Chapter 1

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1.1 Cosmology

Cosmology deals with the large-scale structure, evolution and the origin of the universe. The large-scale mass distribution in the universe tells us about the nature and evolution of the universe and structure described by models is determined by the law of gravitational attraction. As gravitation is the sole long range force binding the constituents of the universe, to understand the nature and evolution of the large scale structures, any theory on cosmos is essentially based on background theory of gravitation. Einstein in 1916 introduced a more powerful theory of gravity, known as general theory of relativity. It has been stated by Chandrashekhar [1] that "the general relativity is like a garden where flowers and weeds grow together. The useless weeds are cut with the desired flowers and separated later". The first application of general theory of relativity to study cosmology was considered by Einstein himself in 1917. He pioneered the subject with static universe, followed by de Sitter cosmological model. Subsequent, developments have been characterized by a series of major ideas. This has evolved from a simple application of space-time geometry with a perfect fluid matter source, to a sophisticated physical theory
for complex matter sources with highly developed observational implications that enable
detailed testing of the realism of cosmological models and making them an important part
of present day astronomical structures, owe their form to mode of evolution of the universe
in general and, probably, to some high-energy processes of fundamental physics occurring
in the early universe. In particular, cosmological models now form the broad framework
for astrophysics, as well as provide tests of different aspects of fundamental physics.

The first expanding model of the universe with spatially homogeneous and isotropic time
dependent geometry was discovered by Friedmann [2, 3] and independently by Lemaitre
[4, 5]. The Friedmann-Lemaitre-Robertson-Walker models [6–10] use to describe large-
scale structure of the universe exactly in the geometry. It follows that co-moving co-
ordinates can be chosen so that the space-time metric takes the form

\[ ds^2 = dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \]

(1.1)

where the surfaces \( t = \text{constant} \) are the surface of homogeneity, the constant curvature
\( k \) of three spaces takes values \( \pm 1 \) or \( 0 \), \( R(t) \) is the scale factor and the world-lines with
tangent vector \( u^a \) represent the motion of fundamental observers. The space sections have
three-curvature \( K = \frac{k}{R^2(t)} \); as \( R(t) \) increases, the distance between co-moving matter in-
creases, and the matter density decreases because of the energy conservation equation [11].

The astronomical observational evidences suggest that the universe on large scale can
be treated as isotropic and homogeneous, but it is not so at smaller scale. Misner [12],
proposed the “Chaotic Cosmology” program according to which initially highly irregular
universe approaches the FLRW model stage only during the process of cosmological evolu-
tion. This program inspired the research for inhomogeneous and anisotropic cosmological
model of the universe. Spatially homogeneous and anisotropic space-time belong either to
Bianchi class or to the Kantowski-Sachs class. The Bianchi space-time possesses a three
dimensional group of isometry \( G_3 \), acting simply transitively on the three dimensional
spaces, denoted as I, II, III, IV, V, VIa, VIIa, VIII, and IX, where ‘a’ is a nonnegative
parameter, subdividing Bianchi classes VI, and VII. The FLRW cosmological models are included in the Bianchi classes as special cases; $k = +1$ type IX, $k = 0$ type I and VII$_0$, and $k = -1$ type V and VII$_a$ with $a > 0$. The other Bianchi types have no isotropic equivalent: they are intrinsically anisotropic. Bianchi and Kantowski-Sachs cosmological models are being intensively studied since quite along time in different contexts as these classes can couple to any theory of gravitation.

### 1.2 Exact Solutions and its Development

The theories of modern physics generally involve a mathematical model, defined by a certain set of differential equations, and supplemented by a set of rules for translating the mathematical results into meaningful statements about the physical world. In the case of theories of gravitation, it is generally accepted that the most successful is Einstein's theory of General Relativity. Here the differential equations consist of purely geometric requirements imposed by the idea that space and time can be represented by the interaction of matter and gravitation contained in Einstein's famous field equations

$$R_{ij} - rac{1}{2} R g_{ij} + \Lambda g_{ij} = \chi_0 T_{ij}$$  (1.2)

($\chi_0$ being Einstein's gravitational constant and $\Lambda$ the cosmological constant) connect the Ricci tensor (with components $R_{ij}$) with the energy-momentum tensor (with components $T_{ij}$).

