Chapter - V

LRS BIANCHI TYPE-I
COSMOLOGICAL MODELS
IN BARBER’S SECOND SELF CREATION THEORY
Chapter 5

LRS BIANCHI TYPE-I COSMOLOGICAL MODELS IN BARBER'S SECOND SELF CREATION THEORY

5.1 Introduction

Several modifications of Einstein's general relativity have been proposed and extensively studied so far by many cosmologists to unify gravitation and many other effects in the universe. Barber [1] has produced two continuous self-creation theories by modifying the Brans-Dicke theory and general relativity. The modified theories create the universe out of self-contained gravitational and matter fields. Brans-Dicke [2] theory develops Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution of matter in motion. However, Barber [1] has included continuous creation of matter in these theories. The universe is seen to be created out of self-contained gravitational, scalar and matter fields. However, the solution of the one-body problems reveals unsatisfactory characteristics of the first theory and in particular the principle of equivalence is severely violated. The second theory retains the attractive features of the first theory and overcomes previous objections. These modified theories create the universe out of self-contained gravitational and matter fields. Recently,
Brans [3] has also pointed out that Barber's first theory is in disagreement with experiment as well as inconsistent, in general, since the equivalence principle is violated. Barber’s second theory is a modification of general relativity to a variable G-theory. The second is an adaptation of general relativity to include continuous creation and is within the observational ambit. In this theory the scalar fields do not directly gravitate, but simply divide the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar field couples with the trade of energy momentum tensor. In view of the consistency of Barber’s second theory of gravitation, we intend to investigate some of the aspects of this theory in this paper.

Several cosmologists have studied various aspects of Robertson-Walker model in Barber’s second self-creation cosmology with perfect fluid satisfying the equation of state $p = (\gamma - 1)\rho$, where $1 \leq \gamma \leq 2$. Pimentel [4] and Soleng [5] have discussed the Robertson-Walker solutions in Barber’s second self-creation theory of gravitation by using power law relation between the expansion factor of the universe and the scalar field. Carvalho [6] studied a homogeneous and isotropic model of the early universe in which parameter gamma of 'gamma law' equation of state varies continuously with cosmological time and presented a unified description of early universe for inflationary period and radiation-dominating era. Singh [7], Reddy [8, 9], and Reddy et al [10] have presented Bianchi type space-times solutions in Barber’s second theory of gravitation. Reddy and Venkateswarlu [11] present Bianchi type-$VI_0$ cosmological solutions in Barber’s second theory of gravitation both, in vacuum as well as in the presence of perfect fluid with pressure equal to energy density. Shanthi and Rao [12] studied Bianchi type II and III space-times in second theory of gravitation, both in vacuum as well as in presence of stiff-fluid. Recently, Shri Ram and Singh [13] have discussed spatially homogeneous and isotropic R-W model of the universe in Barber’s second self-creation theory of gravitation in the presence of perfect fluid by using 'gamma-law' equation of state.
In this paper we obtain cosmological solutions for a LRS Bianchi type-I metric in Barber's second self-creation theory of gravitation with perfect fluid for constant deceleration parameter models for the universe. In section-4, we have also studied the field equations for constructing models for the vacuum universe, Zel'dovich universe and radiation universe and explicit solutions are obtained in these three cases. Some physical consequences of the cosmological models are discussed.

5.2 Field Equations

We consider LRS Bianchi type I space-time

\[ ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2) \]  

(5.1)

where \( A = A(x, t), B = B(x, t) \). The field equations in Barber's second self-creation theory [1] are

\[ R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi T_{ij}}{\phi} \]  

(5.2)

and

\[ \Box \phi = 4\pi \lambda T \]  

(5.3)

where \( \Box \phi \equiv \phi_{ij}^{ik} \) is the invariant d'Alembertian and \( T \) is the trace of the energy momentum tensor. \( \lambda \) is coupling constant to be determined from the experiment (\( |\lambda| \leq 0.1 \)). In the limit \( \lambda \to 0 \) this theory approaches the standard general relativity theory in every respect and \( G = \frac{1}{\phi} \).

