C H A P T E R - I

INTRODUCTION

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1.1 Introduction and Motivation:

In the past decade, several scenarios have been investigated where the gravitational collapse of a massive matter cloud results in the development of a naked singularity as shown by Joshi (2000), Gundlach (1999), Krolak (1999), Penrose (1998), Singh (1998). The final cosequence of gravitational collapse in general relativity is an important problem and interest from the perspective of black hole physics as well as its astrophysical implications. When there is a continual collapse without any final equilibrium, either a black hole forms when the super-dense regions of matter are hidden from the outside observer within an event horizon of gravity, or a naked singularity results as the end product, depending on the nature of the initial data from which the collapse develops. Harada et al (2000, 2001), Joshi et al (2000) have presented the theoretical and observational features of a naked singularity and showed that would be quite different from those of black hole. Hence, it is of crucial importance to understand what are the key physical properties and dynamical characteristics in collapse that give rise to a naked singularity, rather than a black hole.

Iguchi et al (1998), Deshingkar et al (1999), Saraykar et al (1999), Carr and Coley (1999), Wagh et al (1999), Mena et al (2000), Jhingan and Magli (2000), Concalves (2001) Harada et al (2001), Ghosh and Beesham (2001), have obtained and analyzed many models of naked singularity formation within dynamically developing collapse scenarios, but not much has been paid to understanding this important aspect. It is obtained that it is shearing effects which, if sufficiently strong near the central worldline of the collapsing cloud, would delay the formation of apparent horizon so that the singularity becomes visible and communication form the very strong gravity regions to outside observers becomes possible. When the shear is weak and in the extreme case of no shear, the collapse necessarily ends in a black hole, because an early formation of the apparent horizon leads to the singularity being hidden behind an event horizon.

The gravitational collapse of sufficiently massive star, under general conditions, will result into a singularity be the fact established by the singularity theorems by Hawking and Ellis (1973). However, these theorems do not provide whether the singularity will be hidden behind the event horizon or whether it will be visible to an outside observer. The singularity will not be visible is indicated in the relativistic literature by the phrase "Cosmic Censorship Conjecture" (CCC). The weak form of the conjecture leads that it will not be visible to an observer outside the event horizon while the strong form says that it will not be visible to any observer, even the one who is sitting on the collapsing star. This conjecture was articulated by Penrose (1969). Essentially the weak form says that gravitational collapse from a regular initial data never creates the spacetime singularity visible to distant observer i.e., any singularity that forms must be hidden within a black hole. On the other hand, for the strong form, nothing is assumed to emanate out of the singularity and hence, it is not visible to any one even if one is very close to it.

Yet to day, there is no general theory of nature and visibility of singularity. Though, there exit a number of exact solutions of the Einstein equation which admit, depending upon the initial data black holes or naked singularities as shown by Joshi (1993), Wald (1997), Jhingan and Magli (1999) and Joshi (2000). In particular, the Vaidya solution (1951) has been extensively used to show that the final state of collapse for a regular initial data results into a naked singularity. In the absence of a general result, the study of various examples of collapse becomes pertinent to analyse the validity of the phenomenon of occurrence of naked singularity. The two most investigated cases are: inhomogeneous dust and Vaidya null fluid. So, it requires to consider the more general matter distribution.

In fact, as the singularity is approached, matter would be in highly dense and exotic state. The strange quark matter is the densest state of matter known which is produced in quark-hadron phase transition in the early Universe or at ultra high energy neutran stars converting into strange quark matter stars as presented by Witten (1984). Type II fluid has been investigated in the usual 4-dimensional spacetime by Ghosh and Dadhich

(2001, 2002). The presence of strange quark matter tends to shrink the initial data window leading to naked singularity. It is expected, the increase in dimensions would also tend to increase the shrinkage due to gravity getting stronger in higher dimensions as suggested by Ghosh and Dadhich (2001).

Recent investigations and developments in string theory indicate that gravity may be truly higher-dimensional interaction, becoming effectively 4D at lower energies. The main problem is how to bring this theoretical development on the test of observations? One of the ways is to investigate its effects in astrophysical settings, i.e. where the phenomenon of gravitational collapse in higher dimensions obtains relevance. Hence, one has to obtain the effects of higher dimensions as well as strange quark matter on the final product of collapse of null strange quark matter. Therefore, one has to obtain the exact solution describing collapse of null strange quark matter fluid sphere, and then study their effects in the context of cosmic censorship conjecture.