The Einstein’s field equations (1.2) connect the Ricci tensor $R_{ij}$ with the energy momentum tensor $T_{ij}$. The Bianchi identities

$$R_{[\alpha\beta\gamma\delta]} = 0, \quad (1.3)$$

imply the important relation

$$\chi_0 T^{ab}_{;b} = (R^{ab} - \frac{1}{2} R g^{ab})_{;b} = 0. \quad (1.4)$$
1.2 Exact Solutions and its Development

As well as vacuum fields (empty space)

\[ R_{ab} = 0, \quad (1.5) \]

with zero \( T_{ab} \) and \( \Lambda \), we can consider solutions of the field equations (1.2) for the following physically relevant energy-momentum tensor:

(i) electromagnetic field (Maxwell field):

\[
T_{ab} + F_{ac}F^c_b - \frac{1}{4}g_{ab}F_{cd}F^{cd} = \frac{1}{2}(F_{ac}F^c_b + F^c_bF_{ac}) - \frac{1}{2}F^c_dF^d_{bc},
\]

\[
F_{ab}^* \equiv F_{ab} + i\widetilde{F}_{ab},
\]

\[
\widetilde{F}_{ab} \equiv \frac{1}{2}\epsilon_{abcd}F^{cd}, \quad F_{ab}^{*ab} = 0.
\]  \( (1.6) \)

(ii) pure radiation field (null dust):

\[
T_{ab} = \Phi^2k_ak_b, \quad k_ak^a = 0.
\]  \( (1.7) \)

(iii) perfect fluid:

\[
T_{ab} = (\rho + p)u_au_b + \rho g_{ab}, \quad u_au^a = -1.
\]  \( (1.8) \)

In the perfect fluid case we normally assume \( \rho + p \neq 0 \), \( \rho > 0 \). In the particular case where \( T_{ab} = 0 \) and \( \Lambda \neq 0 \), or where \( T_{ab} \) is of perfect fluid type (1.8) but with \( \rho + p = 0 \), we shall say the Ricci tensor is of \( \Lambda \)-term type. Thus the perfect fluid solutions formally include \( \Lambda \)-term case, the Einstein spaces \( R_{ab} = \Lambda g_{ab} \). They also include the combination of a perfect fluid and a \( \Lambda \)-term. The latter can be incorporated in the fluid quantities by putting \((p - \Lambda/\chi_0)\) for \( p \) and \((\rho + \Lambda/\chi_0)\) for \( \rho \); of course this substitution may violate the condition \( \rho > 0 \). In the other non-vacuum cases the cosmological constant \( \Lambda \) is usually set equal to zero. In general we do not consider superpositions of these energy-momentum tensors. By virtue of the field equations (1.2), \( T_{ab} \) has the same algebraic type as \( R_{ab} \).
We do not, of course, set out to discuss all aspects of General Relativity. For any physical theory, there is also the purely mathematical problem of analyzing, as far as possible, the set of differentials and of finding as many exact solutions, or as complete a general solution, as possible. Next comes the mathematical and physical interpretation of the solutions thus obtained; in the case of General Relativity this requires global analysis and topological methods rather than just the purely local solution of the differential equations. In the case of gravity theories, because they deal with the most universal of physical interactions, one has an additional class of problems concerning the influence of the gravitational field on other fields and matter; these are often studied by working within a fixed gravitational field, usually an exact solution.

This thesis deals primarily with the solutions of Einstein’s equations (1.2), and only tangentially with the other subjects. Unfortunately, one cannot say that the study of exact solutions has always maintained good contact with work on more directly physical problems. Kinnersley [13] wrote “Most of the known exact solutions describe situations which are frankly un-physical, and these do have a tendency to distract attention from the more useful ones. But the situation is also partially the fault of those of us who work in this field. We toss in null currents, macroscopic neutrino fields and tachyons for the sake of greater ‘generality’; we seem to take delight at the invention of confusing anti-intuitive notation; and when all is done we leave our newborn metric wobbling on its vierbein without any visible means of interpretation.”

