CHAPTER - III

GRAVITATIONAL COLLAPSE OF RADIATION SHELLS IN HIGHER DIMENSIONAL SPACETIME

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3.1 Introduction:

Of course, general theory of relativity was formulated in spacetime with four-dimensions. However, there are theoretical predictions that we might live in a world with more dimensional spacetime. It has been of considerable interest in recent times to generalise general theory of relativity to higher dimensions. It is believed that the underlying spacetime in the large energy limit of the Planck energy may have higher dimensions. All the basic forces of nature are assumed to unify at this level and hence, it would be pertinent in this context to obtain solutions of the gravitational field equations in higher dimensions. There has been significant attention to study gravitational collapse in higher dimensions by Soda and Duncan (1999), Rocha and Wang (1999). Llha and Lemos (1997), Llha, Kleber and Lemos (1999) have generalized the Oppenhiemer-Snynder collapse model to higher Dimensions.

Penrose (1969) articulated the cosmic censorship conjecture some four decades ago, ever since, gravitational collapse continues to be a very important topic in gravitational research. The cosmic censorship conjecture forbids the existence of naked singularities. The spacetime must be globally hyperbolic in its strong version, whereas an event horizon must form during collapse and the singularity could be invisible to an asymptotic observer in its weaker version. Kuroda (1984) Joshi (1993), Lake (1991), Rajagopal and Lake (1987), Joshi and Dwivedi (1991, 1992), Ghosh and Beesham (2000), Lemos (1992), Lemos (1999), Deshingkar, Joshi and Dwivedi (1999), presented the collapse of spherical matter in the form of dust or radiation forms shells focusing strong curvature singularities violating the cosmic censorship conjecture.

In this chapter, we have presented how these features get modified with extra-dimensions. We have studied impolding radiation in a higher dimensional spacetime. The Vaidya (1951, 1999) metric, representing a spherically symmetric spacetime with radiation, is an exact solution of Einstein field equations. It has been generalised to higher dimensions by Iyer and Vishveshwara (1989), Patel and Dadhich (1999) and we shall call it the higher dimensional Vaidya metric. We have obtained that non self-similar higher dimensional Vaidya spacetime admits strong curvature singularities in the sense of Tipler (1980), for a sufficiently inhomogeneous collapse, giving an explicit counter example to cosmic censorship conjecture.

3.2 Higher Dimensional Vaidya Spacetime:

Kaluza-Klein (1921, 1926) introduced the idea that spacetime should be extended from four to higher dimensions to unify gravity and electromagnetism. Five dimensional spacetime is particularly more relevant because both ten dimensional and eleven dimensional super-gravity theories give solutioins where a five dimensional spacetime results after dimensional reduction as shown by Schwaz (1983). Hence, we are confined to the five dimensional case.

Iyer and Vishveshwara (1989), Patel and Dadhich (1999) presented the higher dimensional Vaidya spacetime which describes an implosion of radiation shells as

(3.1) $$ds^2 = -\left(1-\frac{m(v)}{r^2}\right) dv^2 + 2dvdr + r^2 \, d\Omega^2$$

where

(3.2) $$d\Omega^2 = d\theta_1^2 + \sin^2\theta_1 \left(d\theta_2^2 + \sin^2\theta_3^2 \, d\theta_3^2\right) ,$$

as the metric of the 3-sphere, where v is a null coordinate with

(3.3) $$-\infty < v < \infty ,$$

and r as radial coordinate with

(3.4) $$0 \leq r < \infty ,$$

and the arbitrary function m(v), which is restricted only by the energy conditions, representing the mass at advanced time v. The energy momentum tensor associated with eq. (3.1) may be written as

$$T_{ab} = \frac{3}{2r^3} \frac{dm}{dv} k_a k_b \quad , \tag{3.5}$$

with the null vector k_a satisfying

$$k_a = - \delta^v_a \quad , \tag{3.6}$$

and

$$k_a k^a = 0 \quad . \tag{3.7}$$

We have taken units such that

$$8 \pi G = c = 1 \quad . \tag{3.8}$$

In view of the weak energy condition, we require that

(3.9) $\frac{dm}{dv}$ be non negative.