For spherical gravitational collapse a massive matter cloud, the interior metric in comoving coordinates reads

(1.1) $$ds^2 = -e^{2v(t,r)}\, dt^2 + e^{2\mu(t,r)}\, dr^2 + R^2\,(t,r)\, d\,\Omega^2 \quad .$$

The matter shear assumes the form

(1.2) $$\sigma_{ab} = e^{-v}\left(\frac{\dot{R}}{R} - \dot{\mu}\right)\left(\frac{1}{3}\, h_{ab} - n_a\, n_b\right) \quad ,$$

where

(1.3) $$h_{ab} = g_{ab} + u_a\, u_b \quad ,$$

be the induced metric on 3-surfaces orthogonal to the fluid 4-velocity u^a, and n^a be a unit radial vector. The initial data for collapse are the values on $t = t_i$ of the three metric functions, the density, pressure and mass function that comes from integrating the Einstein equations as presented by Joshi and Dwivedi (1999),

(1.4) $$F\,(t_i,\, r) = \int \rho\,(t_i, r)\, r^2\, dr$$

where

(1.5) $$4\pi F(t_i, r_b) = M \quad ,$$

as the total mass of the collasing cloud, and where

(1.6) $$r > r_b \quad ,$$

is a Schwarzschild spacetime.

Let us rescale to set

(1.7) $$R(t_i, r) = r$$

such that the physical area radius R increases monotonically in r and with

(1.8) $$R'_i = 1 \quad ,$$

there are no shell-crossings on the initial surface. We are interested here only in the central shell-focusing singularity at

$$R = 0, \qquad r = 0, \tag{1.9}$$

which is a gravitationally strong singularity, as opposed to the shell-crossing ones which are weak, and through which the spacetime may sometimes be extended. The evolution of the density and radial pressure read as

$$\rho = \frac{F'}{R^2 R'} , \tag{1.10}$$

$$p_r = \frac{\dot{F}}{R^2 \dot{R}} . \tag{1.11}$$

The central singularity at

$$r = 0 , \tag{1.12}$$

where density and curvature are infinite, be naked if there are outgoing non-spacelike geodesics which reach outside observers in the future and terminate at the singularity in the past. The outgoing radial null geodesics of (1.1) are given by

$$\frac{dt}{dr} = e^{\mu - \nu} . \tag{1.13}$$

Let us consider first the case of homogeneous-density collapse

$$\rho = \rho(t) \quad . \tag{1.14}$$

Let us set

$$f = e^{-2\mu} R'^{2} - 1 \quad , \tag{1.15}$$

the Einstein equations provide

$$f - e^{-2\nu} \dot{R}^{2} = - F/R \quad . \tag{1.16}$$

Hence, the eq. (1.13) assumes the form

$$\frac{dR}{du} = \left(1 - \sqrt{\frac{f+ F/R}{1+f}} \right) \frac{R'}{\alpha r^{\alpha-1}} \quad , \tag{1.17}$$

where

$$u = r^{\alpha} \qquad (\alpha > 1) \quad . \tag{1.18}$$

If there are outgoing radial null geodesics terminating in the past at the singularity with a definite tangent, then at the singularity, one has

$$(1.19) \qquad dR/du > 0 .$$

In the case of homogeneous density, the whole mass of the cloud collapses to the singularity simultaneously at the event

$$(1.20) \qquad t = t_s, \quad r = 0 ,$$

such that

$$(1.21) \qquad F/R \longrightarrow \infty .$$

In view of eq. (1.17)

$$(1.22) \qquad \frac{dR}{du} \longrightarrow -\infty ,$$

so that no radial null geodesics may emerge from the central singularity. It may be shown that all the later epochs $t > t_s$ are similarly covered. Hence, it is obvious that for spherical gravitational collapse with homogeneous density and arbitrary pressures, the final outcome is necessarily a black hole. It may be noted that this conclusion does not require homogeneity of pressures p_r and $p_\perp$, and is independent of their behaviour. This result generalizes the well known Oppenheimer-Suyder result for special case of dust, where the homogeneous cloud collapses to form a black hole always. Therefore, an immediate consequence is that if the final outcome of spherical gravitational collapse is not a black hole, then the density must be inhomogeneous. In any physically realistic scenario, the density must be typically higher at the center, so that generically collapse is inhomogeneous.