In defence of work on exact solutions, it may be pointed out that certain solutions have played a very important role in the discussion of physical problems. Obvious examples are the Schwarzschild and Kerr solutions for black holes, the Friedmann solutions for cosmology, and the plane wave solutions which resolved some of the controversies about the existence of gravitational radiation. It should also be noted that because general Relativity is a high non-linear theory, it is not always easy to understand what qualitative features solutions might possess, and here the exact solutions, including many such as the
Taub-NUT solutions which may be thought un-physical, have proved an invaluable guide. Though the fact is not always appreciated, the non-linearities also mean that perturbation schemes in General Relativity can run into hidden dangers (see e.g. Ehlers et al. [14]). Exact solutions which could be compared with approximate results would be very useful in checking the validity of approximation techniques.

Those who do not work in the field often suppose that the intractability of the full Einstein equations means that very few solutions are known. In a certain sense this is true: we know relatively few exact solutions for real physical problems. In most solutions, for example, there is no complete description of relation of field to sources. Problems which are without an exact solution include the two-body problem, the realistic description of our inhomogeneous universe, the gravitational field of a stationary rotating star, and the generation and propagation of gravitational radiation from a realistic bounded source. There are, on the other hand, some problems where the known exact solutions may be the unique answer, for instance, the Kerr and Schwarzschild solutions for the final collapsed state of massive bodies.

Any metric whatsoever is a "solution" of (1.2) if no restriction is imposed on the energy-momentum tensor, since (1.2) then becomes just a definition of $T_{ij}$; so we must first make some assumption about $T_{ij}$. Beyond this we may proceed, for example, by imposing symmetry conditions on the metric, by restricting the algebraic structure of the Riemann tensor, by adding field equations for the matter variables, or by, imposing initial and boundary conditions. The exact solutions that are known have all been obtained by making some such restrictions. We have used the term "exact solution" without a definition, and we do not intend to provide one. Clearly a metric would be called an exact solution if its components could be given, in suitable coordinates, in terms of the well-known analytic functions (polynomials, trigonometric and hyperbolic functions, and so on). It is then hard to find grounds for excluding any analytic function, even one defined only by some system of differential equations. Thus "exact solution" has a less clear meaning than
one might like, although it conveys the impression that in some sense the properties of the metric are fully determined; no generally-agreed precise definition exists. We have proceeded rather on the basis that what we chose to include was, by definition, an exact solution.

In the first few years (or decades) of research in General Relativity, only rather a small number of exact solutions were discussed. These mostly arose from highly idealized physical problems, and had very symmetry. As examples, one may cite the well-known spherically-symmetric solutions of Schwarzschild, Reissner and Nordsörm, Tolman, and Friedmann (this last using the spatially-homogeneous metric form now associated with the names of Robertson and Walker), the axisymmetric static electromagnetic and vacuum solution of Weyl, and plane wave metrics. Although such a limited range of exact solutions was studied, we must, in fairness, point out that it includes nearly all the exact solutions which are of importance in physical applications; perhaps the only one of comparable importance which was a post-war discovery is the Kerr solution.

In the early period there were comparatively few people actively working on General Relativity, and it seems to us that the general belief that time was that exact solutions would be of little importance, except perhaps as cosmological and stellar models, because of the extreme weakness of the relativistic corrections to Newtonian gravity. Of course, a wide variety of physical problems were attacked, but in a large number of cases they were treated only by some approximation scheme, especially the weak-field, slow-motion approximation.

Moreover, many of the techniques now in common use were either unknown or at least unknown to most relativists. The first to become popular was the use of groups of motions, especially in the construction of cosmologies more general than Friedmann’s. The next, which was in part motivated by the study of gravitational radiation, was the algebraic
classification of the Weyl tensor (into Petrov types) and the understanding of the properties of algebraically special metrics. Both these developments led in a natural way to the use of invariantly-defined tetrad bases, rather than coordinate components. The null tetrad methods, and some ideas from the theory of group representations and algebraic geometry, gave rise to the spinor techniques, and equivalent methods, now usually employed in the form given by Newman and Penrose. Using these methods, it was possible to obtain many new solutions, and this growth is still continuing: there are now over 100 new papers on exact solutions every year.

There are (at least) four schemes for classification of the known exact solutions which could be regarded as having more or less equal importance; these four are the algebraic classification of conformal curvature (Petrov types), the algebraic classification of the Ricci tensor (Plebański types) and the physical characterization of the energy-momentum tensor, the existence and structure of the preferred vector fields, and the group of symmetry “admitted by” (i.e., which exist for) the metric. The four-dimensional presentation of the solutions that would arise from the classification schemes outlined above may be acceptable to relativists.

