Stabilization of Rayleigh-Taylor Instability in a non-Newtonian Incompressible Complex Plasma

The stabilization of Rayleigh-Taylor (RT) instability is investigated in a non-Newtonian unmagnetized dusty plasma with an experimentally verified model of shear flow rate dependent viscosity. It has been found that non-Newtonian property has also a significant role in stabilization of RT instability along with velocity shear stabilization in the short wavelength regime. The effect of the non-Newtonian parameters are more profound in the higher velocity shear rate regime. A detailed investigation has been pursued on the role of non-Newtonian effect on RT instability with conventional dust fluid equations using standard numerical eigenvalue analysis.¹

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7.1 Introduction

The velocity shear parameter is an important parameter for understanding the stability of such a complex system [87–89] as we found in the earlier chapters. The generation of shear flow enables to measure the shear viscosity of those systems which reflects the non-Newtonian signature [97]. The non-Newtonian signature in a complex system like dusty plasma generates a fascinating research field in this direction due to its natural occurrence in various realistic scenarios. In 2005, Liu et al. [98] have reported about the dependence of shear viscosity with the temperature of dust particle, moving randomly in the system, through the Coulomb coupling parameter. Ivlev et. al [99] in their experiment revealed the non-Newtonian signature of the dusty plasma with varying velocity gradient dependent viscosity coefficient over a considerable range of velocity shear rate. Gavrikov et al. [100] have also reported about the non-Newtonian property of the complex dusty plasma fluid.

Along with the non-Newtonian characteristics, gravity also plays an important role in the dust dynamics due to the heavy mass of the dust particles compared to the electron and ion mass. The Rayleigh Taylor instability (RTI), in a multi species neutral fluid with opposing direction of density gradient (i.e. heavy fluid is supported by the lighter one) and gravity, is a classic example of gravity induced instability. Similarly, inverse stratification of dust particles/mass density, influenced by the gravity; leads to a RT like instability in a dusty plasma which has been a subject of research interest for understanding various astrophysical phenomena [109,110]. Although velocity shear drives the instability in most of the cases, but the stabilization of RTI both in magnetized and un-magnetized plasma in presence of velocity shear and curvature is an interesting issue [193] and many linear and nonlinear theories have suggested that, indeed there is a stabilizing effect [93, 94]. Guzdar et al. [95] showed in 1982 that, velocity shear can substantially reduce the growth rate of RTI in the shorter wavelength regime. Hassam [194] also reported the influence of viscous damping and velocity shear effect on RTI. All these previous works on RTI, mentioned above, were made on Newtonian plasmas. The non-Newtonian properties of plasmas in presence of velocity shear are emerging as an interesting field of study in plasma physics to obtain the information about the structure and properties of such complex systems under consideration as viscosity, in general, always shows non-
Newtonian characteristics in most of the fluids and complex systems. This motivates us to study the RTI in a non-Newtonian dusty plasma which may have a significant importance in various dense astrophysical objects, where shear flows are important along with density stratification and shear flow dependent viscosity.

In this work, we have found that the velocity shear gives furthermore stabilization of RTI for a non-Newtonian fluid in shorter wavelength regime, compared with that of the Newtonian one. The effect would be more profound in a situation where the velocity shear rate is high; because the non-Newtonian viscosity, having a complex dependence with the velocity shear rate, shows sharp changes in such regimes. The chapter is organized in the following manner: section 7.2 contains the governing equations for the study and a description about the non-Newtonian viscosity model. In section 7.3, we present the linear stability analysis where the local dispersion relation shows the apparent stabilization effect of velocity shear along with the critical condition for RTI. Thereafter, the non local analysis with the eigenvalue equation has been carried out and the numerical results show further stabilization on RTI in presence of velocity shear for a non-Newtonian fluid. Finally, in section 7.4, a summary of conclusions derived are made and the physical consequences are discussed.

