CHAPTER - IV

BLACK HOLES AND NAKED SINGULARITIES IN HIGHER DIMENSIONAL VAIDYA SPACETIMES

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4.1 Introduction:

There has been considerable interest in recent times to obtain solutions of the Einstein equation in higher dimensional spacetime i.e. more than four, keeping in view of the work done in string theory as well as other field theories. It is assumed and believed that the underlying spacetime in the large energy limit of Planck energy may have higher dimensions more than four. At this level, all the basic forces of nature are supposed to unify and, hence, it would be necessary in this context to obtain solutions of gravitational field equations in higher dimensional spacetime. In fact, such considerations would be more relevant when the usual four-dimensional spacetime becomes inapplicable. This would perhaps happen as we approach a singularity, whether in gravitational collapse or in cosmology.

Since the formulation of the singularity theorems by Hawking and Ellis (1973) and cosmic censorship conjecture by Penrose (1969), the gravitational collapse became the centre strage in gravitational research. The singularity theorems presented that the occurrence of singularities, is a generic feature of spacetimes in classical general relativity. However, these theorems provide nothing about the detailed properties of the singularities, such as their visibility to an external observer as well as their strength. In general relativity, the cosmic censorship conjecture still remains as one of most outstanding unresolved problems. However, there are several known examples in literature showing that both naked singularities and black holes may be formed in gravitational collapse as investigated by Joshi (1993), Clarke (1993), Wald (1997), Singh (1999), Joshi (2000). The central shell focusing singularity may be naked or covered depenting upon the selection of initial data. There is a critical point of solutions, where a transition from naked singularity to black hole takes place.

In this chapter we have investigated the role of dimensionality of spacetime in the context of cosmic censorship conjecture. It turned out that as dimension increases, the window for naked singularity shrinks. It means, gravity appears to get strengthened with an increase in dimensions of spacetime.

4.2 Singularities in Higher-dimensional Vaidya Spacetime:

The metric for this purpose is well known by the works of Iyer and Vishveshwara (1989) and Patel and Dadhich (1999) as

$$(4.1) \qquad ds^2 = -\left[1 - \frac{2m(v)}{(n-1)r^{(n-1)}}\right]dv^2 + 2dvdr + r^2\, d\Omega_n^2 \quad ,$$

where

$$(4.2) \qquad d\Omega_n^2 = d\theta_1^2 \sin^2\theta_1 \;(d\theta_2^2 + \sin^2\theta_2 d\theta_3^2 + \ldots \sin^2\theta_{n-1}\, d\theta_n^2\,) \quad .$$

with

(4.3) $v \in (-\infty, \infty)$

be the null coordinate representing advanced Eddington time, $r \in (0, \infty)$ be the radial cordinate. Again the total number of dimensins are

(4.4) $D = n+2$,

where a metric on the n spheres. The arbitrary functin $m(v)$, which is restricted only by energy conditions, represents mass at advanced time v. One may obtain the energy momentum tensor as

(4.5) $$T_{ab} = \frac{n}{(n-1)r^n} \frac{dm}{dv} k_a k_b$$

where k_a be the null vector

(4.6) $$k_a = -\delta_a^v ,$$

and

(4.7) $$k_a k^a = 0 .$$

Again we have also taken

(4.8) $$8\pi G = c = 1 .$$

It is obvious that for the weak energy condition may be satisfied, one requires dm(v)/dv must be non-negative. Hence, the mass function m(v) is a non-nevative increasing function of v for imploding radiation.

4.3 Self-Similar Models:

In order to obtain an analytical solution for higher dimensional case, let us select mass function as

(4.9) $$m(v) = \begin{cases} 0 & v < 0 \\ \lambda(n-1)v^{(n-1)} , & \lambda>0,\ 0 \le v \le T, \\ m_o > 0 , & v > T \end{cases}$$

with this choice of m(v) the spacetime is self-similar as shown for a general symmetric spacetime if there exist a radial coordinate and the orthogonal time coordinate t for which

$$(4.10) \qquad g_{tt}\,(ct,\, cr) = g_{tt}\,(t,r) \qquad ,$$

$$(4.11) \qquad g_{rr}\,(ct,\, cr) = g_{rr}\,(t,r) \qquad ,$$

for every c > 0. The self-similar spacetime admits a homothetic Killing vector, which is given by Lie derivative as

$$(4.12) \qquad L_{\xi}\, g_{ab} = \xi_{a;b} + \xi_{b;a}$$

$$= 2g_{ab} \quad .$$

Here, the physical situation is that of a radial influx of null fluid in an empty region of the higher dimensional Minkowskian spacetime. The first shell reaches at r = 0 at time v = 0 and the final at v = T. A central singularity of growing mass is developed at $r = 0$. For

(4.13) $$v < 0 ,$$

we get

(4.14) $$m(v) = 0 ,$$

i.e. higher dimensional Minkowskian spacetime, and for

(4.15) $$v > T ,$$

(4.16) $$\frac{dm(v)}{dv} = 0$$

i.e. $m(v)$ is psitive definite. The metric fr $v = 0$ t $v = T$ is higher dimensional Vaidya, and for $v > T$, one obtains higher dimensional Schwarzschild.