Ever since the 1930s, it has been conventional wisdom in cosmology that the Friedmann [2,3]-Lemaitre [4,5]-Robertson [6–8]-Walker [9,10] (FLRW) models describe the large-scale properties of our observed universe faithfully. At the same time, it has been conventional wisdom in relativity theory that finding exact solutions of the Einstein equations is extremely difficult and possible only for exceptionally simple cases. Both these views where challenged been educated with this conventional wisdom solidly incorporated into their minds. As a result of this situation, a large body of literature has come into existence in which exact solutions generalizing FLRW have been derived and applied to the description of our observed Universe, but most of it remains unknown to the physics community and is not being introduced into textbooks. An exact solution of the Einstein equations is termed “cosmological” if it can reproduce a FLRW metric when its arbitrary
1.3 The Cosmological Constant $\Lambda$

The characteristic feature of general relativity is that the source for gravitational field is the entire energy-momentum tensor. In nongravitational physics, only changes in energy from one state to another are measurable; the normalization of the energy is arbitrary. For example, the motion of a particle with potential energy $V(x)$ is precisely the same as that with a potential energy $V(x) + V_0$, for any constant $V_0$. In gravitation, however, the actual value of the energy matter, not just the difference between states. This behaviour opens up the possibility of vacuum energy: an energy density characteristic of empty space. One feature that we might want the vacuum to exhibit is that it not pick out a preferred direction: it will still be possible to have a nonzero energy density if the associated energy-momentum tensor is Lorentz invariant in locally inertial coordinates. Lorentz invariance implies that the corresponding energy-momentum tensor should be proportional to the metric,

$$T^\text{vac}_{\mu\nu} = -\rho\text{vac}\eta_{\mu\nu},$$  \hspace{1cm} (1.9)

since $\eta_{\mu\nu}$ is the only Lorentz invariant $(0, 2)$ tensor. This generalizes straightforwardly from inertial coordinates to arbitrary coordinates as

$$T^\text{vac}_{\mu\nu} = -\rho\text{vac}\delta_{\mu\nu}.$$  \hspace{1cm} (1.10)

comparing to the perfect-fluid energy-momentum tensor $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$, we find that the vacuum looks like a perfect fluid with an isotropic pressure opposite in sign to the energy density,

$$P\text{vac} = -\rho\text{vac}.$$  \hspace{1cm} (1.11)

The energy density should be constant throughout spacetime, since a gradient would not be Lorentz invariant.
If we decompose the energy-momentum tensor into a matter piece $T^M_{\mu \nu}$ and a vacuum piece $T^\text{vac}_{\mu \nu} = -\rho_{\text{vac}} g_{\mu \nu}$, Einstein's equation is

$$R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} = 8\pi G (T^M_{\mu \nu} - \rho_{\text{vac}} g_{\mu \nu}).$$  \hspace{1cm} (1.12)

Soon after inventing GR, Einstein tried to find a static cosmological model, since that was what astronomical observations of the time seemed to imply. The result was the Einstein static universe. In order for this static cosmology to solve the field equation with an ordinary matter source, it was necessary to add a new term called the cosmological constant, $\Lambda$, which enters as

$$R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} + \Lambda g_{\mu \nu} = 8\pi G T_{\mu \nu}.$$  \hspace{1cm} (1.13)

From comparison with Eq. (1.12), we see that the cosmological constant is precisely equivalent to introducing a vacuum energy density

$$\rho_{\mu \nu} = \frac{\Lambda}{8\pi G}.$$  \hspace{1cm} (1.14)

The term “cosmological constant” and “vacuum energy” are essentially interchangeable.