### 7.2 Governing Equations and non-Newtonian Viscosity Model

We consider a non-uniform plasma consisting of electrons, ions and negatively charged dust grains with a plasma density gradient lying along the positive $x$-axis. The acceleration due to gravity is assumed to act along the negative $x$-direction. Since the primary motivation of this work is to study the effect of non-Newtonian viscosity of dust grains on Rayleigh-Taylor instability, we consider the ions, electrons and dust grains to have a finite temperature. As we are interested in very low frequency phenomena i.e. $\omega \ll kv_{th(e/i)}$ (where $v_{th(e/i)}$, the electron and ions are governed by the Boltzmann distribution. The dust density and momentum evolution equations are as follows

$$\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{v}_d) = 0,$$

(7.1)
\[
\rho_d \left( \frac{\partial}{\partial t} + \mathbf{v}_d \cdot \nabla \right) \mathbf{v}_d + \nabla p - \rho_d \mathbf{g} = \frac{\partial \sigma_{ij}}{\partial x_j},
\]  
(7.2)

where \( \rho_d = n_d m_d \), \( n_d \) is the number density, \( Z \) is the number of charge present on a dust particle which is assumed to be negative, \( m_d \) is the mass of the dust particle, \( p \) is the total pressure of the system, \( \sigma_{ij} \) is the viscous stress tensor, \( \mathbf{v}_d \) is the dust fluid velocity and \( \mathbf{g} \) is the gravitational acceleration. It should be noted here that the dust-neutral collision frequency \( (\nu_{dn}) \) is neglected in Eq. (7.2). Although it is an important parameter in some dusty plasma experiment but in this work we are looking for RT instability whose growth rate is much higher than the magnitude of dust-neutral collision frequency \( \nu_{dn} \); therefore we have assumed that \( \nu_{dn} \) cannot arrest the RT growth rate and that is why we ignored it for simplicity. A quantitative estimate follows in the conclusion section about the justification for dropping the term. The viscous stress tensor \( \sigma_{ij} \) is expressed as \([169]\)

\[
\sigma_{ij} = \eta(|\gamma|) \left[ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} (\nabla \cdot \mathbf{v}) \right], \quad \delta_{ij} = 1(0) \text{ for } i = j(i \neq j);
\]

where \( \eta(|\gamma|) \) is the non-Newtonian viscosity coefficient (which is constant for a Newtonian fluid) and \( |\gamma| \) is a scalar invariant quantity which depends upon the rate of strain tensor i.e.

\[
\gamma_{ij} = \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right).
\]

In 2-D; \( i, j \) varies like as \( x, y \) in \((x - y)\) geometry and \( \sigma, \gamma \) can be written as \((2 \times 2)\) matrix structure as,

\[
\sigma = \begin{pmatrix}
\sigma_{xx} & \sigma_{xy} \\
\sigma_{yx} & \sigma_{yy}
\end{pmatrix}, \quad \gamma = \begin{pmatrix}
\gamma_{xx} & \gamma_{xy} \\
\gamma_{yx} & \gamma_{yy}
\end{pmatrix};
\]

where few components can be visualized explicitly as follows

\[
\sigma_{xx} = 2\eta(|\gamma|) \left( \frac{\partial v_x}{\partial x} - \frac{1}{3} \nabla \cdot \mathbf{v} \right), \quad \sigma_{yx} = \eta(|\gamma|) \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right),
\]

\[
\gamma_{yy} = 2 \frac{\partial v_y}{\partial y}, \quad \gamma_{xy} = \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right).
\]

The viscosity coefficient \( \eta \) is a scalar, but depends on the scalar invariant quantity of \( \gamma \). In a \((2 \times 2)\) tensor, two invariant quantity (scalar) can be formed by taking trace
of $\gamma$ and $\gamma^2$, which are independent of the choice of co-ordinate system to which the components of $\gamma$ are referred \[195\]. These two invariants are,

\begin{equation}
I = \sum_i \gamma_{ii} \quad \text{and} \quad II = \sum_i \sum_j \gamma_{ij} \gamma_{ji}, \quad (7.3)
\end{equation}

where $i, j$ varies as $x, y$. Now, for an incompressible medium we get

\begin{equation}
I = \gamma_{xx} + \gamma_{yy} = 2 \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) = 2(\nabla \cdot \mathbf{v}) = 0.
\end{equation}

Therefore, $|\gamma|$ can be modeled as

\begin{equation}
|\gamma| = \sqrt{II/2} = \sqrt{\left( \gamma_{xx}^2 + \gamma_{yy}^2 + 2\gamma_{xy}\gamma_{yx} \right)/2}. \quad (7.4)
\end{equation}

The effect of bulk viscosity is not considered here. In the present problem, the equilibrium dust velocity is considered to be inhomogeneous in $x$ and directed along $y$ i.e. $\mathbf{v}_{d0} = \hat{e}_y v_{0y}(x)$, where $\hat{e}_y$ is a unit vector along the $y$ direction. The dust flow is also considered incompressible so that we have $\nabla \cdot \mathbf{v}_d = 0$. Therefore $\gamma = d v_{0y}/dx$, with $\sigma_{xy} = \sigma_{yx} = \eta(\gamma) \gamma$.