Let us define tangent vector K^a to a null geodesic

(4.17) $$K^a = \frac{dx^a}{dk} ,$$

where k be an affine parameter. Hence, one obtain

$$(4.18) \qquad \xi^a K_a = r\, K_r + v\, K_v = C \;.$$

Let us put

$$(4.19) \qquad K^v = P/r \;,$$

as presented by Dwivedi and Joshi (1989, 1991), we get

$$(4.20) \qquad K^r = \left(1 - \frac{2m(v)}{(n-1)r^{n-1}}\right) \frac{P}{2r} \;.$$

Using the values for K^v and K^r, the eq. (4.18) gives

$$(4.21) \qquad P = \frac{2C}{2-(v/r) + \lambda(v/r)^n} \;,$$

in this way the geodesics are cmpletely determined. By virtue of K^v and K^r, the radial null gedesics of the metric must satisfy

(4.22) $$\frac{dr}{dv} = \frac{1}{2}\left(1 - \frac{2m(v)}{(n-1)r^{n-1}}\right) .$$

The eq. (4.22) has a singularity at

(4.23) $$r = 0, \qquad v = 0 .$$

The nature of the collapsing solutions may be characterised by the existence of radial null geodesics coming out of singularity i.e. a naked singularity or a black hole. In view of the mass function m(v), the eq. (4.22) assumes the form

(4.24) $$\frac{dr}{dv} = \frac{1}{2} \quad (1- \lambda X^{n-1})$$

where

(4.25) $$X = v/r ,$$

be the tangent to a possible outgoing geodesic. The cental shell focusing singularity is locally naked if and only if there exist $X_o \in (o, \infty)$ which satisfies

$$X_o = \lim_{r \to 0,\ v \to 0} X \tag{4.26}$$

$$= \lim_{r \to 0,\ v \to 0} \frac{v}{r}$$

$$= \lim_{r \to 0,\ v \to 0} \frac{dv}{dr} = \frac{2}{1 - \lambda X_o^{(n-1)}} ,$$

or

$$\lambda X_o^n - X_o + 2 = 0. \tag{4.27}$$

Hence, any solution of eq. (4.27)

$$X = X_o > 0 , \tag{4.28}$$

would correspond to a naked singularity of the spacetime i.e. to future directed null geodesics emanating from singularity i.e.

$$r = 0, \qquad v = 0 . \tag{4.29}$$

Let now examine the condition for occurrence of naked singularity. It may be shown that eq. (4.27) always admits two real positive roots for $\lambda \leq \lambda_c$, where λ_c be the critical value of the parameter λ deciding the existence of a naked singularity or a black hole. The values of λ_c and X_o are obtained as

$$\lambda_c = \frac{1}{n} \left(\frac{n-1}{2n}\right)^{n-1} \tag{4.30}$$

$$X_o = \frac{2n}{n-1} \quad . \tag{4.31}$$

Table 4.1

D = n+2 (Dimensions)	$\lambda_c = \frac{1}{n} \left(\frac{n-1}{2n}\right)^{n-1}$ (critical value)	$X_o = \frac{2n}{n-1}$ (Tangent)
4	1/8	4
4	1/27	3
6	27/2048	2.6667
7	256/5000	2.5

Table 4.2

D Dimensions	$\lambda < \lambda_c$	Two tangents (X_o)
4	0.11	2.9708, 6.1200
5	0.035	2.6551, 3.4978
6	0.013	2.5476, 2.8065
7	0.005	2.4524, 2.5514

Hence, it follows that singularity be naked if

(4.32) $$\lambda \leq \lambda_c$$

Again if

(4.33) $$\lambda > \lambda_c \quad ,$$

the naked singularity will not form and gravitational collapse would result in black hole. It is to be noted that X_o is bounded below the value 2, $X_o \longrightarrow 2$ as $\lambda \longrightarrow 0$

or D $\longrightarrow \infty$. It is obvious from table No. (4.1) that λ_c decreases as D is increased. Hence, the naked singularity gets covered with the introduction of extra dimensions. From Table (4.2), the two roots indicate the naked singularit window in the slope of the tangent to geodesics emanating from the singularity, which pinches with an increase in dimensions. The degree of inhomogeneity is measured by

$$(4.34) \qquad \mu = 1/\lambda \quad .$$

Hence, the inhomogeneity factor increases with D. An increase in inhomogeneity should favour naked singularity and hence, should increase the spectrum.