The cosmological constant $\Lambda$ was originally introduced into cosmology by Einstein [15] in 1917 under the influence of Mach's principle. Mach postulated that all the matter in the universe including the distant stars provided a ‘background’ against which motion could be measured and that unless there was a material background which provide a reference frame, it was not meaningful to talk about rest or motion of a body in any absolute sense. In order to satisfy Mach's principle, Einstein assume that space is globally closed and the universe may be static. These two assumptions were not compatible with general invariance and the energy momentum law. No such static solution of Einstein's original equations could be found hence he modified the field equations by adding a new term involving a free parameter $\Lambda$ known as cosmological “constant”. The static Einstein universe satisfies Mach's principle since it demonstrate that without matter there can be no space against which background inertial effect can be measured. A positive cosmological “constant” plays the role of a repulsive force which can counterbalance the
attractive force of gravity leading to a static Einstein universe. During the same year 1917 de Sitter [21] proposed another cosmological model without any matter distribution and suggested that the cosmological "constant" would cause test particles to accelerate away from one another. After five years of these discoveries, Friedmann [17] obtained a matter dominated expanding model of the universe without a cosmological "constant". The possibility that the universe may be expanding led Einstein to give up the idea of a static universe and the cosmological "constant". The remarkable discovery by Hubble (1929) of a linear expansion law relating redshift to distance made realization that the universe is expanding. After these investigations of cosmic expansion, once Einstein said "The introduction of cosmological "constant" is biggest blunder of my life". Introduced, then rejected, the cosmological "constant" has its time coming - - and going- - - and coming again- - - and so on. The cosmological "constant" remains the focal point of cosmology and particle theory. It is widely believed that the value of \( \Lambda \) was large during the early stages of the universe and strongly influenced its expansion. In the context of quantum field theory a cosmological "constant" corresponds to the energy density of the vacuum. The theoretical value of \( \Lambda \) exceeds observational limits by some 120 orders of magnitude below the value for the vacuum energy density (It is customary to associate a positive cosmological "constant" \( \Lambda \) with a vacuum density \( \rho_v = \frac{\Lambda}{8\pi G} \) predicted by quantum field theory [18]. Zeldovich [19] showed that the vacuum energy-momentum tensor generated by one-loop quantum effect in an expanding space-time geometry had exactly the form of a cosmological "constant". Linde [20] has suggested that the cosmological "constant" may be considered as a function of temperature and hence it varies with time in a homogenous universe as temperature depends on time. The time varying cosmological "constant" resolves the dilemma between a very large cosmological "constant" predicted by field theory and extremely small value (\( \Lambda_0 \leq 10^{-56} \text{cm}^{-2} \)) suggested by observations. In recent years the cosmological "constant" problem is considered as one of the most important problem in cosmology as it resolves many outstanding problems in natural way. Many aspects of \( \Lambda \) - cosmologies such as the age problem, classical tests, observational constraints on structure formation and gravitational lenses have been discussed in the literature. The first concrete mechanism for a variable \( \Lambda \) term based on a non minimally coupled scalar field,
was proposed by Dolgov [?]. Tian [22] has obtained static cylindrically symmetric vacuum solutions of Einstein's equations with cosmological "constant" which represent the exterior metric of a cosmological string. The first review of history of cosmological "constant" problem was presented by Weinberg [23]. He proposed five different approaches (viz (1) super-symmetry, super-gravity, super-string, (2) anthropic consideration, (3) adjustment mechanism, (4) changing gravity and (5) quantum cosmology) as solution of the cosmological "constant" problem. The effects of varying cosmological "constant" on the baryogenesis and primordial nucleosynthesis was considered by Sato et al. [24]. It was speculated that the universe inflates extremely rapidly by the slowly-decaying cosmological "constant" in the period between baryogenesis and the primordial nucleosynthesis. Considering general relativistic approach in which $\Lambda$ is a dynamical variable, Chen and Wu [25] argued that the cosmological "constant" $\Lambda$ varies in time as $R^{-2}$, $R$ being the scale factor of the expanding universe. Their argument is based on dimensional considerations in the spirit of quantum cosmology in which the vacuum energy density can be written as $M_{pl}^4$ times a dimensionless quantity where $M_{pl} = (\frac{\hbar c}{G})^{\frac{1}{4}}$ is the Plank mass. They therefore considered

$$\Lambda \propto \frac{1}{\ell_{pl}^2} \left[ \frac{\ell_{pl}}{R} \right]^n,$$

where $\ell_{pl} = (G\hbar/c^3)^{\frac{1}{2}}$ is the Plank length. Carvalho et al. [26] have suggested that the relation (1.15) is not the only possible dynamic law for $\Lambda$ given by the Chen and Wu ansatz. One may for example, also write