The non-Newtonian viscosity $\eta(\gamma)$ has a specific functional dependence on equilibrium velocity shear flow rate and a mathematical model is required for carrying out the analytical work. In literature of non-Newtonian fluids, there exists many models which are applicable in different physical scenarios, accordingly. The common idea of all the models is that; it should converge to the Newtonian limit i.e., constant viscosity for very small shear rate. In this perspective, we have taken the experimentally verified model for the dependence of kinematic viscosity $\nu(\gamma)$ on velocity shear rate $\gamma$, given by the Ivlev et al. \[99\]. The mathematical expression for the kinematic viscosity i.e. $\nu = \eta/\rho$ is given by

\begin{equation}
\nu(\gamma) = \frac{2(1 + \epsilon)}{\sqrt{1 + 4\gamma^2 - 4\epsilon\gamma^4 + 1 - 2\epsilon\gamma^2}} \tilde{\nu}, \quad (7.5)
\end{equation}

where $\tilde{\nu}$ is the value of Newtonian kinematic viscosity, $\gamma$ (normalized) is the equilibrium velocity shear rate and $\epsilon$ is the parameter that characterizes the non-Newtonian property \[99\]. In the limiting case when $\gamma, \epsilon \to 0$, the viscosity chosen in this model converges to the general Newtonian viscosity i.e. $\nu \to \tilde{\nu}$. In figure (7.1) the non-
Newtonian viscosity is shown plotted against the velocity shear rate $\gamma$ for different values of non-Newtonian parameter $\epsilon$. The figure clearly shows that for increasing $\epsilon$ the fluid property changes from shear thinning (where viscosity decreases with increasing velocity shear) to shear thickening (where viscosity increases with increasing velocity shear).

The non-Newtonian behavior of complex plasmas over a wide range of shear flow is well known and in order to study such effects on RT instability, and to keep the Kelvin-Helmholtz instability outside the scope of this work, we assumed the equilibrium shear flow rate $\gamma = dv_0/dx$ to be constant. We also assumed that the $x$ dependence of equilibrium dust density as $\rho_0(x) = \bar{\rho}_0 \exp(x/L_n)$, where $\bar{\rho}_0$ is constant [196].
7.3 Linear Stability Analysis

We perturbed the system around the equilibrium such that all the perturbations are in the $x - y$ plane. As mentioned earlier the dust velocity and density can be written as follows $v_d = v_0(y) \hat{e}_y + v_d(x, y, t)$, and $\rho_d = \rho_0(x) + \rho_1(x, y, t)$. Let us first linearize the non-Newtonian viscous force term (R.H.S. of the Eq.(7.2)). The linearized form of the rate of strain tensor $\gamma$ takes the form as

$$\gamma = \begin{pmatrix} 2 \frac{\partial v_x}{\partial x} & \epsilon_0 + \epsilon_1 \\ \epsilon_0 + \epsilon_1 & 2 \frac{\partial v_y}{\partial y} \end{pmatrix};$$

where

$$\epsilon_0 = \frac{dv_0y}{dx}, \quad \epsilon_1 = \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}.$$

Now using equation(7.4), $|\gamma|$ can be expressed in linearized form as

$$|\gamma| = \sqrt{2 \left( \frac{\partial v_x}{\partial x} \right)^2 + 2 \left( \frac{\partial v_y}{\partial y} \right)^2 + (\epsilon_0 + \epsilon_1)^2}.$$

Now neglecting the quadratic shear rate fluctuation terms, the above expression can be written as

$$|\gamma| = \epsilon_0 \sqrt{1 + 2 \frac{\epsilon_1}{\epsilon_0}}.$$

In the limit, $\epsilon_1 \ll \epsilon_0$, the above expression can be expanded binomially as $|\gamma| = \epsilon_0 + \epsilon_1$. Now, expanding the viscosity $\eta(|\gamma|)$ in Taylor series, we have

$$\eta(\epsilon_0 + \epsilon_1) \simeq \eta(\epsilon_0) + \frac{d\eta}{d\epsilon_0} \epsilon_1 = \eta_0 + \eta'_0 \epsilon_1;$$

where $\eta'_0$ is the gradient of viscosity with the unperturbed velocity shear rate $v'_0 y$.

