Chapter-5

Modified Injection Synchronized Balanced Optical Phase-Locked Loop

5.1 Introduction

Radio-Over-Fiber is an attractive technology for millimeter-wave wireless applications, because it reduces the complexity of the necessary numerous base stations. Several methods exist to generate mm-wave modulated optical signals for such a system. Semiconductor laser diodes cannot be efficiently modulated at mm-wave frequencies. Modulation bandwidths exceeding 40 GHz have been reported for unpackaged 1100-nm lasers [1] and 25 GHz for a packaged 1550-nm distributed feedback (DFB) laser [2].

Optical heterodyning is an attractive method of generating mm-wave modulated optical carriers. The efficiency of the optical to mm-wave transduction is high, ideally 100%, the range of frequencies that can be generated is very large, limited by the bandwidth of the detector. The limitation of this method is that the purity of the generated mm-wave signal is determined by the linewidth of the lasers used. Therefore, narrow linewidth lasers need to be used to produce beats of acceptable purity, such as ECL laser. Narrow linewidth lasers are larger in size than wide linewidth DFB lasers and consume more power. For high spectral purity and absolute frequency stability, the phase fluctuations of the two lasers need to be correlated. The most commonly used technique is optical injection locking (OIL) where one or two lasers are injection locked to an optical comb, such as generated by frequency modulation (FM) sidebands of a master laser. The main practical limitations of optical sideband injection locking are: (1) the locking range is small (typically a few hundred MHz) so that the laser temperature must be controlled with milli-kelvin precision, as the temperature sensitivity is typically 10.0 GHz/K, and (2) increase of locking range beyond the limit set by locking ratio -50 dB [3] after which the system becomes unstable. Moreover, the increase of the strength of the injection signal decreases the effective Q-value of the injection locked oscillator. In this
connection it is important to note that the two lasers should be locked where in knowledge parameter, called ‘Locking Range’ is an important and crucial one. It appears to the authors that quite a few papers have been written on the injection locking of lasers [4]-[6], but none has dealt with the dependence of the locking range on the laser phase noise.

In order to overcome instability problem of OIL, the traditional alternative of optical injection synchronization i.e., an optical phase-locked loop (OPLL) is recommended. Had there been no loop propagation delay in an optical phase-locked loop, the required phase-error variance can be easily realized by increasing the locking range of the OPLL. Unfortunately, the presence of the finite loop propagation delay does not only increase the phase-error variance but also makes the system unstable and manifests as a spurious locking. Frequency search halts and the loop appears to lock at a frequency that bears no obvious relation to the input frequency. Even if the loop is stable, the presence of loop propagation delay can cause acquisition difficulties in the form of false locks. In order to overcome the instability problem, a modified OPLL has been suggested [7]. Few papers have been written on the acquisition performance of the optical phase-locked loop in presence of loop propagation delay [8]-[11]. Also, a split-loop technique which overcomes many of the problems associated with conventional long-loop phase-locked radio receivers has been proposed [12].

Thus, we note that though there is limitation on the locking range of the injection synchronization yet it does not add loop-delay. Whereas the OPLL apparently does not have limitation on the locking range till the AM noise due to bias current predominates over the shot noise. But it is prone to instability due to the unavoidable loop-delay, which in turn increases the phase-error variance. Thus the solution is to combine the principle of injection synchronization and phase locking technique [13], [14]. This technique has been used to generate 36 GHz mm-waves of very narrow linewidth of a few kHz only. Optical injection phase-locked loop (OIPLL) has been shown to have the ability to increase the locking range of an OIL system from 2 GHz to 30 GHz when OPLL was activated [15]. A Radio-over-Fiber system employing the OIPLL technique to generate a 36 GHz mm-wave signal modulated with 140 Mbps was reported [16]. Also, generation and
transmission of mm-wave data-modulated optical signals was presented using an OIPLL [17]. Millimeter-wave signal generation was demonstrated with wide locking range, 30-GHz low phase noise level, -93 dBC/Hz, and a wide frequency tuning range; 4-60 GHz generation demonstrated using OIL only, verified by using OIPLL in the 26-40 GHz range. The combination results in a system with low values of phase-error variance, (though not up to the desired value) over a much wider stable locking range. But detailed analysis shows that the effect of loop-delay, although minimized, still remains. It is known that wider the locking range, longer the mean time between cycle-slips in a tracking system. As a result, the restrictions imposed on the linewidth of the lasers are relaxed and the use of commercially available DFB lasers in place of very expensive and bulky narrow linewidth lasers is seen commercially viable. However, the need of a very low value of phase-error variance for the (semi) coherent reception of digital signals with the error probability of 10^{-9} is not fulfilled with this arrangement. It is easily appreciated from the above discussion that the addition of the injection synchronizations path effectively increases the loop bandwidth. As a result the tracking system becomes more faithful in following phase variation of the input signal and because of the increase of the loop bandwidth noise rejection capability becomes weak.

In this chapter, we propose a modified injection synchronized balanced optical phase-locked loop to generate a 60 GHz mm-wave signal using a wide linewidth DFB lasers for broadband mobile communication. The detailed theory is studied for the modified injection synchronized balanced OPLL, considering the effect of laser phase noise and with non-negligible loop propagation delay. The pull-in behavior of the proposed OPLL is studied. The locking range of the proposed loop in presence of loop-delay is calculated. Also, the effect of finite linewidth on optical injection locking for the homodyne OPLL is investigated.
5.2 Millimeter-wave signal generation using modified injection synchronized balanced optical phase-locked loop

5.2.1 Signal generation

Fig. 5.1 shows a system to generate a 60-GHz mm-wave signal using two cascaded Mach-Zehnder modulators (MZM) and the modified injection synchronized balanced OPLL. The modified OPLL consists of the arrangements by means of which instantaneous frequency of the slave laser is controlled by two mechanisms, namely, through injection locking and optical phase-locked principle using an additional arrangement for controlling the output phase of the VCO laser in correspondence to a measure of the instantaneous phase-error. The outputs from such two systems are heterodyned to generate the required mm-wave signal.