$$\Lambda \propto \frac{1}{t_{pl}^2} \left[ \frac{t_{pl}}{t_H} \right]^n,$$

where $t_{pl}$ and $t_H = H^{-1}$ are the Plank and Hubble times respectively. With the same argument as stated above, one finds $\Lambda \propto H^2$. Further, the Chen and Wu [25] ansatz was generalized by Carvalho et al. [26] by considering $\Lambda = \alpha S^{-2} + \beta H^2$ and later on by Waga [27] by considering $\Lambda = \alpha S^{-2} + \beta H^2 + \gamma$, where $\alpha$, $\beta$ and $\gamma$ are adjustable dimensionless parameter. Decaying-vacuum singularity-free cosmological models based on the Chen and Wu [25] ansatz of a cosmological term varying as a $R^{-2}$ have been presented by Abdel-Rahman [28]. These models describe a closed ever-expanding universe with density parameter $\Omega \geq 1$ and no entropy, horizon, or monopole problems. Chatterjee et al. [29]
discussed the dynamics of inhomogeneous cosmological models with a cosmological "constant". Overduin and Cooperstock [30] have re-examined the evolution of the scale factor with a variable Λ term, and also generalized the treatment to include non zero pressure. They have obtained new solutions and evaluated them using a variety of observational criteria. Recent observational consequences by Perlmutter et al. [31,32] and Riess et al. [33] strongly favoured a significant and positive Λ. Their findings arise from the study of more than 50 type Ia supernovae with redshifts in the range $0.16 \geq z \geq 0.83$ and suggest Friedmann models with negative pressure models such as cosmological "constant", domain walls or cosmic strings (Vilenkin [34], Garnavich et al. [35]). By modifying Chen and Wu ansatz, Vishwakarma [36] has investigated some cosmological models with which fit to the angular size redshift relation data very well and demand cosmic expansion with a positive decreasing Λ. The luminosity distance also plays a crucial role in determining cosmological parameter once the absolute brightness of a class of object is known. In a spatially flat universe the presence of a Λ term increases the luminosity distance to a given redshift, leading to interesting astrophysical consequences. The physical volume associated with a unit redshift interval increases in models with Λ > 0. In such models light from a quasar which will encounter a lensing galaxy. Consequently the probability that a quasar is lensed by intervening galaxies increases appreciably in a Λ - dominated universe. The recent luminosity redshift observations of type Ia supernovae reveal that apart from the gravitationally clustered matter (including dark matter), the universe also consists of some weird form of matter with negative pressure commonly referred to as 'dark energy'. An obvious candidate for dark energy is a positive cosmological "constant" Λ. Sahni [38,39] has reviewed the observational evidence for small cosmological "constant" which mainly comes from high redshift observation of type Ia supernovae. When the results are combined with CMB observations it strongly support a flat universe with $\Omega_m + \Omega_\Lambda \approx 1$. A number of authors have constructed models of a more phenomenological character in which specific decay laws are postulated for Λ within framework of general relativity. Sahni and Starobinsky [18,40] have suggested that the phenomenological methods may be classified into three main groups

1. Kinematic models where it is simply assumed to be a function of either the cosmic time $t$ or the scale factor $R(t)$ of
1.4 Bulk Viscosity and its Role

The role of dissipative effects in the evolution of the universe during early stages is a subject of growing importance. These dissipative processes may well account for the present high degree of isotropy and also the huge value of the number of photons to baryons [86]. In the early universe viscosity may arise due to various processes such as decoupling of neutrinos during the radiation era, the decoupling of matter from radiation during the recombination era, creation of super-string during the quantum era, particle collision involving gravitation, particle creation process and the formation of galaxies [87—89]. It has been suggested that in large class of homogenous but anisotropic universe, the anisotropy dies away rapidly. The most important mechanism in reducing the anisotropy is neutrons viscosity at temperature just above $10^{10}$ K. It is important to develop a model of dissipative cosmological processes in general, so that one can analyze the overall
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dynamics of dissipation without getting lost in the details of complex processes.