Hence, the mass function m(v) is a non-negative increasing function of v. For the metric (3.1), one may obtain the Kretschmann scalar

$$K = R_{abcd} R^{abcd} , \tag{3.10}$$

where R_{abcd} be the Riemann tensor. The Kretschmann scalar assumes the form

$$K = \frac{72m^2(v)}{r^8} , \tag{3.11}$$

which diverges along

$$r = 0 , \tag{3.12}$$

giving a scalar polynomial singularity. The Weyl scalar reads

$$C = C_{abcd} C^{abcd} , \tag{3.13}$$

where C_{abcd} be the Weyl tensor has the same expression as the Kretschmann scalar and hence, the Weyl scalar also diverges whenever the Kretschmann scalar diverges and so the singularity is physically significant as shown by Barve and Singh (1997).

Here, the physical situation is that of a radial influx of a null fluid in an initially flat and vacuum region of the higher dimensional spacetime. For $v < 0$, one obtains

(3.14) $$m(v) = 0 ,$$

i.e. higher dimensional flat spacetime, and for $v > T$

(3.15) $$\frac{dm}{dv} = 0 ,$$

it means $m(v)$ is positive definite. Hence, the metric for

(3.16) $$v = 0 \quad \text{to} \quad v = T ,$$

be higher dimensional Vaidya, and for

(3.17) $$v > T ,$$

one obtains higher dimensional Schwarzschild solution. The first shell comes at $r = 0$ at time $v = 0$ and the final at v = T. Hence, a central singularity of growing mass comes in picture at r = 0.

3.3 Nature of naked Singularities:

Let us put

$$K^a = dx^a/dk \quad , \tag{3.18}$$

be the tangent to the null geodesic, where k be the affine parameter. The null condition gives

$$K^a K_a = 0 \quad . \tag{3.19}$$

Hence, the geodesic equations assume the form

$$\frac{dK^v}{dk} + \frac{m(v)}{r^2} (K^v)^2 = 0 \; , \tag{3.20}$$

$$\frac{dK^r}{dk} + \frac{1}{2r^2} \frac{dm}{dv} (K^v)^2 = 0 \; . \tag{3.21}$$

Let us introduce

$$K^v = \frac{P(v,r)}{r} \; , \tag{3.22}$$

and in view of null condition, one obtains

$$K^r = \left(1 - \frac{m(v)}{r^2}\right) \frac{P}{2r} (v,r) \quad . \tag{3.23}$$

The function P(v,r) must satisfy the differential equation

$$\frac{dP}{dk} - \left(1 - \frac{3m(v)}{r^2}\right) \frac{P^2}{2r^2} = 0 \quad . \tag{3.24}$$

However, in general, the eq. (3.24) may not give an analytical solution.

Now one may obtain the radial null geodesics of the metric (3.1) as

$$\frac{dr}{dv} = \frac{1}{2} \left(1 - \frac{m(v)}{r^2}\right) . \tag{3.25}$$

It is obvious, the eq. (3.25) has a singularity at

$$r = 0, \quad v = 0 \tag{3.26}$$

The nature i.e. a naked singularity or a black hole, of the collapsing solution may be characterised by the existence of radial null geodesics coming out from the singularity. The nature of singularity depends on the exact form of mass function m(v). For example, as shown by the spherical symmetric spacetime is self similar if

$$g_{tt}\,(ct,cr) = g_{tt}\,(t,r) \quad , \tag{3.27}$$

and

$$g_{rr}\,(ct,cr) = g_{rr}\,(t,r) \quad , \tag{3.28}$$

for every $c > 0$. i.e.

$$m(v) \sim \lambda\, v^2 \quad , \tag{3.29}$$

showing that spacetime is self-similar, admitting a homothetic Killing vector and singularities may be analysed with case. However, self-similarity is a strong geometric condition on the spacetime. Hence, we are interested to examine more general forms of the mass function $m(v)$. Here, we are presenting specific examples satisfying weak energy condition but give strong curvature naked singularities.

Example 1.

