CHAPTER-VII

ABSOLUTE OF R-PROXIMITY SPACES

7.1 Introduction

Extremally disconnected spaces play a crucial role in the theory of Boolean algebras, in axiomatic set theory and in some branches of functional analysis as well.

In the classical context, there are several methods of obtaining the absolute of a regular topological space. In one of these methods, the absolute of a regular topological space X is realized as the Stone space of the complete Boolean algebra of the family of regular closed sets; while in another, the absolute is obtained as a dense subspace of an extremally disconnected subspace E(X) of the product space $\Pi\{\check{a}:a\in\mathscr{U}(X)\}$,where ǎ is the discrete topological space and $\mathscr{U}(X)$ is the collection of all regular covers of X.

In the present chapter, the proximal absolute (p-absolute) of an R-proximity space is obtained as a dense proximal subspace of the proximity space T(X), a closed proximal subspace of the product proximity space T_0 of all discrete proximity spaces.

7.2 Definitions and Results

In the present section , by X we shall mean an R-proximity space, RC(X) will denote the set of all p-regular covers of X, $RC_0(X)$ is the refined and directed family of all p-regular covers. For $\alpha \in RC_0(X)$, $\hat{\alpha}$ denotes the discrete proximity space. Further, $T_0(X)$ is the collection of proximal threads in $RC_0(X)$, endowed with the induced proximity by T_0. Let $\dot{X}_0$ be the set of all distinguished proximal threads in $RC_0(X)$ together with the induced proximity by T_0.

DEFINITION 7.2.1: A mapping $\prod_X^0 : \dot{X}_0 \to X$ of the proximity space $\dot{X}_0$ of all distinguished proximal threads in $RC_0(X)$ into the R-proximity space X defined by $\prod_X^0(\xi) = \cap\{A : A \in \xi\}$, is called a natural mapping generated by the family $RC_0(X)$.

REMARK 7.2.2: If we consider RC(X), we obtain the proximity space T(X) of all proximal threads in RC(X) and the proximity space $\dot{X}$ of all distinguished proximal threads in RC(X). Obviously, $\dot{X} \subseteq T(X) \subseteq T_0 = \prod\{\hat{\alpha}, \alpha \in RC(X)\}$.

DEFINITION 7.2.3: A proximally continuous mapping f from a proximity space X onto a proximity space Y is called p-irreducible if Y is not the image under f of a closed set F in X, other than X.

DEFINITION 7.2.4: A proximity space X is called extremally disconnected if for every open set U in $\tau(\delta)$ in X the set $\overline{U}$ is not only closed but also open in X with respect to the induced topology $\tau(\delta)$.

DEFINITION 7.2.5: A proximity space $\dot{X}$ is called a proximal absolute or p-absolute of the R-proximity space X if, $\dot{X}$ is a p-irreducible perfect pre image of X and every p-irreducible perfect pre image of $\dot{X}$ is p-homeomorphic to $\dot{X}$.

THEOREM 7.2.6: *Let* $f : X \rightarrow Y$ *be a p-irreducible, p-closed mapping and* $f(X) = Y$. *Then for every open set* U *in* X, *the set* $f^{-1}f^{\#}(U)$ *is open, non-empty, is contained in* U *and is proximally dense in* U, *where* $f^{\#}(U) = \{y \in Y : f^{-1}y << U\}$.

PROOF: Obviously, $f^{-1}f^{\#}(U)$ is non-empty. Let $x \in f^{-1}f^{\#}(U)$. Then $f(x) \in f^{\#}(U)$. Hence $f^{-1}f(x) << U$ (by definition of $f^{\#}$) or $f^{-1}(f(x)) \not\delta$ X-U. Since f is p-closed, $f(f^{-1}(f(x))) \not\delta f(X\text{-}U)$ or $f(x) \not\delta f(X\text{-}U)$ i.e. $f(x) << Y - f(X\text{-}U)$ or $f(x) << f^{\#}(U)$, since $f^{\#}(U) = Y - f(X\text{-}U)$. Thus, by the p-continuity of f, $x \subseteq f^{-1}(f(x)) << f^{-1} f^{\#}(U)$ and hence $x << f^{-1} f^{\#}(U)$. Consequently, $f^{-1} f^{\#}(U)$ is open in the induced topology.

Next, we have to show that $f^{-1} f^{\#}(U) \subseteq U$.