Phase coherence between the two outputs from the slave lasers means phase coherence between the master laser and the slave laser also, and cycle slipping is an annoying problem. Improving the phase coherence means two things simultaneously, namely, increasing the tracking bandwidth of the locked oscillator and improving the noise squelching property of the system. Unfortunately, an attempt to increase one deteriorates the other [18]. However, the tracking system as shown in Fig. 5.1, achieves this property (i.e., to increase the locking range and to decrease the noise bandwidth at the same time) to a great extent. Before we come to the question of phase noises of the master laser and the slave laser, that disturb phase tracking, it is also necessary that there is no unwanted modulation component close to the center frequency of the slave laser. Otherwise, this exerts pulling and pushing force on the slave laser. Thus this will cause serious disturbances in the synchronization process.

To solve this problem, let us take the following example. Let the modulation bandwidth of the master laser is close to 10.0 GHz, and it is desired to generate 60.0 GHz mm-wave signal. Referring to the new modulation scheme depicted in Fig. 5.1 and using the drive voltages with $m\pi = 3.843 \left( m = \frac{V_{RF}}{V_\pi}\right)$, where $V_{RF}$ and $V_\pi$ are as defined in
Chapter-3), the modulated laser field can be expressed as

\[ E_R(t) = E_i \left\{ 0.43 \sin \left[ (\omega_i \pm 3\omega) t + \phi_{nR}(t) \right] + 0.114 \sin \left[ (\omega_i \pm 5\omega) t + \phi_{nR}(t) \right] \right\} \] (5.1)

Equation (5.1) has been derived in section 4.2. The third harmonic components have been picked up, because the output mm-wave signal is required to be of 60.0 GHz with modulating frequency of 10.0 GHz. Disturbing components are away by 20.0 GHz, and 6.0 dB less in amplitude, causing almost no pulling and pushing force on the slave laser.

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**Fig. 5.1** Millimeter-wave signal generation using two cascaded Mach-Zehnder modulators and modified injection synchronized balanced optical phase-locked loop.
5.2.2 Theoretical analysis of modified injection synchronized balanced optical phase-locked loop

In this sub-section, we shall investigate the detailed theory of the injection synchronized balanced OPLL considering the effect of laser phase noise and with non-negligible loop propagation delay.

The electric fields of the incoming signal and the phase modulated VCO light can be expressed, respectively, as

\[
E_R(t) = \sqrt{P_R} \exp \left( j \left( \omega_R t + \phi_{nR}(t) \right) \right) \tag{5.2}
\]

\[
E_{LO}(t) = \sqrt{P_{LO}} \exp \left( j \left( \omega_{LO} t + \phi_{PM}(t) + \phi_{VCO}(t) + \phi_{INJ}(t) + \phi_{nLO}(t) \right) \right) \tag{5.3}
\]

where \( \phi_{INJ}(t) \) is the phase modulation of the local laser VCO due to injection signal and the other symbols have their usual meanings.

For the present we assume that VCO’s phase is in lock to the received signal phase, it can then be assumed that the phase-error \( \phi_E(t) \ll 1 \). Assuming a balanced photodetector, it is easily shown that the photodetector output is given by

\[
V_\phi(t) = K_{PD} \phi_E(t) + n(t) \tag{5.4}
\]

where \( \phi_E(t) = (\phi_{nR}(t) - \phi_{nLO}(t)) - \phi_{PM}(t) - \phi_{VCO}(t) - \phi_{INJ}(t) \)

is the total phase-error. \( K_{PD} \), the phase detector gain is given by equation (3.6).

The control voltage at the output of the loop filter is given by

\[
V_f(t) = V_\phi(t) * f(t) \tag{5.6}
\]

where \( f(t) \) denotes the impulse response of the loop filter, and \( * \) signifies the convolution operation.

The different phase modulation components of the laser source are given by [3], [7]

\[
\frac{d\phi_{INJ}}{dt} = \frac{1}{2\tau_{pm}} \sqrt{\frac{P_R}{P_{LO}}} \sin \phi_E(t) = K_{PD} K_{INJ} \sin \phi_E(t) \tag{5.7}
\]

\[
\frac{d\phi_{PM}}{dt} = K_{PM} \frac{dV_f}{dt} \tag{5.8}
\]
\[
\frac{d\phi_{VCO}}{dt} = K_{VCO} V_f(t) \tag{5.9}
\]

where \( \tau_{pn} \) is the photon life time of the laser cavity, and \( K_{INJ} = \frac{1}{4\tau_{pn} RR \tau L_0} \) is the equivalent gain of the injection locking in the first-order phase-lock loop equivalent model in rad/(s.V).

Using equation (5.5), the phase-error between the incoming signal and the laser VCO output can be written as

\[
\gamma(t) = \phi_{nR}(t) - \phi_{nLO}(t) - \phi_{VCO}(t) - \phi_{INJ}(t) = \phi_E(t) + \phi_{PM}(t). \tag{5.10A}
\]

Using equations (5.4) through (5.10), and simply linearising the loop (i.e., replacing \( \sin \phi_E \) by \( \phi_E \)), it can be easily shown that

\[
\gamma(s) = \left[ \phi_{nR}(s) - \phi_{nLO}(s) \right] \left[ 1 - \frac{K_{INJ} + K (1 + s \tau_p) F(s) \exp(-s \tau)}{s + K_{INJ} + K (1 + s \tau_p) F(s) \exp(-s \tau)} \right] + \frac{n(s)}{K_{PD}} \tag{5.10B}
\]

where \( K = K_{PD} K_{VCO} \) is the dc loop gain of the PLL part of the OPLL.