Eckart [91] was the first one who described the cosmic viscous fluid. Subsequently an equivalent formulation was given by Landau and Lifschitz [92]. The Eckart's theory was the most used theory to describe the effect of bulk viscosity during evolution of the universe. Due to the work of a number of researchers [93–98] it became clear that Eckart theory suffers from serious shortcomings namely causality and stability. The problem arises from the first order nature of the theory i.e. it considers only the first order deviations from equilibrium. In fact, to prevent non-causal and unstable behaviour it is necessary to consider second order terms. These terms transform the equations governing dissipative quantities from the algebraic first order type to differential evolution equations. To overcome the shortcomings of Eckart theory [91], Israel and Stewart [95], Pavon et al. [96], developed a fully relativistic formulation of the theory by considering higher order theories i.e. extended irreversible thermodynamics (EIT). The EIT theory has a number of advantages which, as listed by Hiscock and Lindblom [97,98], are as follows: (1) for stable equilibrium configurations, the dissipative signals propagate with subluminal speeds; (2) unlike Eckart theory, there is not a generic short-wavelength secular instability inherent in it; (3) even for rotating fluids, the perturbations have a well posed initial value problem. There exists extensive literature dealing with viscous dissipation in the early universe which makes use of the relativistic Eckart theory [91] of transport phenomena as well as EIT.

It has been pointed out by Padmanabhan and Chitre [99] that the presence of bulk viscosity leads to inflationary-like solutions in general relativistic FRW models. Another peculiar characteristic of bulk viscosity is that it acts like a negative energy field in an expanding universe [100]. A detailed review of non-causal viscous cosmological models was presented by Grøn [101]. Lima and Germano [102] have studied the equivalence of bulk viscosity and matter creation. They have shown that the matter creation process can generate the same dynamic behaviour as of the FRW universe with bulk viscosity, but the models are quite different from thermodynamical point of view. Spatially homo-
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geneous cosmological model containing a viscous fluid has been studied by Chakraborty and Nandi [103]. With the help of causal thermodynamics, Romano and Pavon [104] have analyzed the Bianchi type-I cosmological model. The results obtained indicate that the initial anisotropy gets rapidly dissipated and for an ample range of values of the parameters describing the cosmic fluid one gets inflationary expansion. Wolf [105] has studied the inflation driven solutions by energy and curvature dependent bulk viscosity. In the study of dark matter and dissipation, Pavon and Zimdahl [106] have suggested that there may exist a dissipative bulk stress on a cosmological scale which can be ascribed to the presence of dark matter. The effect of bulk viscosity in the context of open thermodynamic system, which during evolution of the FRW models is investigated by Sudharsan and Johri [107]. Elst et al. [108] have examined constraints on many of the inflationary models in the presence of shear and bulk viscosity. Considering a critical review of previous results of dissipative cosmology, Maartens [109] has discussed Israel-Stewart causal theory of bulk viscosity and shown that truncated versions of the theory leads to a pathological temperature evolution. Pavon et al. [110] have examined particle decay and bulk dissipative stress in the early universe. It has been found that the combined action of particle decay and a dissipative bulk stress can lead to a period of generalized inflation and temperature increase. This effect might have occurred at the baryogenesis era. Zimdahl [111] has presented a brief survey on bulk viscous cosmology and discussed the possibility of bulk viscosity-driven inflationary solutions of the full causal Muller-Israel-Stewart (MIS) theory. The expression for the bulk viscous pressure of FRW and Bianchi type-I space-times dominated equilibrium was proposed in the reference [112]. It has been realized that if the relaxation is toward collision-free equilibrium, the bulk viscosity vanishes but there is still entropy production. A viscous cosmological model with variable gravitational and cosmological "constants" have been considered by Arbab [113]. Barreto et al. [114] have studied self-similar dynamics viscous spheres. Recently, viscous cosmological models in (1+1) dimensions and higher derivative theory have been presented by Paul et al. [115]. Mak and Harko [116] have considered bulk viscosity coefficient proportional to Hubble function and obtained the general solution for FRW model of the universe in the framework of the full causal Israel-Stewart-Hiscock theory. Singh and Beesham [117] have analyzed the effect of
bulk viscosity on evolution of the flat FRW model in the context of open thermodynamic systems, which allow particle creation within the framework of Brans-Dicke theory. The stability of cosmic inflationary expansions driven by a changing dissipative fluid is studied by Chimento et al. [118]. They have found that the de Sitter solution is asymptotically stable for a wide set of reasonable fluid quantities of the dissipative cosmological medium. Very recently Belinchen [119] has presented flat FRW cosmological models with a time dependent bulk viscous coefficient, $G$ and $\Lambda$, in the presence of adiabatic matter creation.