The energy momentum tensor \( T_{ij} \) for a perfect fluid distribution is given by

\[ T_{ij} = (p + \rho)u_i u_j - pg_{ij} \]  

(5.4)
where \( p \) is the pressure, \( \rho \) the energy density and \( u_i \) represents the four velocity vector. Corresponding to metric (1), the four velocity vector \( u_i \) satisfies the equation

\[
g_{ij} u^i u^j = 1
\]  
(5.5)

The Bianchi identities in contravariant form applied to equation (2) are

\[
W_{\mu} T^{\mu}_{ij} + W T_{ij} = 0
\]  
(5.6)

where \( W = -8\pi\phi^{-1} \) and in general relativity \( W = -8\pi G \). A comma and a semicolon denote ordinary and covariant differentiation, respectively. In a comoving coordinate system, the surviving components of the field equations (2)-(6) for metric (1) are

\[
\frac{2B}{B} + \frac{\dot{B}^2}{B^2} - \frac{B^2}{A^2 B^2} = -8\pi\phi^{-1} \rho
\]  
(5.7)

\[
\frac{\dot{B}'}{A} - \frac{B'\dot{A}}{A} = 0
\]  
(5.8)

\[
\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\ddot{A}B}{AB} - \frac{B''}{A^2 B} + \frac{A'B'}{A^3 B} = -8\pi\phi^{-1} \rho
\]  
(5.9)

\[
\frac{2B''}{A^2 B} - \frac{2A'B'}{A^3 B} + \frac{B^2}{A^2 B^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} = 8\pi\phi^{-1} \rho
\]  
(5.10)

\[
\ddot{\phi} + \frac{\dot{A}\dot{\phi}}{A} + \frac{2B\ddot{\phi}}{B} + \frac{A'\phi'}{A^3} - \frac{2B'\phi'}{A^2 B} - \frac{\phi''}{A} = (\rho - 3p)\left(\frac{8\pi\lambda}{3}\right)
\]  
(5.11)

Here and in what follows a prime and a dot indicate partial differentiation with respect to \( x \) and \( t \) respectively.

### 5.3 Solutions of the field equations

Equation (8), after integration, yields
\[ A = \frac{B'}{\sigma} \quad (5.12) \]

where \( \sigma \) is an arbitrary function of \( x \).

Equations (7) and (9), with the use of equation (12), reduce to

\[ \frac{B}{B'} \frac{d}{dx} \left( \frac{\dot{B}}{B} \right) + \frac{\dot{B}}{B'} \frac{d}{dt} \left( \frac{B'}{B} \right) + \frac{\sigma^2}{B^2} \left( 1 - \frac{B_0'}{B'} \right) = 0 \quad (5.13) \]

If we assume \( \frac{B'}{B} \) as a function of \( x \) alone, then \( A \) and \( B \) are separable in \( x \) and \( t \), equation (13) gives after integration

\[ B = \sigma S(t) \quad (5.14) \]

where \( S(t) \) is an arbitrary function of \( t \).

With the help of equation (14), equation (12) becomes

\[ A = \frac{\sigma'}{\sigma} S \quad (5.15) \]

Now the metric (1) takes the form

\[ ds^2 = dt^2 - S^2(t) [dX^2 + e^{2X}(dy^2 + dz^2)] \quad (5.16) \]

where \( X = \ln \sigma \).

With the use of equations (14) and (15), equations (7), (10) and (11) yield

\[ \frac{2\dot{S}}{S} + \frac{\dot{S}^2}{S^2} - \frac{1}{S^2} = -8\pi\phi^{-1}p \quad (5.17) \]

\[ \frac{3}{S^3} - \frac{3\dot{S}^2}{S^2} = 8\pi\phi^{-1} \rho \quad (5.18) \]

\[ \ddot{\phi} + \frac{3\dot{\phi} \dot{S}}{S} - \frac{3\phi' \sigma}{\sigma' S^2} - \frac{\sigma^2}{\sigma' S^2} \frac{d}{dx} \left( \frac{\phi'}{\sigma} \right) = \frac{8\pi \lambda}{3} (\rho - 3\rho) \quad (5.19) \]
For the sake of simplicity, we assume $\phi$ is a function of $t$ only, then equation (19) with the use of equations (17) and (18) reduces to

$$\frac{\ddot{\phi}}{\phi} + \frac{3\dot{\phi} \dot{S}}{\phi S} - \frac{2\lambda \ddot{S}}{S} = 0$$  \hspace{1cm} (5.20)$$