Thus, the closed-loop transfer function and error function are, respectively, given by

\[
H(s) = \frac{K_{INJ} + K (1 + s \tau_p) F(s) \exp(-s \tau)}{s + K_{INJ} + K (1 + s \tau_p) F(s) \exp(-s \tau)} \tag{5.11}
\]

\[
1 - H(s) = \frac{s}{s + K_{INJ} + K (1 + s \tau_p) F(s) \exp(-s \tau)}. \tag{5.12}
\]

It may be seen that on substituting \( \tau_p = 0 \) and \( K_{INJ} = 0 \) in (5.11) and (5.12), we arrive at the expressions for closed-loop transfer function and error function for conventional OPLL given in [11]. Putting \( s = j2\pi f \) in (5.11) and (5.12), respectively, we get

\[
H(f) = \frac{K_{INJ} + K (1 + j2\pi f \tau_p) F(j2\pi f) \exp(-j2\pi f \tau)}{j2\pi f + K_{INJ} + K (1 + j2\pi f \tau_p) F(j2\pi f) \exp(-j2\pi f \tau)} \tag{5.13}
\]

\[
1 - H(f) = \frac{j2\pi f}{j2\pi f + K_{INJ} + K (1 + j2\pi f \tau_p) F(j2\pi f) \exp(-j2\pi f \tau)}. \tag{5.14}
\]
5.2.3 **Minimization of frequency of slipping cycles**

In the following we neglect the shot noise contribution to the phase-error, as it is small over the range of the laser linewidth (say, 10.0 MHz) with the laser power on the order of 100 µW and the detector sensitivity of about 0.5 A/W [3]. In view of this noise bandwidth gives an estimate of the phase noise contribution at laser output. In this connection, it is important to note that noise output is considered at the output of the laser VCO from where the reference signal is taken and not at the output of the phase modulator after the laser VCO (Fig. 5.1). To demonstrate the validity of the concept in the simplest form, we consider only first-order system (i.e. $F(s) = 1$). In this connection it is important to note that an injection locked oscillator is a first-order phase-locked system and its locking range $K_{\text{INJ}}$ is limited to about 360.0 Mrad by the stability condition [3]. The loop phase-error variance is inversely related to $K_{\text{INJ}}$. Because of the phase noise, the slave lasers will slip cycles even when the phase-error variance is small, and in doing so the phase coherence is lost during the period of slipping cycles. The average time between cycle slips is approximately given by [3]

$$T_{av} = \frac{\pi}{4B_n} \exp \left[ \frac{2}{\sigma_E^2} \right].$$

(5.15)

Noise bandwidth $B_n$ and the phase-error variance $\sigma_E^2$ can be calculated using the expressions given by equations (4.19), and (4.32), respectively.

Equation (5.15) clearly demonstrates that in order to increase $T_{av}$ (a strong function of $\sigma_E^2$), it is necessary that $B_n$ as well as $\sigma_E^2$ should be reduced. When the loop-delay is negligible (i.e. $\tau = 0$), substituting (5.13) into (4.19), the noise bandwidth of the modified injection synchronized balanced OPLL is given by

$$B_n = \frac{K_{\text{INJ}} + K}{4(1 + K\tau_p)}.$$

(5.16)

Substituting (5.14) and (3.10) into (4.32), we obtain the phase-error variance of the modified loop in the absence of loop delay
\[
\sigma^2_E = \frac{\pi \Delta \nu}{(K_{INJ} + K)(1 + K \tau_p)}.
\] (5.17)

The beauty of inclusion of the phase modulator (i.e., \( \tau_p \)) is that noise bandwidth \( (B_n) \) as well as phase-error variance \( (\sigma^2_E) \) decreases with the increase of normalized phase modulator sensitivity \( \tau_p \). This is in contrast to the operation of conventional OPLL [18]. Had there been no limitation on the value of loop gain \( K \), the summed linewidth \( \Delta \nu \) can be increased to a large value. But due to presence of the loop-delay, the maximum permissible value of loop gain \( K \) is limited.

### 5.2.4 Maximum permissible value of loop gain

Because of the loop propagation delay, the loop becomes susceptible to instability when the loop gain is increased beyond a certain limit. For the sake of simplicity, we consider the first-order system (i.e. \( F(s) = 1 \)) with delay. From equation (5.11), we write the characteristic equation of the system as

\[
jz + b + (1 + jzp)\exp(-jzd) = 0
\] (5.18)

where \( z = \frac{\omega}{K} \) is the frequency normalized with respect to \( K \), \( b = \frac{K_{INJ}}{K} \) is the injection locking range normalized with respect to \( K \), \( d = K \tau \) is the normalized delay time of the loop and \( p = \tau_p K \) is the normalized phase modulation index. The stability criteria of the loop can be found by finding the roots of the equation (5.18).