Let us consider a higher dimensional analogue of Lake (1987) solution, which requires

$$m(v) = \lambda v^2 + f(v) \tag{3.30}$$

where λ be a constant and

$$f(v) = O(v^2) \tag{3.31}$$

as $v \longrightarrow 0$. It follows, hence, that for

$$0 < \lambda \leq 1/27 , \tag{3.32}$$

the singular point becomes an unstable node and the family of null geodesics meets the singularity with definite tangents. One may obtain the possible values of tangents by the roots of eq. (3.25)

(3.33) $$\lambda \gamma_o^3 - \gamma_o^2 + 2 = 0$$

where

(3.34) $$\gamma_o = \lim_{r \to 0,\ v \to 0} \gamma$$

$$= \lim_{r \to 0,\ v \to 0} \frac{v}{r} \quad .$$

The two positive roots of eq. (3.33) are

(3.35) $$\gamma_o = 2.21833 \quad ,$$

and

(3.36) $$\gamma_o = 5.69593 \quad ,$$

corresponding to

(3.37) $$\lambda = 1/50 \quad .$$

For all such values, the singularity is naked. Hence, gravitational collapse of null fluid in higher leads to a naked singularity if

$$\lambda \leq 1/27 \quad , \tag{3.38}$$

and to formation of black hole otherwise.

In view of Lemos (1992, 1999), the degree of inhomogeneity of the collapse is defined as

$$\mu = 1/\lambda \quad . \tag{3.39}$$

It is observed that for a collapse sufficiently inhomogeneous, naked singularities develop. Let us compare with the analogous 4-dimensional case, we obtain that naked singularity occurs for a slightly larger value of the inhomogeneity factor in higher dimensions. The global nakedness of the singularity may be observed by making a junction onto the higher dimensional Schwarzschild spacetime. The Kretschmann scalar K reads

$$K = \frac{a\,\lambda^2}{r^4} \quad , \tag{3.40}$$

where a be the some constant. Hence as $r \longrightarrow 0$, the collapse forms a scalar polynomial curvature singularity. The critical direction with the node reads

$$r = \mu v + g(v) \tag{3.41}$$

where

$$\mu = 1/\gamma_o \quad , \tag{3.42}$$

and

$$f = (1-2\mu)\, g(2\mu v+g) - g' \,(\mu v+g)^2 \quad . \tag{3.43}$$

For a spacetime to be self-similar, one requires that

$$g = 0 \quad . \tag{3.44}$$

The radial null geodesics given by eq. (3.41) be the Cauchy horizon associated with the node, which is a strong curvature singularity. One may observe that the weak energy condition is satisfied for

$$2\lambda v \geq (2\mu-1)\, 2\mu g + 2g' \,(3\mu+g'-1)\,(\mu v+g) + g''\,(\mu v+g)^2 \tag{3.45}$$

and the Cauchy horizon expanding for

$$g' > -\mu \quad . \tag{3.46}$$

Deshingkar, Joshi and Dwivedi (1999) made an attempt to relate the strength of a singularity to stability. A singularity is defined gravitationally strong or simply strong, if it destroys by crushing or stretching any object which falls into it. Along a null geodesics let

$$\psi = R_{ab} K^a K^b \quad , \tag{3.47}$$

where R_{ab} be the Ricci tensor and where the geodesic terminates at

$$\lambda = 0 \quad . \tag{3.48}$$

We obtain the following condition

(3.49) $$\lim_{k \to 0} k^2 \psi > 0 ,$$

where k be the affine parameter. The condition (3.49) is equivalent to the termination of a geodesic in a strong curvature singularity. In view of the above equations, one obtains

(3.50) $$\lim_{k \to 0} k^2 \psi = \lim_{k \to 0} \frac{3}{2r} \frac{dm}{dv} \left(\frac{kP}{r^2}\right)^2$$

The fact that the singularity is approached for k—> 0 and r——>0 and in view of L'Hopitalis rule, we get

(3.51) $$\lim_{k \to 0} kP/r^2 = 2/1 + \lambda \gamma_o^2$$

for

(3.52) $$P_o = \infty$$

where

(3.53) $$P_o = \lim_{k \to 0} P$$

and

(3.54) $$\lim_{k \to 0} kP/r^2 = 1/1- \lambda \gamma_o^2$$

for

(3.55) $$P_o \neq \infty \; .$$

Hence, the eq. (3.50) gives

(3.56) $$\lim_{k \to 0} k^2 \psi = \frac{12 \lambda \gamma_o}{(1+\lambda \gamma_o^2)} > 0 \quad \text{for } P_o = \infty$$