Let $x \in f^{-1} f^{\#}(U)$ or $x \in f^{-1}(Y - f(X\text{-}U))$. So $f(x) \in Y - f(X\text{-}U)$. This gives $x \notin X\text{-}U$ or $x \not\delta X\text{-}U$ or $x \ll U$. Consequently, $f^{-1} f^{\#}(U) \subseteq U$.

That $f^{-1} f^{\#}(U)$ is proximally dense in U, follows by using the fact that for any open set U' contained in U, $f^{\#}(U') \neq \phi$.

RESULT 7.2.7: Let $f : X \to Y$ be a p-irreducible, p-closed mapping. Then $f\,\overline{U} = \overline{f^{\#} U}$ for every open set U in X and the image of every p-regular closed set in X is a p-regular closed set in Y.

RESULT 7.2.8: A dense proximal subspace X_0 of a extremally disconnected R-proximity space X is also extremally disconnected.

7.3 Absolute of R-Proximity Spaces

THEOREM 7.3.1: *Let* X *be a* R*-proximity space and* RC(X) *is the family of all p-regular covers of* X*. Consider the Tychonoff product* $T_0 = \prod\{\hat{\alpha}, \alpha \in RC_0(X)\}$ *.Then* $\dot{X} \subseteq T(X) \subseteq T_0$.

Suppose that $\dot{X}$ *and* T(X) *are assigned the subspace proximity induced by* T_0. Then the following hold-

(i) T(X) *is closed in* T_0;

(ii) T(X) *is compactum (compact and separated)*;

(iii) *the subspace* $\dot{X} \subseteq$ T(X) *is proximally dense in* T(X).

PROOF: (i) Let $\xi_0 \in T_0 = \prod\{\hat{\alpha}, \alpha \in RC_0(X)\}$ and $\xi_0 \in \left(\overline{T(X)}\right)_{T_0}$. We have to show that $\xi_0 \in$ T(X) i.e. ξ_0 is a proximal thread. Let $\alpha_1, \alpha_2 \in$ RC(X) and $\alpha_2 > \alpha_1$. Let us consider the coordinates $A^0_{\alpha_1}$ and $A^0_{\alpha_2}$ of ξ_0 in the discrete proximity spaces $\hat{\alpha}_1$ and $\hat{\alpha}_2$ respectively, i.e. $\{A^0_{\alpha_1}\} = \xi_0 \cap \alpha_1$, $\{A^0_{\alpha_2}\} = \xi_0 \cap \alpha_2$. It is required to show that $A^0_{\alpha_2} \subseteq A^0_{\alpha_1}$. Let $N(A^0_{\alpha_1}, A^0_{\alpha_2})$ be a proximal neighbourhood of ξ_0 in T_0. Since $\xi_0 \in \left(\overline{T(X)}\right)_{T_0}$, then every proximal neighbourhood of ξ_0 in T_0 meets T(X). Let ξ be a member in the intersects $T(X) \cap N(A^0_{\alpha_1}, A^0_{\alpha_2})$. It follows that ξ is a proximal thread and $A^0_{\alpha_1}, A^0_{\alpha_2} \in \xi$. Since $\alpha_2 > \alpha_1$, it follows that $A^0_{\alpha_2} \subseteq A^0_{\alpha_1}$.

(ii) Obvious.

(iii) Follows from the fact that

$$\langle\overline{A_0}\rangle \cap \dot{X} = \langle A_0\rangle,$$

where $\langle\overline{A_0}\rangle = \{\xi \in T(X) : A_0 \in \xi\}$, $\langle A_0\rangle = \{\xi \in \dot{X} : A_0 \in \xi\}$.

THEOREM 7.3.2 : *A binary relation* Π^* *on* $P(\dot{X})$ *defined by: for* $\mathcal{A}$, $\mathcal{B} \in P(\dot{X})$ "$\mathcal{A}\ \Pi^*\ \mathcal{B}$ *if and only if there exist*

$\langle A\rangle, \langle B\rangle \in P(\dot{X})$, A, B $\in$ R(X) *such that* $\mathcal{A} \subseteq \langle A\rangle$, $\mathcal{B} \subseteq \langle B\rangle$ *and* A $\not\delta$ B" satisfies the following properties –

(i) Π^* *is symmetric*;

(ii) $(\mathcal{A} \cup \mathcal{B})\ \not\Pi^*\ \mathcal{C}$ *iff* $\mathcal{A}\ \not\Pi^*\ \mathcal{C}$ *and* $\mathcal{B}\ \not\Pi^*\ \mathcal{C}$;

(iii) $\mathcal{A} \cap \mathcal{B} \neq \phi$ *implies* $\mathcal{A}\ \Pi^*\ \mathcal{B}$;

where $\langle A\rangle = \{\xi \in \dot{X} : A \in \xi\}$.