1.2 Inhomogeneous Dust:

Let us now consider a collapsing inhomogeneous dust cloud with

$$p = 0 \quad , \tag{1.23}$$

with density higher at the centre. The metric is Tolman-Bondi-Lemaitre, as (1.1) with $v = 0$ and

(1.24) $$e^{2\mu} = R'^2/ 1+ f \quad ,$$

and

(1.25) $$\dot{R}^2 = f(r) + \frac{F(r)}{R} \quad .$$

These models are fully characterised by the initial data, specified on the initial surface $t = t_i$ from which the collapse develops which consists of two free functions

(1.26) $$\rho_i (r) = \rho(t_i, r)$$

or equivalently, the mass function F(r), and f(r) which describes the initial velocity of collapsing matter shell. hence, one obtains

(1.27) $$F(r) = r^3 \bar{F} (r)$$

where

(1.28) $$o < \bar{F} (o) < \infty \quad .$$

The initial density $\rho_i(r)$ reads

$$(1.29) \qquad \rho_i(r) = r^{-2} F'(r) \quad .$$

The shell-focusing singularity appears along the curve

$$(1.30) \qquad t = t_s(r) \quad ,$$

defined as

$$(1.31) \qquad R(t_s(r), r) = 0 \quad .$$

The trapped surfaces develop within the collapsing cloud, as the density grows without bound. These may be traced explicitly through the null outgoing geodesics, and the equations of the apparent horizon,

$$(1.32) \qquad t = t_{ah}(r) \quad \cdot$$

which marks the boundary of the trapped region reads

(1.33) $$R\,(t_{ah}(r),r) = F(r) \quad .$$

If the apparent horizon begins developing before than the epoch of singularity formation, then the event horizon may fully cover the strong gravity regions including the final singularity, hence, it will be hidden within a black hole. Again, if trapped surfaces develop sufficiently later during the evolution of collapse, then it may be possible for the singularity to communicate with outside observer. Let us now consider marginally bound collapse

(1.34) $$f = 0 \quad .$$

The eq. (1.25) may be integrated to obtain

(1.35) $$R^{3/2}\,(t,r) = r^{3/2} - \frac{3}{2}\,(t-t_i)\,F^{\frac{1}{2}}\,(r) \quad .$$

The eqs. (1.31) and (1.33) give

(1.36) $$t_s\,(r) = t_i + \frac{2}{3}\quad \left(\frac{r^3}{F(r)}\right)^{\frac{1}{2}} \quad ,$$

(1.37) $$t_{ah}\,(r) = t_s(r) - \frac{2}{3}\,F(r) \quad .$$

The central singularity at $r = 0$ comes at time

$$t_o = t_s(0) = t_i + \frac{2}{\sqrt{3 \rho_c}} , \tag{1.38}$$

where

$$\rho_c = \rho_i(0) \tag{1.39}$$

For inhomogeneous dust, one obtains

$$\sigma^2 = \tfrac{1}{2} \sigma_{ab} \sigma^{ab} \tag{1.40}$$

$$= \frac{r^2}{6R^4 R'^2 F} (3F - rF'^2) ,$$

and the mass function $F(r)$ reads as

$$F(r) = F_o r^3 + F_1 r^4 + F_2 r^5 + \ldots\ldots\ldots \tag{1.41}$$

Now near $r = 0$, we obtain

(1.42) $$F_o = \rho_c/3 \quad .$$

Homogeneous dust collapse has

(1.43) $$F_n = 0 \quad \text{for} \quad n > 0 \quad .$$

Hence,

(1.44) $$\sigma = 0 \quad .$$

The converse is also true, in this case. For higher density at centre

(1.45) $$F_n \neq 0 \quad \text{for } n > 0 \quad ,$$

it follows from eq. (1.40) that the shear is then necessarily nonzero.