### 5.2.5 Calculation of phase-error variance

In this sub-section, we investigate the effect of loop propagation delay on the phase-error variance of the modified injection synchronized balanced OPLL. We now calculate the phase-error variance \( (\sigma^2_E) \) due to the white frequency induced phase noise in the presence of loop propagation delay. \( \sigma^2_E \) can be calculated using the expression given by equation (4.32). Considering active low pass loop filter transfer function given by (2.12), and substituting (2.12) into (5.13) and then (5.13), (3.10) into (4.32), and after
few steps of mathematical simplifications, we obtain the phase-error variance of the modified injection synchronized balanced OPLL in the presence of loop-delay

\[
\sigma_E^2 = \frac{\Delta v}{f_n} \int_0^\infty \frac{1 - H(x)^2}{x} dx
\]

(5.19A)

where

\[
\frac{1 - H(x)^2}{x} = \frac{1}{\left[(1-2p\xi x)^2 \cos xd + x(p+2\xi)\sin xd\right]^2 + \left[ax + x(p+2\xi)\cos xd - (1-2p\xi x^2)\sin xd\right]^2}
\]

(5.19B)

where \( a = \frac{K_{\text{PLOJ}}}{\omega_n} \) and the other symbols are as defined in Chapter-4 (sub-section 4.4.2).

Then equation (5.19A) is numerically integrated in order to evaluate the modified OPLL performance.

5.2.6 Results and discussion

The plot of critical value of normalized delay \( (K\tau) \) against normalized locking range \( b \) (i.e. injection to OPLL locking ratio) is shown in Fig.5.2. An appropriate choice of the phase modulation index \( p = K\tau_p \) can improve the value of \( K\tau \) to a large extent. That is, either higher value of \( K \) can be used or larger value of loop-delay \( \tau \) can be accommodated. It is seen that the introduction of the injection signal for direct synchronization and the additional phase control loop, aside directly decreasing the phase-error variance and noise bandwidth, it also increases the stability of the tracking system. To illustrate let us consider an OPLL with a loop delay of 1.6 ns. Let us choose a value of 2.2 for \( K\tau \) with \( p = 0.35 \). Therefore, the permitted value of the \( K \) (open-loop gain) is 1375.0 Mrad/s. Thus, \( b = 0.2618 \). Hence to realize the same phase-error variance for the two cases, namely with \( p = 0.0 \) and \( p = 0.35 \), the ratio of the summed linewidths of the lasers with and without phase control can be found to 1.545. That is, with the modified injection synchronized balanced OPLL, 50% more linewidth of the laser can be allowed.
The plot of normalized phase-error variance (i.e., phase-error variance/summed linewidth) against loop-delay is shown in Fig. 5.3. When the loop propagation delay is taken into account, the noise bandwidth and the phase-error variance increase rapidly after a certain values of the loop-delay. Simple relations for the noise bandwidth and phase-error variance do not hold good. However, when the delay is small, say up to about 0.7 ns, the variation nearly obeys the rule as given in equations (5.16) and (5.17). But it is important to observe that by properly adjusting the value of the phase modulation index, both the noise bandwidth and phase-error variance can be kept small over a larger value of the loop propagation delay. It is also interesting to note that the performance of the loop will become worse than that of the simple OPLL, if a large value of the phase modulation index (p) is taken.

![Normalized delay versus injection to OPLL locking ratio for the cases p = 0, 0.35, and 0.70.](image)

**Fig. 5.2** Normalized delay versus injection to OPLL locking ratio for the cases p = 0, 0.35, and 0.70.
Fig. 5.3 Normalized phase-error variance (i.e. phase-error variance/summed linewidth) versus loop-delay for the cases $p = 0, 0.35, \text{and } 0.70$.  

5.3 False locking in modified injection synchronized balanced optical phase-locked loop

The annoying circuit component in optical phase-locked loop is the loop propagation delay. The presence of this delay along with the finite linewidth of the laser adds instability to the loop and manifests as a spurious locking. Frequency search halts and the loop appears to lock at a frequency that bears no obvious relation to the input frequency. Even if the loop is stable, the presence of loop propagation delay can cause acquisition difficulties in the form of false locks. In this section, the role of the optical injection in the pull-in behavior of the modified OPLL will be investigated in presence of loop propagation delay.
5.3.1 Calculation of phase detector output voltage

A modified injection synchronized balanced OPLL is considered. We assume that the loop is operating in the unlocked mode. Let us assume that the incoming signal is $A\sin[\omega t + \phi_{R}(t)]$ and the free-running laser VCO output is $2\cos[\omega t + \phi_{LO}(t)]$, where the symbols have their usual meanings.

It can be shown that the governing equation of the homodyne OPLL is

$$\frac{d\phi_{E}(t)}{dt} = \Omega_{0} - AKF(s)(1+s\tau_{p})\sin(\phi_{E}(t-\tau)) - AK_{INJ}\sin\phi_{E}(t) + \frac{d}{dt}\left(\phi_{R}(t) - \phi_{LO}(t)\right)$$

(5.20)

where $\Omega_{0} = \omega_{i} - \omega_{0}$ is the open-loop frequency error. Consider a standard first order passive lag-lead loop filter with transfer function $F(s) = \frac{1+s\tau_{4}}{1+s\tau_{3}}$, where $\tau_{3}$ and $\tau_{4}$ are filter time constants.

We assume a solution of Eq. (5.20) of the form given by

$$\phi_{E} = \phi_{s} + \phi_{n}$$

(5.21)

where $\phi_{s}$ is the steady state solution and $\phi_{n}$ is a random variable due to the incoming noise having zero mean. From equations (5.20) and (5.21), we get

$$\frac{d\phi_{s}}{dt} + \frac{d\phi_{n}}{dt} = \Omega_{0} - \{AK_{INJ} + AKG_{F}(s)\}[\sin\phi_{s}\cos\phi_{n} + \cos\phi_{s}\sin\phi_{n}] + n_{\phi}(t)$$

(5.22)

where $n_{\phi}(t)$ is the laser phase noise component, and $G_{F}(s) = (1+s\tau_{p})\left(\frac{1+s\tau_{4}}{1+s\tau_{3}}\right)\exp(-s\tau)$.