PROOF: We shall prove only (ii).

(ii) Suppose $(\mathcal{A} \cup \mathcal{B})\ \not\Pi^*\ \mathcal{C}$. To show $\mathcal{A}\ \not\Pi^*\ \mathcal{C}$ and $\mathcal{B}\ \not\Pi^*\ \mathcal{C}$.

Since $(\mathcal{A} \cup \mathcal{B})\ \not\Pi^*\ \mathcal{C}$, there exist $\langle A\rangle, \langle B\rangle \in P(\dot{X})$ such that $(\mathcal{A} \cup \mathcal{B}) \subseteq \langle A\rangle$, $\mathcal{C} \subseteq \langle B\rangle$ and A $\not\delta$ B i.e. $\mathcal{A} \subseteq \langle A\rangle$, $\mathcal{B} \subseteq \langle A\rangle$, $\mathcal{C} \subseteq \langle B\rangle$ and A $\not\delta$ B. It follows that $\mathcal{A}\ \not\Pi^*\ \mathcal{C}$ and $\mathcal{B}\ \not\Pi^*\ \mathcal{C}$.

Conversely, suppose that $\mathcal{A}\ \not\Pi^*\ \mathcal{C}$ and $\mathcal{B}\ \not\Pi^*\ \mathcal{C}$. Then there exist $\langle A\rangle, \langle B\rangle, \langle C\rangle, \langle D\rangle \in P(\dot{X})$, where A, B, C, D $\in$ R(X) satisfying $\mathcal{A} \subseteq \langle A\rangle$, $\mathcal{C} \subseteq \langle B\rangle$ and A $\not\delta$ B. Also, $\mathcal{B} \subseteq \langle C\rangle$, $\mathcal{C} \subseteq \langle D\rangle$ and C $\not\delta$ D. Since A $\not\delta$ B and C $\not\delta$ D, we get $(A \cup C)\ \not\delta\ (B \cap D)$.

Now, $\mathcal{A} \subseteq \langle A\rangle$, $\mathcal{B} \subseteq \langle C\rangle$ imply $\mathcal{A} \cup \mathcal{B} \subseteq \langle A\rangle \cup \langle C\rangle$ and $\mathcal{C} \subseteq \langle B\rangle$, $\mathcal{C} \subseteq \langle D\rangle$ gives $\mathcal{C} \subseteq \langle B\rangle \cap \langle D\rangle$. It follows that $(\mathcal{A} \cup \mathcal{B})\ \not\Pi^*\ \mathcal{C}$.

THEOREM 7.3.3: *Let* (X, δ) *be a* R-*proximity space. If* RC(X) *is the family of all p-regular covers of* X, $\dot{X}$ *is the space of all distinguished proximal threads in* RC(X) *and* $\Pi_X : \dot{X} \to X$ *is the natural mapping. Then, for every p-regular closed set* $A_0 \in R(X)$*, we have equalities:*

(a) $\Pi_X(\langle A_0 \rangle) = A_0$;

(b) $\Pi_X^{\#}(\langle A_0 \rangle) = \mathrm{Int} A_0$, where $\Pi_X^{\#}(\langle A_0 \rangle) = \{x \in X : \Pi_X^{-1} x << \langle A_0 \rangle\}$.

PROOF: (a) Let $\xi \in \langle A_0 \rangle$. Then $A_0 \in \xi$. But by the definition of Π_X, $\Pi_X \xi = \bigcap\{A : A \in \xi\} = \{x\} \subseteq A_0$ i.e. $\Pi_X \xi \in A_0$. Thus,

$$\Pi_X(\langle A_0 \rangle) \subseteq A_0 \qquad (1)$$

Conversely, let $x_0 \in A_0$. Then there exist a proximal thread ξ_0 distinguished at x_0 (cf. VI, Theorem 6.4.7), for which $A_0 \in \xi_0$ i.e. $\xi_0 \in \dot{X}$, $A_0 \in \xi_0$ and $\{x_0\} = \bigcap\{A : A \in \xi_0\}$. Thus $\xi_0 \in \langle A_0 \rangle$ and $\Pi_X \xi_0 = \{x_0\}$. Hence