It is to be noted that if one wishes the density profile to be analytic, one may set all odd terms F_{2n-1} to zero. Now the important problem is: what is the effect of such a shear on the evolution and development of the trapped surfaces? In other words, one wants to obtain the behaviour of apparent horizon in the vicinity of the central singularity at $R = 0$, $r = 0$. Let us consider nonvanishing derivative of density at $r = 0$ as the n-th one for $n > 0$, i.e.

$$(1.46) \quad F(r) = F_0\, r^3 + F_n\, r^{n+3} + \ldots\ldots, \quad F_n < 0 \quad ,$$

near the centre. Hence, we get

$$(1.47) \quad \sigma^2\,(t,r) = \frac{n^2 F_n^2}{6F_o}\left[1-3\,F_o^{\frac{1}{2}}\,(t-t_i) + \frac{9}{4}\,F_o\,(t-t_i)^2\right] r^{2n} \quad ,$$

$$(1.48) \quad t_{ah}\,(r) = t_o - \frac{2}{3}\,F_o\,r^3 - \frac{F_n}{3F_o{}^{3/2}}\quad r^n, + O\,(r^{n+1}) \;.$$

From eq. (1.47), the time dependent factor in square bracket on the right decreases from 1 at $t = t_i$ to 0 at $t = t_o$. The initial shear

(1.49) $\sigma_i = \sigma \ (t_i, r)$

on the surface $t = t_i$ grows as r^n, $n \geq 1$, near $r = 0$. The volume expansion Θ reads

(1.50) $\Theta = 2 \frac{\dot{R}}{R} + \frac{\dot{R}'}{R'}$.

Hence, one obtains

(1.51) $\left|\frac{\sigma}{\Theta}\right|_i = \frac{-nF_n r^n}{3\sqrt{6}F_o} \left[1 + O(r)\right]$.

1.3 Gravitational Collapse in Higher dimensional Spacetime:

Let us consider (n+2) dimensional spherically symmetric spacetime, in advanced Eddington time coordinate v, with metric

(1.52) $ds^2 = - e^{\psi(v,r)} dv \, [f(v,r) \, e^{\psi(v,r)} dv + 2dr]$

$+ r^2 \, d\Omega_n^2$,

where

(1.53) $$0 \leq r \leq \infty ,$$

(1.54) $$-\infty \leq v \leq \infty .$$

(1.55) $$d\Omega^2 = \sum_{i=1}^{n} \left[\prod_{j=1}^{i-1} \sin^2\theta_j \right] d\theta_i^2$$

$$= d\theta_1^2 + \sin^2\theta_1 \, d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 \, d\theta_3^2 +$$

$$\ldots\ldots + \sin^2\theta_1 \sin^2\theta_2 \ldots\ldots \sin^2\theta_{n-1} \, d\theta_n^2 .$$

Let us introduce a mass function $m(v,r)$ such that

(1.56) $$f = 1 - 2m/r.$$

Hence, $m(v,r)$ be an arbitrary function of advanced time v and radial coordinate r. When $m = m(v)$, we get the Vaidya solution in higher dimensions as obtained by Ghosh and Dadhich (2001). The usual Vaidya solution in four-dimensional spacetime is obtained for $m = m(v)$ and $n = 2$.

On may put the metric (1.52), without loss of generality, to the form

$$(1.57) \quad ds^2 = -\left[1 - \frac{2m(v,r)}{(n-1)r^{n-1}}\right] dv^2 + 2\, dv\, dr + r^2\, d\Omega_n^2 \quad .$$

The energy momentum tensor for type II (null strange quark matter SQM) fluid reads

$$(1.58) \quad T_{ab} = \mu\, l_a l_b + (\rho + p)(l_a \eta_b + l_b \eta_a) + p\, g_{ab},$$

$$(1.59) \quad l_a = \delta_a^o \quad ,$$

$$(1.60) \quad \eta_a = \frac{1}{2}\left[1 - \frac{2m(v,r)}{r}\right] \delta_a^o - \delta_a^1 \quad ,$$

$$(1.61) \quad l^a = \delta_1^a \quad ,$$

(1.62) $n^a = -\delta^a_o - \frac{1}{2}\ [1 - \frac{2m(v,r)}{r}]\ \delta^a_1$,

(1.63) $l_a l^a = n_a n^a = 0$,

(1.64) $l_a n^a = -1$,

where ρ , p be the strange quark matter energy density and thermodynamic pressure and μ be the energy density of the Vaidya null radiation. The null vector l_a be a double null eigenvector of T_{ab}. Physically distribution is null radiation in radial direction for