Thus using quasi-linearization technique in the locked state, one finds

$$\frac{d\phi_{s}}{dt} = \Omega_{0} - \{AK_{INJ} + AKG_{F}(s)\}\exp\left(-\sigma_{E}^{2}/2\right)\sin\phi_{s}$$

(5.23)

$$\frac{d\phi_{n}}{dt} = -\{AK_{INJ} + AKG_{F}(s)\}(\cos\phi_{s})G_{F}(s)\cos\left(-\sigma_{E}^{2}/2\right)\phi_{n}(t) + n_{\phi}(t).$$

(5.24)

Now when the loop is beating, the output of the VCO can assumed as

$$2\cos\left[\left(\omega_{0} + \Delta\Omega\right)t - M\cos(\Omega t) + \phi_{nLO}(t)\right],$$

where the closed loop frequency-error is given by $\Omega = (\omega_{i} - \omega_{0}) - \Delta \Omega = \Omega_{0} - \Delta \Omega$. $\Delta \Omega$ is the frequency shift of the laser VCO.
and $M \cos(\Omega t)$ appears because of the fluctuating component. $M$ is the input-output index error. In this case, it comprises of two parts: i) $M_i$ due to injection component and ii) $M_p$ due to the loop.

The phase detector output can be expressed by

$$V_\phi(t) = A\sin[\Omega t + M \cos(\Omega t) + \Psi_n(t)]$$

(5.25)

where $\Psi_n(t)$ represents the phase due to the noise component.

The output of the filter is given by

$$V_j(t) = G_F(s)V_\phi(t)$$

$$= F_0\sqrt{1 + \Omega^2\tau_p^2}\exp(-j\Omega\tau - \tan^{-1}(\Omega\tau_p))V_\phi(t)$$

(5.26)

where $F_0$ is the asymptotic gain of the loop filter, and $\Psi = (\Omega\tau - \tan^{-1}(\Omega\tau_p))$ is the phase angle introduced due to loop propagation delay and the phase modulator.

Equation (5.25) further simplifies to

$$V_\phi(t) = A\sin(\Omega t + \Psi_n)\cos\{M \cos(\Omega t)\} + \cos(\Omega t + \Psi_n)\sin\{M \cos(\Omega t)\}$$

$$\approx AJ_1(M)\exp(-\sigma_k^2/2) + AJ_0(M)\exp(-\sigma_k^2/2)\sin(\Omega t).$$

(5.27)

where $J_1(.)$ and $J_0(.)$ are the Bessel functions of the first kind of order one and zero, respectively.

Thus, the output of the phasemeter comprises of a dc voltage and an alternating voltage component. In deriving the above equation, all the higher order terms are removed due to low-pass filtering. Using equations (5.26) and (5.27), we obtain the loop filter output

$$V_j(t) = AF_\phi\sqrt{1 + \Omega^2\tau_p^2}\{K_{inj} + KG_F(s)\}[J_1(M) + J_0(M)\sin(\Omega t)]\exp(-j\Psi)\exp(-\sigma_k^2/2)$$

(5.28)

Thus, the output of the filter consists of a dc component and only the fundamental component of the ac voltage. Since the output of the filter modulates the instantaneous frequency of the VCO, we have

$$\Delta\Omega = AJ_1(M)(K_{inj} + K)\exp(-\sigma_k^2/2)$$

(5.29)
and

\[ M \Omega \sin(\Omega t) + \frac{d\Psi}{dt} = V_f(t) \bigg|_{AC} = \]

\[ AF_0 J_0^2 \left( \frac{1}{1 + \Omega^2 \tau_p^2} \left( K_{INJ} + KG_p(s) \right) \right) \exp\left( -\sigma_E^2 / 2 \right) \sin(\Omega t - \Psi). \]  

(5.30)

This ac component will cause VCO phase modulation. If we ignore the noise modulation of VCO, then

\[ M\Omega = AK_{INJ} \exp\left( -\sigma_E^2 / 2 \right) \]

\[ M_p\Omega = AKG_p(s) \exp\left( -\sigma_E^2 / 2 \right) \]

and thus

\[ M = \frac{A \left( K_{INJ} + K \sqrt{1 + \Omega^2 \tau_p^2} \right) F_0 \cos \Psi}{\Omega} \exp\left( -\sigma_E^2 / 2 \right). \]

(5.32)

For small M, \( J_1(M) \approx \frac{M}{2} \) and thus, the dc output of the phase detector, which is responsible for pull-in phenomenon, is given by

\[ V_{\text{dc}} = \left( \frac{A}{2} \right) \exp\left( -\sigma_E^2 / 2 \right) \left[ K_{INJ} \right] \left[ K_{INJ} + \sqrt{1 + \left( \Omega \tau_p \right)^2} \right] F_0 \cos \Psi]. \]  

(5.33)

Since \( \Delta \Omega \) is due to the dc voltage at the output of the phase detector,

\[ \Delta \Omega = \left( \frac{AK}{2} \right) \exp\left( -\sigma_E^2 / 2 \right) \left[ K_{INJ} \right] \left[ K_{INJ} + \sqrt{1 + \left( \Omega \tau_p \right)^2} \right] F_0 \cos \Psi]. \]  

(5.34)

### 5.3.2 Calculation of locking range

We now calculate the locking range of the modified injection synchronized balanced OPLL in presence of loop propagation delay. Since the beat error frequency is given by \( \Omega = \Omega_0 - \Delta \Omega \), equation (5.34) reduces to

\[ 2\Omega^2 = 2\Omega_0^2 - \left( AK \right)^2 \exp\left( -\sigma_E^2 \right) \left[ 1 + K_{INJ} \left[ K_{INJ} + \sqrt{1 + \left( \Omega \tau_p \right)^2} \right] F_0 \cos \Psi \right]. \]

(5.35)

We approximate \( \cos[\Omega \tau - \tan^{-1} \Omega \tau_p] \) as \( \frac{1 - \left( \Omega \tau_p \right)^2}{\sqrt{1 + \left( \Omega \tau_p \right)^2}} \), where \( m_1 = \left( \frac{1}{2} \frac{\tau_p}{\tau} \right) \).