The $x$ component of the viscous force in linearized form can therefore be written as,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \frac{\partial}{\partial x} \left[ (\eta_0 + \eta'_0 \epsilon_1) \frac{\partial v_{1x}}{\partial x} \right] + \frac{\partial}{\partial y} \left[ (\eta_0 + \eta'_0 \epsilon_1) (\epsilon_0 + \epsilon_1) \right],$$

$$= \eta_0 \nabla^2 v_x + \eta'_0 v'_0y \frac{\partial}{\partial y} \left( \frac{\partial v_{1x}}{\partial y} + \frac{\partial v_{1y}}{\partial x} \right) + 2 \eta'_0 v''_0 y \frac{\partial v_{1x}}{\partial x}.$$
Similarly, the $y$ component of viscous force in linearized form can be written as,

$$
\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yx}}{\partial x} = \eta_0 \nabla^2 v_{1y} + \left\{ 2\eta_0' v''_{0y} + \eta_0 v''_{0y} \frac{\partial v_{1y}}{\partial x} \right\} \left( \frac{\partial v_{1x}}{\partial y} + \frac{\partial v_{1y}}{\partial x} \right),
$$

where $v'_{0y}$ and $v''_{0y}$ denote 1st and 2nd order derivative of velocity with $x$. Therefore, the linearized momentum equation in $x, y$ components and the continuity equation can be written

$$\rho_0 \left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) v_{1x} + \frac{\partial p_1}{\partial x} - \rho_1 g = \eta_0 \nabla^2 v_{1x} + \eta_0' v'_{0y} \frac{\partial}{\partial x} \left( \frac{\partial v_{1x}}{\partial y} + \frac{\partial v_{1y}}{\partial x} \right) \quad (7.6)$$

$$\rho_0 \left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) v_{1y} + \frac{\partial p_1}{\partial y} + \rho_0 v'_{0y} v_{1x} = \eta_0 \nabla^2 v_{1y} + \eta_0' v'_{0y} \frac{\partial}{\partial x} \left( \frac{\partial v_{1x}}{\partial y} + \frac{\partial v_{1y}}{\partial x} \right) \quad (7.7)$$

$$\left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) \rho_1 + \rho'_0 v_{1x} = 0 \quad (7.8)$$

where $v'_{0y}$ denotes $dv_{0y}/dx$, $\eta_0^*$ denotes $d\eta_0/dv'_{0y}$ and $\mathbf{v}_{d1}(x, y, t) = v_{1x}(x, y, t)\hat{e}_x + v_{1y}(x, y, t)\hat{e}_y$. Since the equilibrium velocity shear profile is chosen to be linear so the second and higher order derivatives of $v_{0y}$ do not occur in the above equations. The density in a dust fluid changes mainly due to compressibility and convection. Since incompressibility has been assumed in the present case, density changes can occur only due to the convection. This is clearly reflected in the perturbed density equation i.e. Eq.(7.8).

Now taking derivatives of Eq.(7.6) and Eq.(7.7) with respect to $y$ and $x$ respectively and subtracting them, we have

$$\rho_0 \left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) \frac{\partial v_{1y}}{\partial x} - \frac{\partial v_{1x}}{\partial y} + \rho'_0 \left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) v_{1y} + \rho'_0 v'_{0y} v_{1x} + g \frac{\partial \rho_1}{\partial y} = \eta_0 \nabla^2 \left( \frac{\partial v_{1y}}{\partial x} - \frac{\partial v_{1x}}{\partial y} \right) + \eta_0^* v'_{0y} \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \left( \frac{\partial v_{1x}}{\partial y} + \frac{\partial v_{1y}}{\partial x} \right) \quad (7.9)$$

The problem considered here is linear and inhomogeneous in $x$ so any arbitrary disturbance may be decomposed into normal modes assuming wave propagation along the $y$ direction as $f(x, y, t) = f(x)e^{i(k_0 y - \omega t)}$, where $\omega$ is the mode frequency,
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$k_y$ is the wave vector along $y$ and $f \equiv (v_{1x}, v_{1y}, \rho_{d1})$. Thus from the perturbed incompressibility condition i.e. $\nabla \cdot \mathbf{v}_{d1} = 0$, we can write

$$v_{1y} = \frac{1}{k_y} \frac{\partial v_{1x}}{\partial x},$$

(7.10)

and from the density evolution equation we have

$$\rho_1 = -i \frac{\rho_0}{(\omega - k_y v_{0y})} v_{1x}.$$  

(7.11)