(1.65) $\rho = p = 0$,

which is the Vaidya spacetime of radiating star. When $\mu = 0$, T_{ab} reduces to degenerate Type I fluid, and represents string dust for $\mu = 0 = p$. The energy conditions are :

(1) The weak and strong energy conditions,

(1.66) $\mu > 0$, $\rho \geq 0$, $p \geq 0$.

(2) The dominant energy condition,

(1.67) $\mu > 0, \quad \rho \geq p \geq 0.$

By selecting suitably the mass function m(v,r), it is possible to satisfy the energy conditions. In particular for m = m(v), the energy condition reduces to $\mu \geq 0$. One may obtain the field equations as

(1.68) $$8\pi \mu = \frac{n\dot{m}}{(n-1)\, r^n} ,$$

(1.69) $$8\pi \rho = \frac{nm'}{(n-1)\, r^n} ,$$

(1.70) $$8\pi p = - \frac{m''}{(n-1)\, r^{n-1}} .$$

The dash and dot denote derivative $\partial/\partial r$ and $\partial/\partial v$ respectively. In T_{ab} the part $\mu\, l_a\, l_b$ be the component of matter field that moves along the null hypersurface v = constant. When $p = \rho = 0$, one obtains Vaidya solution in higher dimensions. When $\mu = 0$, it is static perfect fluid spacetime giving regular spacetime only

(1.71) $$m \propto r^{n+1} \quad ,$$

i.e. de Sitter space.

Let us take the equation of state as

(1.72) $$p = \frac{1}{k} \ (\rho - 4B)$$

for k > 0, a constant. Hence, one obtains

(1.73) $$m'' = - \frac{nm'}{kr} + \frac{32\pi B}{k} \ r^{n-1} \quad .$$

Let us select the mass function as

(1.74) $$m(v,r) = m_o \ (v,r) + \frac{\Delta}{(n+1)} \ r^{n+1} \quad ,$$

where m_o (v,r) is unknown function, then

(1.75) $$m'' = - \frac{n}{k} \quad m_o'$$

(1.76) $$B = \frac{32\pi\Delta}{n(k+1)} \quad ,$$

admitting a general solution

$$m_o = Q(v)\, r^{-n/k+1} + M(v). \tag{1.77}$$

In view of the above, one may evaluate m, μ, ρ and p as

$$m = M(v) + Q(v)\, r^{-n/k+1} + \frac{\Lambda}{n+1}\, r^{n+1} \quad , \tag{1.78}$$

$$\mu = \frac{n}{8\pi(n-1)r^n}\left(\dot{M}(v) + \dot{Q}(v)\, r^{-n/k+1}\right) \quad , \tag{1.79}$$

$$p = \frac{n}{8\pi\,(n+1)r^n}\left(-\frac{(k-n)}{k^2}\, Q(v)\, r^{-n/k+1} - \Lambda\, r^n\right), \tag{1.80}$$

$$\rho = \frac{n}{8\pi(n-1)r^n}\left(\frac{k-2}{k}\, Q(v)\, r^{-n/k} + \Lambda\, r^n\right), \tag{1.81}$$

which is the general solution of collapsing null strange quark matter in higher dimensions. The corresponding metric reads

$$ds^2 = -\left[1 - \frac{2M(v)}{(n-1)r^{n-1}} - \frac{2Q(v)}{(n+1)r^{[(n+2)k+n]/k}} - \frac{\Lambda\, r^2}{n^2-1}\right] dv^2 + 2dv\, dr + r^2\, d\Omega^2. \tag{1.82}$$

All the energy conditions are satisfied for $n \geq 2$ leading to $\rho \geq 0$, and $p \geq 0$. For Q(v) i.e.

