at a loop-delay of 1 ns, linewidth $\Delta\nu$ is 1.8 MHz when $K_{PM} = 15$ rad/V, while $\Delta\nu$ becomes 2.1 MHz when $K_{PM} = 20$ rad/V. Thus, linewidth requirement of the receiver can be relaxed by increasing the phase modulator sensitivity, and also the proposed receiver can be easily built with the help of commercially available DFB lasers. Laser linewidth requirement conditions for different OPLL configurations are presented in Table 6.1.
### Table 6.1

Laser linewidth requirements in presence of loop-delay

<table>
<thead>
<tr>
<th>Technique</th>
<th>$\Delta \nu \times \tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Homodyne PSK)</td>
<td></td>
</tr>
<tr>
<td>Costas OPLL</td>
<td>$\leq 3.13 \times 10^{-3}$ [21]</td>
</tr>
<tr>
<td>Decision-Driven OPLL with optimum power splitting ratio</td>
<td>$\leq 2.99 \times 10^{-3}$ [22]</td>
</tr>
<tr>
<td>Modified Balanced OPLL with</td>
<td></td>
</tr>
<tr>
<td>(a) $K_{PM} = 20$ rad/V</td>
<td>$\leq 2.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>(b) $K_{PM} = 15$ rad/V</td>
<td>$\leq 1.8 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Modified balanced loops impose more stringent requirements on the laser linewidth than decision-driven loops. The basic reason is the data-to-phaselock-crosstalk, which represents an additional source of performance degradation in balanced loops. Thus, from the point of view of laser linewidth requirements, decision driven loops are a better choice than modified balanced loops. However, this advantage has to be weighted against the increased system complexity. Decision driven loops are more complex than modified balanced loops because they employ more complicated post-detection processing.

Using equation (6.47), normalized phase detector output voltage $\frac{V_{\phi, DC}}{AK}$ is plotted by changing the normalized open-loop frequency error $\frac{\Omega}{AK}$ for two different phase modulator sensitivities in Fig. 6.12. It is observed in Fig. 6.12 that for modified balanced OPLL, the pull-in voltage is larger than the ordinary balanced OPLL. This means that the pull-in will be faster in the modified balanced OPLL.

In Fig. 6.13, normalized lock-in range $\frac{\Omega_{\phi}}{AK}$ is plotted as a function of normalized loop propagation delay $AK \tau$ for various normalized phase sensitivities. The
lock-in range should be as large as possible in order to improve the tracking capability. It can be easily seen from fig. 6.13 that phase control improves the lock range of the homodyne OPLL in presence of loop propagation delay.

**Fig.6.3** Normalized maximum allowable delay $d$ as a function of normalized phase control parameter $p$ for four different phase deviations $\psi$: $0^\circ$, $10^\circ$, $20^\circ$ and $30^\circ$. 
Fig. 6.4 Variation of phase-error standard deviation $\sigma_E$ with the loop natural frequency $f_n$ for four different values of loop-delay $\tau$: 1 ns, 2 ns, 3 ns and 4 ns. Here, $P_p=-53$ dBm, $\psi=10^\circ$, $\Delta \nu=1$ MHz, $K_{pm}=15$ rad/V and $K_{VCO}=5$ GHz/V.
Fig. 6.5 Variation of minimum phase-error standard deviation $\sigma_{E_{\text{min}}}$ with the loop delay $\tau$ for three different phase modulator sensitivities. Here, $P_R = -53$ dBm, $\psi = 10^0$, $\Delta \nu = 1$ MHz, and $K_{\text{VCO}} = 5$ GHz/V.
Fig. 6.6 Receiver sensitivity as a function of phase deviation $\psi$ for different laser linewidths. Here, $\tau = 2$ ns and $K_{pm} = 15$ rad/V.
Fig. 6.7 Optimum phase deviation as a function of laser linewidth for different values of loop propagation delay.
Fig. 6.8 Residual-carrier penalty as a function of laser linewidth for different values of loop-delay and phase modulator sensitivities.
Fig. 6.9 Imperfect phase recovery induced power penalty $e_{\text{imperfect}}$ as a function of laser linewidth $\Delta \nu$ for different values of loop-delay $\tau$. Here, $K_{PM} = 15$ rad/V.
Fig. 6.10 Normalized beat line-width $\frac{\Delta \nu}{R_b}$, which permits 1dB power penalty due to imperfect phase recovery at a BER of $10^{-10}$, versus normalized delay $\tau R_b$ for two different phase modulator sensitivities.
Fig. 6.11 Imperfect phase recovery induced power penalty $\varepsilon_{\text{imperfect}}$ versus normalized loop-delay (i.e. loop-delay x linewidth) $\Delta U \times \tau$ for two different phase modulator sensitivities.
Fig. 6.12 Normalized phase detector output voltage versus normalized open-loop frequency error for two different normalized phase control parameters $p$. 
Fig. 6.13 Normalized lock-in range versus normalized delay $d$ for three different normalized phase control parameters $p$. 
6.9 Conclusion

We have envisaged a modified balanced OPLL receiver, which contains all the components of a standard balanced OPLL in conjunction of an additional phase modulator and analyzed its performance in comparison to a ordinary OPLL receiver accounting for the effects like phase noise from lasers, photodetector shot noise, data-to-phase-lock cross talk and loop-delay. This modified receiver provides (i) a narrow-band tracking and wideband demodulation bandwidth, (ii) more relaxed line-width requirement, (iii) improved tolerance and performance against delay. The only disadvantage is the increased residual carrier penalty due to the use of wide line-width laser. The stability performance has been assessed for the modified second order OPLL and it has been found that the proposed OPLL has better stability than conventional balanced OPLL. Phase-error variance for the proposed receiver has been computed. The loop natural frequency $f_n$ has been optimized for minimum phase error variance, and the corresponding optimum loop performances have been evaluated. Bit error rate has been computed for the proposed system in the presence of loop-delay, and power penalty due to the imperfect phase tracking has been calculated. The maximum permissible laser line-width has been assessed and found to be $\Delta \nu \leq 2.1 \times 10^{-3} \nu / \tau$. This number refers to the imperfect carrier recovery induced power penalty of 1dB at BER$=10^{-10}$ with phase modulator sensitivity $K_{PM} = 20$ rad/V. Also, it has been found that extra phase control improves the lock range of the modified receiver in the presence of loop propagation delay.
References