Now eliminating $v_{1y}$ and $\rho_1$ from Eq.(7.9), we get the normalized single equation in $v_{1x}$ as follows

$$\left[ \omega^2 \left( \frac{\partial^2}{\partial x^2} - \hat{k}_y^2 \right) + \hat{\omega} \left\{ -2\hat{k}_y \hat{v}_{0y} \left( \frac{\partial^2}{\partial x^2} - \hat{k}_y^2 \right) + \hat{k}_y \hat{v}_{0y}' - \left\{ i\hat{v}_0 \left( \frac{\partial^2}{\partial x^2} - \hat{k}_y^2 \right)^2 \right. \right. \right. \right. \right.$$  

$$+ i \hat{v}_0 \hat{v}_{0y}' \left( \frac{\partial^2}{\partial x^2} + \hat{k}_y^2 \right)^2 \right\} + \left[ \hat{k}_y^2 \hat{v}_{0y} \left( \frac{\partial^2}{\partial x^2} - \hat{k}_y^2 \right) - \hat{k}_y^2 - \hat{k}_y \hat{v}_{0y} \hat{v}_{0y}' \right. \right. \right.$$  

$$+ \hat{k}_y \hat{v}_{0y} \left\{ i\hat{v}_0 \left( \frac{\partial^2}{\partial x^2} - \hat{k}_y^2 \right)^2 + i \hat{v}_0 \hat{v}_{0y}' \left( \frac{\partial^2}{\partial x^2} + \hat{k}_y^2 \right)^2 \right\} \right] \hat{v}_{1x} = 0$$  

(7.12)

The normalized variables used in Eq.(7.12) are $\hat{\omega} = \omega/\sqrt{g/L}$, $\hat{x} = x/L$, $\hat{k}_y = k_y L$, $\hat{v}_{0y} = v_{0y}/\sqrt{gL}$, $\hat{v}_{0y}' = v_{0y}' L/\sqrt{gL}$, $\hat{v}_{1x} = v_{1x}/\sqrt{gL}$, $\hat{v}_0 = (\eta_0/\rho_0)/L\sqrt{gL}$ and $\hat{v}_{0y}' = (\eta_0/\rho_0)/L^2$, where $L \equiv L_n$.

7.3.1 Local Analysis

The local dispersion relations are valid only for the perturbations whose scale length in the $x$-direction are much smaller than the inhomogeneity scale length. In this limit, we treat $v_{0y}$, $\eta_0$ and their derivatives as parameters. This enables Fourier analysis of Eq.(7.12) with respect to $x$ also and we obtain the dispersion relation as

$$\hat{\omega}^2 - \hat{\omega} \left[ \frac{\hat{k}_y^2}{\hat{k}_x^2} \hat{v}_{0y}' - \left\{ i\hat{v}_0 \hat{k}_x^2 + i \hat{v}_0 \hat{v}_{0y}' \left( \frac{\hat{k}_y^2 - \hat{k}_x^2}{\hat{k}_x^2} \right) \right\} \right] + \frac{\hat{k}_y^2}{\hat{k}_x^2} = 0$$  

(7.13)

where $\hat{k}_x^2 = \hat{k}_x^2 + \hat{k}_y^2$ and $\hat{k}_y = k_y L$. Here $\hat{\omega} = \omega - \hat{k}_y \hat{v}_{0y}$ is the Doppler shifted frequency. If the flow is inviscid, the effect of velocity shear on RT can obviously be
visible from the above equation as
\[
\bar{\omega} = \frac{\hat{k}_y}{k} \left[ \frac{\hat{v}'_0 y}{k} \pm \frac{1}{2} \left( \frac{\hat{v}'_0^2}{k^2} - 4 \right)^{1/2} \right]
\] (7.14)

with a condition for instability as \( \hat{v}'_0^2 / \hat{k}^2 - 4 < 0 \) [95] and also if there is no equilibrium flow then we get back the normal RTI as \( \omega = i \sqrt{(g/L) (k_y/k)} \). Thus Eq.(7.14) reveals the stabilization of RTI due to velocity shear as well as the \( k \)-dependence of the stabilization. In higher wavelength regime, velocity shear tries to maximize the RTI whereas in the shorter wavelength regime it stabilizes the RTI.