4.4 Strength of Singularity:

In view of Clark and Krolak (1986), let us assume the null geodesics affinely parametrized by k and terminating at shell focusing singularity

$$(4.35) \qquad r = v = k = 0.$$

Hence, it must be a strong curvature singularity as investigated by Tipler (1980), such that

$$(4.36) \quad \lim_{k \to 0} k^2 \psi = \lim_{k \to 0} k^2 R_{ab} K^a K^b > 0$$

where R_{ab} be the Ricci tensor. In view of the above the equation (4.36) assumes the form

$$(4.37) \quad \lim_{k \to 0} k^2 \psi = \lim_{k \to 0} n \lambda X^{n-2} \left(\frac{kP}{r^2}\right)^2 .$$

Let us note that

$$(4.38) \quad \frac{dX}{dk} = \frac{1}{r} K^v - \frac{X}{r} K^r$$

$$= (2 - X + \lambda X^n) \frac{P}{2r^2}$$

$$= \frac{C}{r^2} .$$

In view of the fact that as singularity is reached,

$$(4.39) \quad k \to 0, \quad r \to 0$$

and

(4.40) $$x \longrightarrow a_+$$

where a_+ be a root of eq. (4.27) and also using L'Hopital Law, one obtains

(4.41) $$\lim_{k \to 0} \frac{kP}{r^2} = \frac{2}{1+(n-2)\lambda X_o^{n-1}} ,$$

so the eq. (4.37) yields

(4.42) $$\lim_{k \to 0} k^2 \psi = \frac{4n\lambda X_o^{n-2}}{\left(1+(n-2)\lambda X_o^{n-2}\right)^2} > 0$$

Hence, along the radial null geodesics, a strong curvature condition is satisfied and therefore, it is a strong curvature singularity.

4.5 Non self-similar Models:

Let us investigate the non self-similar models in higher dimensional Vaidya spacetimes. It has been presented that in 4-dimensional non self-similar spacetimes, naked singularity occurs as shown by Joshi (1993), Wald (1997) Singh (1999), Jhingan (2000)

Rajagopal and Lake (1987, 1991) Dwivedi and Joshi (1991). Hence, we studied a similar situation in higher dimensional Vaidya spacetimes. Let us define the mass function as

(4.43) $$3m(v) = (n-1)\, \beta^{n-1}\, v^{\alpha(n-1)} \left(1-2\alpha\beta\, v^{(\alpha-1)}\right) ,$$

where α and β are constants such that

(4.44) $$\alpha > 1 .$$

In this case the condition of self-similarity is not satisfied i.e.

(4.45) $$g_{tt}(ct, cr) \neq g_{tt}(t,r)$$

(4.46) $$g_{rr}(ct, cr) \neq g_{rr}(t,r)$$

for every $c > 0$.

Rajagopal and Lake (1987, 1991), Joshi and Dwivedi (1991, 1992) have presented this class of solutions for 4-dimensional spacetimes. As stated above, the null radiation shells imploding at v = 0 and the final shell comes at v = T. The weak energy condition requires

$$T^{\alpha-1} < \frac{n-1}{2\beta(n\alpha-1)} \quad . \qquad (4.47)$$

It is obvious that for

$$v = 0, \quad m(v) = 0, \qquad (4.48)$$

and one obtains a higher dimensional Minkowskian spacetime and for

$$v = T, \quad \frac{dm}{dv} = 0 \qquad (4.49)$$

and

$$m(v) = m_o(T) > 0 \quad , \qquad (4.50)$$

One obtains higher dimensional Schwarzschild spacetime. For the mass function (4.43), one may obtain the radial null geodesics

$$r = \beta v^{\alpha} \qquad (4.51)$$

This integral curve meets the singularity with a tangent at

$$r = 0 \quad , \qquad (4.52)$$

Showing the presence of naked singularity.

4.6 Concluding Remarks:

We have investigated the end state of the gravitational collapse of a null fluid in higher dimensional spacetimes, i.e. higher dimensional Vaidya spacetimes. Both naked singularities and black holes are shown to be developing as the final outcome of the collapse. It is also presented that the naked singularity spectrum in a collapsing Vaidya four-dimensional spacetime gets covered with the increase in dimensions and hence higher dimensions favours a black hole in comparision to a naked singularity. We have studied the effect of the increase of dimension of the spacetime on the come in picture: first is an increase in inhomogeneity and the other a strengthening of gravitational field. The former favours naked singularity and the latter a black hole. We have obtained that in final analysis the black hole is formed and leads to the shrinkage of the naked singularity. The motivation for higher dimensional study came from the string theory where the effective actioin involves the dilaton scalar field or antisymmetric tensor field. The dilaton field couples nonminimally to the Ricci curvature. However, it would be trivial in this case, as the scalar curvature vanishes for the Vaidya solution. The case of the antisymmetric tensor field would be similar as well. Hence, the results obtained here would also be relevant and valid for effective supergravity theories.

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