$$Q(v) = 0 , \tag{1.83}$$

One recovers the higher dimensional Vaidya-de Sitter solutions, where Vaidya solution is obtained for

$$Q(v) = 0 = B. \tag{1.84}$$

1.4 About a naked Singularity:

Here, we have the physical system of a radial influx of null fluid in the higher dimensional empty space but non-flat. The first shell comes at $r = 0$ at time $v = 0$ and the last at $v = T$. So, the central singularity of growing mass is developed at $r = 0$. One has

$$m(v) = 0 \qquad \text{for} \quad v < 0 , \tag{1.85}$$

i.e. higher dimensional de Sitter type space, and

(1.86) $\dot{m}(v) = 0 \quad \text{for} \quad v = T$,

i.e. higher dimensional generalised Vaidya space, and for $v > T$ one obtains higher dimensional Schwarzschild space. Let us select, for higher dimensional Vaidya space

(1.87) $$2M(v) = \alpha (n-1) v^{n-1} \qquad (\alpha > 0),$$

(1.88) $$2Q(v) = \beta (n-1)^{[(n-2)k+n]/k} v \qquad (\beta > 0).$$

Now the radial null geodesics are

(1.89) $$\frac{dr}{dv} = \frac{1}{2}\left[1 - \frac{2M(v)}{(n-1)r^{n-1}} - \frac{2Q(v)}{(n-1)r^{[(n-2)k+n]/k}} - \frac{\Lambda}{n^2-1}\right].$$

The above equation shows a singularity at $r = 0$, $v = 0$. Let us investigate the behaviour of radial null geodesics near the singularity. Let us determine the limiting of x at $r = 0$, $v = 0$, on a singular geodesic i.e.

(1.90) $$x_o = \lim_{r \to 0,\ v \to 0} x$$

$$= \lim_{r \to 0,\ v \to 0} \frac{v}{r}$$

$$= \lim_{r \to 0,\ v \to 0} \frac{dv}{dr}$$

$$= \lim_{r \to 0,\ v \to 0} \frac{2}{1 - \alpha x^{n-1} - \beta x^{n(k+1)/k} - \frac{\Lambda r^2}{n^2-1}}$$

or

(1.91) $$\beta x_o^{[(n-2)k+n]/k+1} + \alpha x_o^n - x_o + 2 = 0$$

If one obtains one or more positive roots of x_o, the singularity be naked. Hence, the collapse leads to a black hole in the absence of positive roots.

Let us discuss some other cases of interest.

<u>Case I</u> $k = n$.

In this case, one obtains

(1.92) $$\rho = - p = \frac{n\Lambda}{8\pi(n-1)} ,$$

(1.93) $$\mu = \frac{n}{8\pi (n-1)r^n} [\dot{M}(v) + \dot{Q}(v)] ,$$

and the algebraic equation assumes the form

(1.94) $$(\alpha + \beta) x_o^n - x_o + 2 = 0.$$

In view of the above the metric assumes the form of the higher dimensional Vaidya-de Sitter metric, i.e., the higher dimensional collapse of null fluid in an expanding de Sitter background. The singularity be naked for

(1.95) $$(\alpha + \beta) \leq \lambda_c = \frac{1}{n} (\frac{n-1}{2n})^{n-1} .$$

The equal roots at λ_c is

(1.96) $$x_o = \frac{2n}{n-1} .$$

It is an important to note that both critical parameters and tangents to outgoing geodesics are dependent on dimensions of spacetime. It is obvious $x_o \longrightarrow 2$ as $\lambda \longrightarrow 0$ or $D \longrightarrow \infty$.

Case II $\qquad k \longrightarrow \infty$.

In this case, we get

(1.97) $$p = 0 ,$$

(1.98) $$\rho = \frac{n \beta}{16 \pi r^2} x^{n-2} ,$$

(1.99) $$\mu = \frac{n}{16 \pi (n-1) r^2} [\alpha x^{n-2} + \beta x^{n-3}] ,$$

and the algebraic equation as

(1.100) $$\alpha x_o^n + \beta x_o^{n-1} - x_o + 2 = 0.$$

For n = 2, one obtains a positive roots for $\alpha \leq$ 1/8 (β-1)$_2$. For n = 3, the condition for naked singularity modifies to $\beta^2 < 4\alpha + 108\alpha^2 + 36\alpha\beta + \beta^2$.

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