### 7.3.2 Non-local Analysis:

This nonlocal analysis enables us to study the modes with wavelengths comparable or greater than the inhomogeneity scale length. The matrix eigenvalue analysis of the differential equation (7.12) has been carried out with the previously mentioned velocity shear and density profiles by using the eigenvalue subroutine (\texttt{eig}) in MATLAB software after the proper discretization of the said equation with standard finite difference discretization scheme. After a few algebraic steps, the linearized second order equation (7.12) reduces to a polynomial eigenvalue problem in \( \hat{\omega} \) as

\[
\left[ A_0 - \hat{\omega} A_1 - \hat{\omega}^2 A_2 \right] \hat{v}_{1x} = 0,
\] (7.15)

where \( A_i \)'s are the matrix elements in above equation, where \( i = 0, 1 \) and 2. The polynomial eigenvalue problem can be changed into a more general eigenvalue problem by using a dummy variable \( \chi = \hat{\omega} \hat{v}_{1x} \). Therefore, the new eigenvalue problem can be expressed as

\[
\begin{pmatrix} A_0 & Z \\ Z & I \end{pmatrix} \begin{pmatrix} \hat{v}_{1x} \\ \chi \end{pmatrix} = \hat{\omega} \begin{pmatrix} A_1 & A_2 \\ I & Z \end{pmatrix} \begin{pmatrix} \hat{v}_{1x} \\ \chi \end{pmatrix},
\] (7.16)

where \( I \) is the identity matrix and \( Z \) is the null matrix of the same order with all \( A_i \)'s. This trick simplifies the original polynomial eigenvalue problem into a more
Figure 7.2: Eigenfunctions of $\hat{v}_{1x}$ real (red) and imaginary (blue) are shown for the maximum growth rate point for RTI in presence of velocity shear ($\hat{v}_{0y} = 0.4$) for a non-Newtonian dusty plasma with $k_xL = 0.5$.

simple and well known matrix eigenvalue problem as

$$R\bar{\phi} = \omega S\bar{\phi};$$

where

$$R = \begin{pmatrix} A_0 & Z \\ Z & I \end{pmatrix}; \quad S = \begin{pmatrix} A_1 & A_2 \\ I & Z \end{pmatrix}; \quad \bar{\phi} = \begin{pmatrix} \hat{v}_{1x} \\ \chi \end{pmatrix}.$$  

The eig subroutine is used to solve the eigenvalue equation in MATLAB. We have calculated the imaginary part of eigenvalues $\hat{\omega}$, the positive value of which indicates the growth rate of the RT mode which has a bounded eigenfunction as shown in figure (7.2).

In figure (7.3), the growth rate of RT is shown plotted against wave number ($k_yL$) for different cases such as - RT in inviscid fluid (without and with velocity shear), RT with viscosity both in Newtonian and non-Newtonian (with different values of non-Newtonian parameter) regimes in presence of shear flow. The substantial
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Figure 7.3: Plots of growth rates vs $k_y L$ under various conditions. At the top, the black line with circle marker shows the Rayleigh-Taylor instability without velocity shear. The blue one shows stabilization of RT instability in presence of finite velocity shear ($v_0' y = 0.4$). When viscosity added to the system, the red curve shows further stabilization of RT instability in Newtonian limit. Now in non-Newtonian limit the figure shows further stabilization for different value of non-Newtonian parameter $\epsilon = 0.1, 0.3, 0.5$ and $0.8$ by dark green, magenta, light green and ash respectively. For the Newtonian plot and all the non-Newtonian plots the velocity shear parameter is $v_0' y = 0.4$.

stabilization of RT instability by velocity shear is clearly visible [95]. On the other hand viscosity, being dissipative in nature, also leads to the lowering of the growth rate. In the figure, it has been shown that shear thickening regime (higher $\epsilon$) is more effective for stabilization than the shear thinning regime (smaller $\epsilon$) for the short wavelength modes. The growth rate of the pure RT instability increases with wave number $\hat{k}_y$, however viscosity, being of higher order in spatial derivatives, begins to play role at higher $\hat{k}_y$ i.e. at smaller scales due to which the growth rate begins to decrease. At small values of $\hat{k}_y$, the effect of viscosity is totally insignificant due to which the growth rate does not show any dependence on the non-Newtonian characteristics of the plasma. This leads to increase of the growth rate of RTI with
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Figure 7.4: Plots of growth rates vs $k_y L$ for various velocity shear rate parameter. The red one shows the Rayleigh-Taylor instability without velocity shear. In the non-Newtonian limit for fixed value of non-Newtonian parameter $\epsilon = 0.5$, the figure shows the stabilization of RT instability for different value of shear rate parameter $\dot{\epsilon}_{0y} = 0.2, 0.5$ and $0.8$ by dark green, blue and magenta respectively.