$$A_0 \subseteq \Pi_X(\langle A_0 \rangle) \qquad (2)$$

From (1) and (2)

$$\Pi_X(\langle A_0 \rangle) = A_0.$$

(b) Let $x_0 \in \mathrm{Int}A_0$. Then $x_0 << \mathrm{Int}A_0$ and $A_0 \in \alpha_0$. We have to show that $x_0 \in \prod_X^{\#}(\langle A_0 \rangle)$. Let us consider any distinguished proximal thread ξ, for which $\prod_X \xi = \{x_0\}$ i.e. $\{x_0\} = \bigcap\{A : A \in \xi\}$. The set $A_0 \in \alpha_0$ represents a distinguished proximal thread. Hence $\xi \in \langle A_0 \rangle$. But $\prod_X^{-1} x_0 = \left\{\xi \in \dot{X} : \bigcap\{A : A \in \xi\} = \{x_0\}\right\}$ so that $\prod_X^{-1} x_0 \subseteq \langle A_0 \rangle$. This gives $x_0 \in \prod_X^{\#}(\langle A_0 \rangle)$. Thus, $\mathrm{Int}A_0 \subseteq \prod_X^{\#}(\langle A_0 \rangle)$.

Conversely, let $x_0 \in \prod_X^{\#}(\langle A_0 \rangle)$. Then by definition, $\prod_X^{-1} x_0 = \left\{\xi \in \dot{X} : \bigcap\{A : A \in \xi\} = \{x_0\}\right\} \subseteq \langle A_0 \rangle$. Then $A_0 \in \xi$ for every $\xi \in \prod_X^{-1} x_0$.

We have to show that $x_0 \in \mathrm{Int}A_0$. It suffices to show that $\xi \bigcap \alpha$ consists of exactly one element for each $\alpha_0 \in \mathrm{RC}(X)$.

Suppose that $\alpha_0 \in \mathrm{RC}(X)$ is such that $A_0 \in \alpha_0$. To the contrary, assume that $x_0 \notin \mathrm{Int}A_0$. We would have $A_0' \in \alpha_0$, $A_0' \neq A_0$ for which $x_0 \in A_0'$. Then there exist a distinguished proximal thread ξ_0' such that $\{x_0\} = \bigcap\{A_0' : A_0' \in \xi_0'\}$ and $A_0' \in \xi_0'$. Since $\alpha_0 \in \mathrm{RC}(X)$. Thus, $\xi_0' \in \prod_X^{-1} x_0$ and $A_0' \in \xi_0'$. Also, as $\xi_0' \in \prod_X^{-1} x_0$, $A_0 \in \xi_0'$. Thus we obtain, $A_0 \in \xi_0' \bigcap \alpha_0$, $A_0' \in \xi_0' \bigcap \alpha_0$, $A_0' \neq A_0$, which is a contradiction to the fact that

ξ_0' is a proximal thread iff ξ_0' is a x_p-ultrafilter. Hence $x_0 \in \text{Int}A_0$. Thus $\Pi_X^{\#}(\langle A_0 \rangle) \subseteq \text{Int}A_0$. Consequently, $\Pi_X^{\#}(\langle A_0 \rangle) = \text{Int}A_0$.

THEOREM 7.3.4: *If* X *is a* R*-proximity space, then the natural mapping* $\Pi_X : \dot{X} \to X$ *is p-continuous.*

PROOF: Let A, B $\in$ P(X) such that A $\bar{\delta}$ B, for the p-continuity of Π_X, it is sufficient to show that $\Pi_X^{-1}(A)\ \bar{\delta}\ \Pi_X^{-1}(B)$ in $\dot{X}$. Now A $\bar{\delta}$ B implies A << X-B. Since X is a R-proximity space, there exist C, D, U $\in$ P(X) such that A << $C^0 \subseteq$ C << X-U << D << X-B. Then A << $\Pi_X^{\#}(\langle C \rangle) \subseteq$ C << X-U << D << X-B, since $\Pi_X^{\#}(\langle A_0 \rangle)$= IntA=$A^0$. This gives $\Pi_X^{-1}(A) \subseteq$ C (by the definition $\Pi_X^{\#}$). Now, X-U << X-B i.e. B << U.