and

(3.57) $$\lim_{k \to 0} k^2 \psi = \frac{3 \lambda \gamma_o}{(1-\lambda \gamma_o^2)} > 0 \quad \text{for } P_o \neq \infty$$

where

(3.58) $$\gamma_o \neq 1/\sqrt{\lambda} \; .$$

Hence, we observe that along radial null geodesics, the strong curvature condition is satisfied. Nolan (1999) presented an alternative method to check the nature of singularities without integrating the geodesics equations. He showed that a radial null geodesics which run into r = 0 terminates in a gravitationally weak singularity if and only if $\dot{r}$ is finite in the limit k = 0, the dot represents differentiation along the geodesics. Hence, for weak singularity, we obtain

$$(3.59) \qquad \dot{r} \sim d_o$$

$$(3.60) \qquad r \sim d_o k$$

In view of

$$(3.61) \qquad m(v) \sim \lambda v^2 \qquad (\text{as } k = 0) \quad ,$$

one obtains the geodesic equation as

$$(3.62) \qquad \frac{d^2v}{dk^2} \sim \dot{\delta}\, k^{-1}$$

where

$$(3.63) \qquad \delta = -\lambda \gamma_o^4 d_o = \text{a nonzero constant,}$$

which is inconsistent with

$$(3.64) \qquad \dot{v} \sim d_o \gamma_o$$

i.e. finite. As the coefficient δ of k^{-1} is non-zero, the singularity be the gravitationally strong.

Example 2. Let us consider Joshi and Dwivedi (1991, 1992) type solution for higher dimensional spacetime and let us select mass function m(v) as

$$(3.65) \qquad m(v) = \beta^2 v^{2\alpha} (1-2\beta\alpha \; v^{\alpha-1})$$

where α and β are constants such that

$$(3.66) \qquad \alpha > 1 \quad \text{and} \quad \beta > 0 \quad .$$

It is a representative class of a more general problem

$$(3.67) \qquad m(v) \sim v^n \quad (n > 2)$$

For

(3.68) $$\alpha = 1 ,$$

It corresponds to a self-similar model. The null radiation shells start imploding at

(3.69) $$v = 0 ,$$

and the final shell comes at

(3.70) $$v = T .$$

The weak energy condition is to be satisfied we have

(3.71) $$\frac{dm}{dv} \geq 0,$$

T must satisfy

(3.72) $$T^{\alpha - 1} < \frac{1}{\beta(3\alpha - 1)} ,$$

which also gurantees

(3.73) $$m(v) > 0.$$

Now for the mass function (3.65), an outgoing radial null geodesic, meeting the singularity v = 0, r = 0 in the past reads as

(3.74) $$r = \beta v^{\alpha} .$$

This integral curve meets the singularity with tangent r = 0 and showing that singularity be naked. Since

(3.75) $$dr/dv > 0 ,$$

the null geodesics (3.74) escape to infinity and the singularity is globally naked and the condition

(3.76) $$r^2 > m(v)$$

is satisfied along the trajectories.

3.4 Concluding Remarks:

We have investigated gravitational collapse of radiation shells in a non self-similar higher dimensional spherically symmetric spacetime, showing that strong curvature naked singularities form for a highly inhomogeneous collapse, violating the cosmic censorship cnjecture. As a special case, we have obtained, self-similar models. The proof and the rigorous formulation for either version of the cosmic censorship conjecture is not yet available. Therefore, examples showing the presence of naked singularities remain important and may be valuable if one attempts to formulate the notion of the conjecture in precise mathematical form. The Vaidya metric in the four-dimensional case has been extensively employed to study the occurrence of naked singularities in spherically symmetric gravitational collapse. We have extended this investigation to a higher dimensional Vaidya metric, and obtained that strong curvature naked singularities appear for slightly higher values of the inhomogeneity parameter and only for mass function $m(v) \sim v^2$. We have checked for naked singularities to be gravitationally strong by the approach given by Clarke and Krolak (1986) and by Nolan (1999) as well, and found that both are in well agreement. In general, the models obtained here are not self-similar, and as a special case the self-similar models are obtained. Now, it is a straight forward to extend the above investigation for non radial causal curves, and to spacetime of any dimensions i.e. $n \geq 4$. Hence, we may say that this study offers a counter example to the cosmic censorship conjecture.

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