Thus, equation (5.35) reduces to

\[ \Omega = \Omega_0 - \left( \frac{AK}{2\Omega} \right)^2 \exp\left( -\sigma_E^2 \right) \left[ 1 + K_{INJ} \left[ K_{INJ} + F_0 \right] \right] - \left( m_1 \tau^2 F_0 \right) \Omega^2. \]  

(5.36)
To find the pull-in range, we equate \( \frac{\partial \Omega}{\partial \Omega_0} = \infty \) and thus one obtain the pull-in range as

\[
\frac{\Omega_0}{AK} = \sqrt{2 \left(1 + \frac{K_{INJ}}{K}\right) \left(\frac{K_{INJ}}{K} + F_0\right) \left[1 - 0.5 m_1 F_0 (AK \tau)^2 \left(1 + \frac{K_{INJ}}{K}\right) \exp \left(-\sigma_E^2\right)\right]^{1/2} \exp \left(-\sigma_E^2 / 2\right)}.
\]

(5.37)

Remembering that \( \tau_p \) and \( \tau \) are of the same order one obtains the pull-in range for

\[
\frac{\Omega_0}{AK} = \sqrt{2 \left(1 + \frac{K_{INJ}}{K}\right) \left(\frac{K_{INJ}}{K} + F_0\right) \left[1 + 0.25 m_1 F_0 (AK \tau)^2 \left(1 + \frac{K_{INJ}}{K}\right) \exp \left(-\sigma_E^2\right)\right]^{1/2} \exp \left(-\sigma_E^2 / 2\right)}.
\]

(5.38)

\[5.3.3 \text{ Results and discussion}\]

In Fig. 5.4, normalized phase detector output \( \frac{\Delta \Omega}{AK} \) is plotted against the normalized open-loop frequency error \( \frac{\Omega}{AK} \) with and without injection. From the upper figure it is at once clear that for an ordinary OPLL (i.e., without injection and phase modulator) and modified OPLL (i.e., with phase modulator and without injection) having finite loop propagation delay, dc phase detector output shows null, consequently it appears that the loop is locked, but actually not out of the nulls. This mysterious mode of OPLL operation is known as false locking. In the modified injection synchronized loop, as proposed, this phenomenon will not be observed as there are no nulls of pull-in voltage and pull-in will be faster because the pull-in voltage is larger than the ordinary OPLL.

The pull-in range should be as large as possible in order to improve the tracking capability. It can be easily seen from Fig. 5.5 that injection improves the locking range of the homodyne OPLL in presence of loop propagation delay. Moreover, in presence of injection, phase modulation does not play any role.
Fig. 5.4 Normalized phase detector output as a function of normalized frequency error with injection and without injection. Loop-delay and normalized phase modulator sensitivity are used as the parameters.
5.4 Effect of finite linewidth on optical injection locking

In this section, we shall investigate the effect of finite linewidth of laser on optical injection locking.

5.4.1 Calculation of phase-error variance and locking range in presence of linewidth enhancement factor

A coherent optical beam from a master laser source with an angular frequency $\omega_i$ is injected into a slave laser with an angular frequency $\omega_o$. Let us assume that the free-running electric field of the slave laser to be

$$ e_0 = E_0 \exp\left[ j (\omega_o t + \phi_{\text{dLO}}(t)) \right] $$

(5.39)

where $E_0$ is the envelope function, and $\phi_{\text{dLO}}(t)$ is the slave laser phase noise. We further assume that the electric field of the master (injection) laser and that of the slave laser under the influence of the injection electric field are, respectively, given by

Fig. 5.5 Normalized locking range as a function of phase-error variance.
\[ e_i = E_i \exp \left[ j \left( \omega t + \phi_{nk}(t) \right) \right] \]  \hspace{1cm} (5.40)

\[ e_1 = E_1 \exp \left[ j \left( \omega t + \Psi_{ni}(t) \right) \right] \]  \hspace{1cm} (5.41)

where \( \phi_{nk}(t) \) is due to the laser phase noise and \( \Psi_{ni}(t) \) is the phase modulation of the slave laser under the influence of the injection laser.

Now we require for the analysis of the above-mentioned phenomenon that the basic equations must include the phase and the light injection terms. For this purpose, we employ here the semi-classical theory \([4],[6]\) where we assume that the laser oscillates in a single longitudinal mode and assume the rotating wave approximation under which (5.39), (5.40), and (5.41) are divided by \( \exp \left[ j \left( \omega t + \phi_{nLO}(t) \right) \right] \). Then the rotating wave approximation fields are represented by \( E'_i, E'_i, \) and \( E'_o, \) respectively.