Similarly, since B << $U^0 \subseteq$ U, it follows that $\Pi_X^{-1}(B) \subseteq$ U. Since C $\bar{\delta}$ U, therefore $\Pi_X^{-1}(A)\ \bar{\delta}\ \Pi_X^{-1}(B)$. Hence the map $\Pi_X : \dot{X} \to X$ is p-continuous.

THEOREM 7.3.5: *Let* (X, δ) *be a* R*-proximity space. Then the natural mapping* $\Pi_X : \dot{X} \to X$ *is a p-irreducible and compact.*

PROOF: To show that Π_X is compact. Let $x_0 \in X$. It is required to show that the subspace $\Pi_X^{-1} x_0$ is compact. Since T(X) is compact, it suffices to show that $\Pi_X^{-1} x_0$ is closed in T(X). It is sufficient to show that $\xi \ \delta \ \Pi_X^{-1} x_0$ imply $\xi \in \Pi_X^{-1} x_0$. Now, since Π_X is p-continuous and $\xi \ \delta \ \Pi_X^{-1} x_0$, therefore $\Pi_X(\xi) \ \delta \ \Pi_X \Pi_X^{-1} x_0$ or $\Pi_X(\xi) \ \delta \ \{x_0\}$ or $\Pi_X(\xi) = \bigcap\{A : A \in \xi\} = \{x_0\}$ or $x_0 \in \bigcap A$, $A \in \xi$. Thus $\xi \in \Pi_X^{-1} x_0$. Hence $\Pi_X^{-1} x_0$ is closed. The irreducibility of Π_X is implied by theorem 7.3.1.

THEOREM 7.3.6: *If* X *is a* R*-proximity space, then the natural mapping* $\Pi_X : \dot{X} \to X$ *is a p-irreducible perfect mapping onto* X.

PROOF: The p-continuity, compactness and p-irreducibility of Π_X have already been shown in above theorem. It remains to show that Π_X is p-closed. It suffices to show that

$\mathcal{A} \not\delta^* \mathcal{B}$ implies $\Pi_X(\mathcal{A}) \not\delta \Pi_X(\mathcal{B})$, where $\mathcal{A}, \mathcal{B} \in P(\dot{X})$.

Suppose $\mathcal{A} \not\delta^* \mathcal{B}$, then by definition, there exist $\langle A\rangle, \langle B\rangle \in P(\dot{X})$ such that $\mathcal{A} \subseteq \langle A\rangle$, $\mathcal{B} \subseteq \langle B\rangle$ and $A \not\delta B$. Now, $\mathcal{A} \subseteq \langle A\rangle$ implies that $\Pi_X(\mathcal{A}) \subseteq \Pi_X(\langle A\rangle)$ or $\Pi_X(\mathcal{A}) \subseteq A$ (since

$\Pi_X(\langle A\rangle) = A)$. Similarly, $\Pi_X(\mathcal{B}) \subseteq B$. Since A $\delta\!\!\!/$ B, it follows that $\Pi_X(\mathcal{A}) \,\delta\!\!\!/\, \Pi_X(\mathcal{B})$. Hence the map Π_X is p-closed.

THEOREM 7.3.7: *Let* $f : X \to Y$ *be a p-irreducible, p-open mapping and* $f(X) = Y$. *If* A_1, A_2 *are p-regular closed sets in* X *for which* $f A_1 = f A_2$, *then* $A_1 = A_2$.

PROOF: Since A_1, A_2 are p-regular closed sets, therefore, by definition, $A_1 = \overline{\text{Int } A_1}$, $A_2 = \overline{\text{Int } A_2}$. Using the result 7.2.7,

$$f A_1 = f(\overline{\text{Int } A_1}) = \overline{f^{\#}(\text{Int} A_1)},$$
$$f A_2 = f(\overline{\text{Int } A_2}) = \overline{f^{\#}(\text{Int} A_2)}. \quad (1)$$

Also,

$$f^{-1}(f^{\#}(\text{Int } A_1)) \subseteq f^{-1}(\text{Int } f(A_1)) \subseteq A_1,$$
$$f^{-1}(f^{\#}(\text{Int } A_2)) \subseteq f^{-1}(\text{Int } f(A_2)) \subseteq A_2. \quad (2)$$

By theorem 7.2.6, since the set $f^{-1}(f^{\#}(\text{Int } A_1))$ is dense in A_1 and $f^{-1}(f^{\#}(\text{Int } A_2))$ is dense in A_2, we have

$$f^{-1}(f^{\#}(\text{Int } A_1)) = \overline{f^{-1}(\text{Int } f(A_1))} = A_1,$$
$$f^{-1}(f^{\#}(\text{Int } A_2)) = \overline{f^{-1}(\text{Int } f(A_2))} = A_2. \quad (3)$$

But by (1) and the equality $f A_1 = f A_2$ imply that Int f $(A_1) = \text{Int } f(A_2)$. Hence using above and (3), we obtain $A_1 = A_2$.