$\hat{k}_y$ and at some value of $\hat{k}_y (\sim 1.6)$, viscous effects begin to become important and the growth rate begins to decrease. At these values of $\hat{k}_y (\sim 1.6 - 2.0)$, $\epsilon$ still does not play any role. The non-Newtonian characteristics of viscosity being compounded with other effects such as velocity shear and non-Newtonian parameter begin to show their effect at further smaller scales ($\hat{k}_y \sim 2.0$ onwards) which then lift the degeneracy of growth-rate dependence on $\hat{k}_y$. These results demonstrate that the velocity shear rate dependent viscosity (non-Newtonian regime) has a prominent role in stabilization of RT instability.

In figure (7.4), growth rates are plotted for different values of velocity shear parameter and due to the coupling between shear and non-Newtonian viscosity gradient the increasing velocity shear leads to further stabilization of RTI. Physically, velocity shear extracts energy from gravitational energy thus making lesser energy available for the growth of RT instability.
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Figure 7.5: Plots of growth rates vs $k_y L$ for various types of fluids. The blue one shows the pure Rayleigh-Taylor instability without any velocity shear. The black ones shows the suppression of growth rate due to velocity shear for an inviscid fluid. For a Newtonian fluid ($\dot{\nu} = 0.01$), shown by the red-curve there is further stabilization. In the non-Newtonian limit for fixed value of non-Newtonian parameter $\epsilon = 0.5$, the magenta colored curve shows even more stabilization of RT instability compared to Newtonian-fluid due to velocity shear rate dependence of viscosity.

In figure (7.5), growth rates are plotted for various fluid characteristics. For an inviscid fluid, velocity shear stabilizes the pure RT substantially. In a viscous one with constant kinematic viscosity i.e. Newtonian fluid there is further stabilization of RT compared to velocity shear. For a non-Newtonian fluid with $\epsilon = 0.5$ it is clear from the figure that there is even more stabilization of RTI compared to the Newtonian one which clearly shows that the gradient of viscosity with respect to velocity shear rate contributes to this stabilization.

Similarly in figure (7.6), it is found that with increasing $\epsilon$ the growth rates are decreasing which also shows the effective stabilization of RTI in the non-Newtonian regime. It should also be noted that, when $\dot{\nu}_0 y = 0$, there is still a viscosity dependence on the right hand side of the Eq.(7.9). Also from Eq.(7.5), it is clear that
even if $\gamma = 0$, viscosity depends on the non-Newtonian parameter $\epsilon$. Therefore the curves start at different values of growth rate for different $\epsilon$ at $v_0' = 0$. The growth rate dependence on $\epsilon$ is distinctly visible also for small values of the velocity shear parameter, as throughout this region the fluid remains more or less Newtonian or a shear thinning one as shown in figure (7.1). For high values of shear rate parameter, the fluid behaves as a shear thinning one for small $\epsilon$ but as a shear thickening one at high values of $\epsilon$. This result reflects in figure (7.6) where for small values of $\epsilon$ the growth rate curves merge with each other at high shear rate values but remain distinct for high values of $\epsilon$. For large values of $\epsilon$, the shear thickening fluid has its viscosity increasing with increasing velocity shear so that the enhanced viscous force stabilizes the system against RT instability. These investigations lead us to an understanding about the stabilization of RTI in the non-Newtonian regime in presence of velocity shear.

![Figure 7.6](image-url)

**Figure 7.6:** Plots of growth rates vs $k_y L$ for various non-Newtonian parameter ($\epsilon$). The green one shows the growth rates of Rayleigh-Taylor instability decreases with shear for a Newtonian fluid. In the non-Newtonian limit, the figure also shows the further stabilization of RT instability for different values of non-Newtonian parameter $\epsilon = 0.2, 0.5$ and $0.8$ by red, blue and magenta respectively.
7.4 Conclusions