Now,

\[ E'_i(t) = E_i(t) \exp \left[ j \Psi_i(t) \right] \]  \hspace{1cm} (5.42)

\[ E'_1(t) = E_1(t) \exp \left[ j \Psi_1(t) \right] \]  \hspace{1cm} (5.43)

where

\[ \Psi_i(t) = (\omega_i - \omega_0) t + \phi_{nk}(t) - \phi_{nLO}(t) \]  \hspace{1cm} (5.44)

and

\[ \Psi_1(t) = (\omega_1 - \omega_0) t + \Psi_{ni}(t) - \phi_{nLO}(t). \]  \hspace{1cm} (5.45)

Thus, the instantaneous phase difference between the master laser and the slave laser is

\[ \phi_i(t) = (\omega_i - \omega_0) t + \phi_{nk}(t) - \Psi_{ni}(t) \]  \hspace{1cm} (5.46)

\[ = \Psi_i(t) - \Psi_1(t). \]

The Vander Pol equation under the light injection is given by \([6]\)

\[ \frac{dE'_i}{dt} = - \left( g - \beta \left| E'_i \right|^2 \right) \left( 1 + j\alpha \right) E'_i = \frac{E'_i}{2\tau_p} \]  \hspace{1cm} (5.47A)

where \( g, \) and \( \beta \) are the net gain and the saturation co-efficient, respectively and are given by

\[ g = \frac{I}{2\tau_p I_{th}} \]  \hspace{1cm} (5.47B)

\[ \beta = \frac{1}{2\tau_p \left| E_{sat} \right|^2} \]  \hspace{1cm} (5.47C)
where $\tau_{pm}$ is the photon life time, $I_{th}$ is the threshold current, $E_{sat}$ is the saturation amplitude, and is given by

$$|E_{sat}| = \frac{|E_0|}{\sqrt{\left(\frac{I}{I_{th}}\right)}} - 1.$$  

(5.47D)

In (5.47A), $\alpha$ denotes the linewidth enhancement factor which describes the coupling between the amplitude and phase of the electric field [6]. Using (5.42), (5.43) in equation (5.47A) and separating into real and imaginary parts, we have

$$\frac{dE_i}{dt} - (g - \beta |E_i|^2)E_i = \frac{E_i}{2\tau_p} \cos \phi_E$$  

(5.48)

$$-\alpha \left( g - \beta |E_i|^2 \right) E_i + (\omega_i - \omega_0) + \frac{d\Psi_m}{dt} - \frac{d\phi_{nlO}}{dt} = \frac{1}{2\tau_{pm}} \sin \phi_E.$$  

(5.49)

Assuming that $E_i$ is fairly constant

$$\left( g - \beta |E_i|^2 \right) E_i = -\frac{E_i}{2\tau_{pm}} \cos \phi_E.$$  

(5.50)

Using (5.49), we get

$$(\omega_i - \omega_0) + \frac{d\Psi_m}{dt} - \frac{d\phi_{nlO}}{dt} = \frac{E_i}{2\tau_{pm}E_1} [\sin \phi_E - \alpha \cos \phi_E].$$  

(5.51)

From (5.46) and (5.51), it can be easily shown that

$$\frac{d\phi_E}{dt} = (\omega_i - \omega_0) - \frac{E_i}{2\tau_{pm}E_1} [\sin \phi_E - \alpha \cos \phi_E] + \frac{d\phi_{nlR}}{dt} - \frac{d\phi_{nlO}}{dt}$$  

(5.52)

$$\frac{d\phi_E}{dt} = \Omega - K_{INJ} (\sin \phi_E - \alpha \cos \phi_E) + \frac{d}{dt}(\phi_{nlR} - \phi_{nlO})$$  

(5.53)

where $\Omega = (\omega_i - \omega_0)$ is the open-loop frequency error, $K_{INJ} = \frac{v_g}{2L} \sqrt{\eta \frac{P_s}{P_o}}$ is the OIL injection rate, $L$ is the effective length of the slave laser cavity, $v_g$ is the group velocity, $\eta \frac{P_s}{P_o}$ is the injection locking ratio, $\eta$ is the coupling coefficient, $P_s$ is the injected signal power, and $P_o$ is the master laser power.

Assuming a solution of the form $\phi_E = \phi_o + \phi_n$, where $\phi_o$ is the steady-state value and $\phi_n$ is a random variable, closely related to the Gaussian variable
\[ p(\phi_n) = \frac{1}{\sqrt{2\pi\sigma_E^2}} \exp\left(-\frac{\sigma_E^2}{2}\phi_n^2\right). \] 

(5.54)

Using the principle of quasi-linearization technique [13] and assuming \( \phi_n(t) \) to be a Gaussian process with zero mean and variance \( \sigma_E^2 \), \( \sin(\phi_n) \) and \( \cos(\phi_n) \) can be approximated by the following relations:

\[ \sin(\phi_n) \approx \exp\left(-\frac{\sigma_E^2}{2}\phi_n\right), \quad \cos(\phi_n) \approx \exp\left(-\frac{\sigma_E^2}{2}\phi_n\right). \] 

(5.55)

Under locking condition, the average value of the instantaneous frequency error is zero, i.e.,

\[ \langle \frac{d\phi_E}{dt} \rangle = 0 = \Omega - K_{INJ}\left(\sin\phi_E - \alpha\cos\phi_E\right) + \langle \frac{d}{dt}(\phi_{nR} - \phi_{nLO}) \rangle. \] 

(5.56)

Using (5.55) into (5.56), it can be easily shown that

\[ \Omega = K_{INJ}(\sin\phi_o - \alpha\cos\phi_o)\exp\left(-\frac{\sigma_E^2}{2}\right). \] 

(5.57)

From equation (5.53), it can be shown that

\[ \frac{d\phi_n}{dt} = -K_{INJ}(\cos\phi_o + \alpha\sin\phi_o)\exp\left(-\frac{\sigma_E^2}{2}\phi_n\right) + \frac{d}{dt}(\phi_{nR} - \phi_{nLO}). \] 

(5.58)