THEOREM 7.3.8: *Let* X *be a* R-*proximity space and let* RC(X) *be the family of all* p-*regular covers of* X. *Then*

(i) T(X) *is an extremally disconnected compactum,*

(ii) $\dot{X}$ *is an extremally disconnected completely regular proximity space.*

(iii) $\dot{X}$ *is a proximal absolute of* X.

PROOF: (i) Let G be an open set in T(X). It is to be shown that $\left(\overline{G}\right)_{T(X)}$ is open in T(X). Since $\dot{X}$ is proximally dense in T(X), then

$$\left(\overline{G}\right)_{T(X)} = \left(\overline{G \cap \dot{X}}\right)_{T(X)} = \left(\overline{\left(\overline{G \cap \dot{X}}\right)_{\dot{X}}}\right)_{T(X)} . \qquad (1)$$

Let us write $A_1 = \Pi_X\left(\left(\overline{G \cap \dot{X}}\right)_{\dot{X}}\right)$ and take A_2 the complement of A_1 in X.

Consider $\alpha_0 = \{A_1, A_2\}$ the p-regular cover of X, since both A_1 and A_2 are p-regular closed sets in X. Let $\langle A_1\rangle = \{\xi \in \dot{X} : A_1 \in \xi\}$, $\langle A_2\rangle = \{\xi \in \dot{X} : A_2 \in \xi\}$ be clopen sets in $\dot{X}$. We know that $\Pi_X(\langle A_1\rangle) = A_1$. Now $\langle A_1\rangle$ is a p-regular closed sets in $\dot{X}$ and $\Pi_X(\langle A_1\rangle) = \Pi_X\left(\left(\overline{G \cap \dot{X}}\right)_{\dot{X}}\right)$, where Π_X is perfect and p-irreducible, therefore $\left(\overline{G \cap \dot{X}}\right)_{\dot{X}} = \langle A_1 \rangle$. But $\left(\overline{\langle A_1 \rangle}\right)_{T(X)} = \overline{\langle A_1 \rangle}$, $\overline{\langle A_1 \rangle}$ is a clopen set in T(X); hence $\overline{\langle A_1 \rangle} = \left(\overline{\langle A_1 \rangle}\right)_{T(X)} = \left(\overline{\left(\overline{G \cap \dot{X}}\right)_{\dot{X}}}\right)_{T(X)} = \left(\overline{G}\right)_{T(X)}$. Consequently, $\left(\overline{G}\right)_{T(X)}$ is clopen in T(X).

(ii) Since T(X) is extremally disconnected and by result 7.2.8, the dense proximal subspace $\dot{X}$ is also extremally disconnected. That $\dot{X}$ is completely regular follows from the fact that it is a separated proximity space.

(iii) Follows from the fact that $\dot{X}$ is a dense proximal subspace of an extremally disconnected compact separated proximity space.

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6. E.M. Alfsen And O. Njastad, Proximity And Generalised Uniformity, *Fund. Math.* 52 (1963) 235-252.

7. E. Čech, *Topological Spaces*, Interscience Publishers, Revised Edition By Zdeněk Froli'k And Miraslav Katětov, John Wiley And Sons Ltd., 1966.

8. E. Čech, On Bicompact Spaces, *Ann. of Math.* 38(1937) 823-844.

9. A. Di Concillio And S.A. Naimpally, Function Space Topologies On (Partial) Maps, Recent Progress In Function Spaces, *Quaderni Di Matematica* (G. Di Maio And L'Hola,Eds.), Vol.3,1999.

10. J. Dugundji, *Topology*, Allyn And Becon, Boston (1986).

11. V.A. Efremovič, Infinitesimal Spaces, *Dokl. Akad. Nauk, SSSR* 76(1951) 341-343.

12. V.A Efremovič, The Geometry of Proximity I, *Mat. Sb.* 31, 73 (In Russian) (1952) 189-200.

43. S. Willard, *General Topology*, Addison Wesley, Reading, Mass (1970).

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