In summary, we have investigated the stabilization characteristics of a non-Newtonian dusty plasma against RT instabilities. The experimentally verified model of velocity shear rate dependent viscosity has been taken to demonstrate the effect of velocity shear on RTI. The linear stability of the system has been investigated by using the standard dust dynamics equations. In the momentum equation, we have neglected the dust-neutral collision frequency \( (\nu_{dn}) \). Although it is an important parameter in dusty plasma experiments but since we are looking for RT instability whose growth rate magnitude is much higher \( (\sim \sqrt{(g/L)}k_y/k \approx 30 \text{ s}^{-1}; \text{for} \ L \sim 1 \text{ cm}, g \sim 981 \text{ cm/s}^2 \text{ and} \ k_y/k \sim 1) \) than the magnitude of dust-neutral friction \( \nu_{dn} \ (\sim 2.4 \text{ s}^{-1}) \) [163,197–199], therefore we have assumed that \( \nu_{dn} \) cannot arrest the RT growth rate and hence it is ignored for simplicity. Also, we are looking for the stabilization of RTI by the scale length dependent parameters i.e. non-Newtonian parameter \( (\epsilon) \) compounded with velocity shear \( (\dot{\epsilon}_{0y}) \), so that the constant parameter \( \nu_{dn} \), which is dissipative in nature can only moderately lower the effective growth rate does not generate any interest in this present context.

In nonlocal analysis, by taking all the inhomogeneities into account, we have derived the nonlocal differential equation and solved that matrix eigenvalue equation instead of Fourier decomposition along the inhomogeneity direction. The results indicate the prominent effect of velocity shear on RTI in a non-Newtonian dusty plasma in the shorter wavelength regime. The effect of the velocity shear on RT is \( k \) dependent which is obvious from the local dispersion results, therefore for sufficiently long \( k \) the effect is very small. In a non-Newtonian fluid with a high value of non-Newtonian parameter \( \epsilon \), viscosity increases with velocity shear rate causing the fluid to behave as a shear thickening fluid and the coupling term between the velocity shear and viscosity gradient further reduces the growth rate of the RTI compared to the velocity shear reduction of growth rates of RTI. However, in the shear thinning regime i.e. small \( \epsilon \), the effect of stabilization of RT in presence of velocity shear is not so much profound although as soon as \( \epsilon \) increases, the system goes from shear thinning to shear thickening regime and viscosity further stabilizes the RTI as viscosity always act as dissipative agent in a system compared to the normal Newtonian fluid.
7.4. Conclusions

The growth rate of the pure RT instability increases with wave number $k_y$ initially and for higher $k_y$ where $k_y/k \sim 1$ it saturates to $\sim \sqrt{g/L}$. However viscosity, being of higher order in spatial derivative, begins to play role at smaller scales due to which the growth rate begins to decrease. At higher scale lengths, the effect of viscosity is totally insignificant due to which the growth rate of RTI does not show any dependence neither on viscosity nor on non-Newtonian characteristics of

Figure 7.7: The top figure shows the contour plot of the growth rate in the plane of wave number $(k_y L)$ and velocity shear $(v'_0 y)$. In the bottom figure, the surface plot of growth rate is drawn in the parametric space of wave number $(k_y L)$ and velocity shear $(v'_0 y)$. In both cases the value of the non-Newtonian characteristic parameter $\epsilon$ is set to 0.5.
the plasma. The viscous effects begin to become important and the growth rate begins to decrease only in smaller scale lengths. The non-Newtonian characteristics of viscosity being compounded with other effects such as velocity shear and non-Newtonian parameter becomes significant at even more smaller scales which can be seen clearly in figure (7.3). In the shear thinning regime, as the viscosity decreases with increasing velocity shear rate, so the stabilization due to dissipative effect is not so effective. On the other hand in the shear thickening regime, as viscosity increases with increasing shear rate, the RTI is stabilized in the shorter wavelength regime. In context of fusion plasma, dense astrophysical plasmas, interstellar clouds, neutron stars etc. where both density and shear flow are important parameters, the result of the present study about the stability properties of RTI with non-Newtonian characteristics may have relevant significance.

Normally viscosity acts as a dissipative agent and velocity shear acts as energy source which triggers the instability in a plasma system. In RTI velocity shear substantially reduces the growth rate of the RTI mode by extracting gravitational energy which reduces the available effective energy for the instability. The viscosity also being a dissipative parameter as mentioned earlier contributes to the suppression of RTI along with velocity shear which is quite evident. The interesting part of this work is that the non-Newtonian viscosity of the system which has a complex dependence on the velocity shear rate $\gamma$ significantly suppresses the RTI in shorter wave length regime. Individually both viscosity and shear stabilize the RTI mode in shorter wave length regime but the coupling term between the shear and viscosity makes the work more relevant and interesting. This result can be very useful for understanding the properties of the high velocity plasma jets emerging in various astrophysical phenomena.

Now, the question arises that; what will be the effect of strong correlation between dust particles in RTI with finite shear? To find this, we have pursued stability analysis of RTI in case of a strongly correlated dust fluid in presence of finite velocity shear, which we are going to discuss it in the next chapter in detail.