From which one can easily get

\[ \phi_n(s) = \frac{s(\phi_{nR} - \phi_{nLO})}{s + K_{INJ}(\cos\phi_o + \alpha\sin\phi_o)\exp\left(-\frac{\sigma_E^2}{2}\right)}. \] 

(5.59)

Thus, transfer function of the loop is given by

\[ H(s) = \frac{s}{s + K_{INJ}(\cos\phi_o + \alpha\sin\phi_o)\exp\left(-\frac{\sigma_E^2}{2}\right)}. \] 

(5.60)

In frequency domain (5.60) can be written as

\[ H(f) = \frac{j2\pi f}{j2\pi f + K_{INJ}(\cos\phi_o + \alpha\sin\phi_o)\exp\left(-\frac{\sigma_E^2}{2}\right)}. \] 

(5.61)

Phase-error variance can be calculated using the expression given by equation (4.32). Substituting (2.12) and (5.61) into (4.32), we obtain the phase-error variance due to the white frequency induced phase noise.
\[
\sigma_E^2 = \frac{\pi \Delta \nu}{K_{\text{INJ}} (\cos \phi_o + \alpha \sin \phi_o) \exp \left( -\frac{\sigma_E^2}{2} \right)}. \tag{5.62}
\]

Now from equation (5.57), we have
\[
\Omega_{\text{max}} = K_{\text{INJ}} \left( \sin[\phi_o]_{\text{max}} - \alpha \cos[\phi_o]_{\text{max}} \right) \exp \left( -\frac{\sigma_E^2}{2} \right). \tag{5.63}
\]

\(\phi\) is maximum when \(\tan \phi = \frac{1}{\alpha}\) and thus \(\left[ \phi_o \right]_{\text{max}} = \left[ \tan^{-1} \left( \frac{1}{\alpha} \right) - \sigma_\phi \right]\).

Therefore,
\[
\left( \frac{\Omega_{\text{max}}}{K_{\text{INJ}}} \right)^\alpha = \sqrt{1+\alpha^2} \cos(\sigma_E) \exp \left( -\frac{\sigma_E^2}{2} \right). \tag{5.64}
\]

Similarly, the phase-error variance is given by
\[
\sigma_E^2 = \left( \frac{\pi \Delta \nu}{K} \right) \frac{1}{\cos \left\{ \tan^{-1} \left( \frac{1}{\alpha} \right) - \sigma_E \right\} + \alpha \sin \left\{ \tan^{-1} \left( \frac{1}{\alpha} \right) - \sigma_E \right\} \exp \left( -\frac{\sigma_E^2}{2} \right)}. \tag{5.65}
\]

From which it can be easily shown that
\[
\left( \sigma_E^2 \right)^\alpha = \left( \frac{\pi \Delta \nu}{K} \right) \times \frac{0.5}{\sqrt{1+\alpha^2} \sin(\sigma_E) \exp \left( -\frac{\sigma_E^2}{2} \right)}. \tag{5.66}
\]

In the absence of linewidth enhancement factor (i.e.), the phase-error variance and maximum locking range are given, respectively, by
\[
\left( \frac{\Omega_{\text{max}}}{K_{\text{INJ}}} \right)^\alpha = \cos(\sigma_E) \exp \left( -\frac{\sigma_E^2}{2} \right). \tag{5.67}
\]
\[
\left( \sigma_E^2 \right)^\alpha = \left( \frac{\pi \Delta \nu}{K} \right) \times \frac{0.5}{\sin(\sigma_E) \exp \left( -\frac{\sigma_E^2}{2} \right)}. \tag{5.68}
\]

### 5.4.2 Results and discussion

In Fig. 5.6, normalized locking range \(\frac{\Omega_{\text{max}}}{K_{\text{INJ}}}\) is plotted against phase-error variance \(\sigma_E^2\). As seen from Fig. 5.6 that the normalized locking range decreases with increase in variance but the locking range increases with the increase in linewidth enhancement factor (\(\alpha\)). From this figure, it is concluded that more than 3 times increase in normalized locking range is possible with linewidth enhancement factor = 3 at 0.04 rad\(^2\) phase-error variance condition.
The linewidth to locking range ratio variation with phase-error variance is shown in Fig. 5.7. The linewidth to locking range ratio increases with an increase in both the variance and the linewidth enhancement factor ($\alpha$) as seen from Fig. 5.7.

**Fig. 5.6** Normalized locking range as a function of phase-error variance in presence of linewidth enhancement factor.
5.5 Conclusion

A modified injection synchronized balanced optical phase-locked loop having an additional arrangement for injection synchronization and phase modulation has been proposed. Also, a method to generate 60-GHz mm-wave signal using wide linewidth lasers for broadband mobile communication has been presented. This method is based on external modulation using two cascaded MZMs and the modified injection synchronized balanced OPLL. The detailed theoretical studies have been performed for the proposed OPLL considering the effect of laser phase noise with non-negligible loop propagation delay. The noise bandwidth and phase-error variance of the proposed OPLL have been calculated. Both first and second-order modified injection synchronized balanced OPLL can allow a large loop-delay within the stability limit. The proposed loop offers more than 50% increase in the laser linewidth requirement and reduces the risk of false locking. Assuming the rotating wave approximation and using the principle of quasi-linearization, the locking range and phase-error variance of the homodyne OPLL have

![Fig. 5.7 Linewidth to locking range ratio as a function of phase-error variance.](image)
been calculated in presence of linewidth enhancement factor. It has been found that the locking range increases with the increase in linewidth enhancement factor. The linewidth to locking range ratio increases with an increase in both the phase-error variance and the linewidth enhancement factor.
References