The function $S(t)$ remains undetermined. To obtain its explicit dependence on $t$, one may have to introduce additional assumptions. In the following we assume the deceleration parameter to be constant to achieve this objective i.e.

$$q = -\frac{\ddot{S}}{S^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b(\text{constant}),$$  \hspace{1cm} (5.21)$$

where $H = \frac{\dot{S}}{S}$ is the Hubble parameter. The above equation may be rewritten as

$$\frac{\ddot{S}}{S} + b \frac{\dot{S}^2}{S^2} = 0$$  \hspace{1cm} (5.22)$$

On integration equation (22) gives the exact solution

$$S(t) = \begin{cases} (d + ct)^{(1+b)} & \text{when } b \neq -1 \\ m_1 e^{m_2 t} & \text{when } b = -1 \end{cases}$$  \hspace{1cm} (5.23)$$

where $c, d, m_1$ and $m_2$ are constants of integration.

Using eq. (21) in eqs. (17), (18) and (20) lead to

$$8\pi \phi^{-1} p = \frac{1}{S^2} + (2b - 1)H^2$$  \hspace{1cm} (5.24)$$

$$8\pi \phi^{-1} \rho = 3\left(\frac{1}{S^2} - H^2\right)$$  \hspace{1cm} (5.25)$$

$$\frac{\ddot{\phi}}{\phi} + \frac{3\dot{\phi} H}{\phi} + 2\lambda b H^2 = 0$$  \hspace{1cm} (5.26)$$

5.3.1 Non-flat Models

Case (i): $b \neq -1$. For singular models since $S(0) = 0$, equation (23) leads to

$$S = m t^{\frac{1}{1+b}}$$  \hspace{1cm} (5.27)$$
Using (27) in equations (26), (24) and (25) yield

$$\phi = m^{-3} t^{-3(1+4k)} \left[ c_1 e^{\frac{1}{2} (1+\sqrt{1-4k})} + c_2 e^{\frac{1}{2} (1-\sqrt{1-4k})} \right]$$  \hspace{1cm} (5.28)

$$8\pi p = \phi \left[ \frac{1}{m^2 t^{3(1+4k)}} + \frac{(2b-1)}{(1+b)^2 t^2} \right]$$  \hspace{1cm} (5.29)

$$8\pi p = 3\phi \left[ \frac{1}{m^2 t^{3(1+4k)}} - \frac{1}{(1+b)^2 t^2} \right]$$  \hspace{1cm} (5.30)

where

$$k_1 = \frac{[2b(4\lambda + 3) - 3]}{4(1+b)^2}$$  \hspace{1cm} (5.31)

and $c_1$ and $c_2$ are arbitrary constants of integration. Therefore the geometry of the universe, in this case, is described by the line-element

$$ds^2 = dt^2 - m^2 t^{3(1+4k)} [dX^2 + e^{2X} (dy^2 + dz^2)]$$  \hspace{1cm} (5.32)

where the Barber's scalar function $\phi$ is given by equation (28) and the corresponding physical parameters $p$ and $\rho$ are given by the equations (29) and (30) respectively.

The energy conditions given by Ellis [14]

(i) $(\rho + p) > 0$

(ii) $(\rho + 3p) > 0$

(iii) $\rho > 0$

and the dominant energy conditions given by Hawking and Ellis [15]

(i) $(\rho - p) \geq 0$

(ii) $(\rho + p) \geq 0$
are satisfied provided \( m^2 < 1 \) and \( b = 0 \) with positive \( c_1 \) and \( c_2 \).

From equation (30), we observe that the energy density \( \rho(t) \) is decreasing function of time. As \( t \) tends to infinity, energy density will vanish. From equation (28), it is observed that Barber's scalar function \( \phi \) decreases as time increases and will vanish when \( t \) tends to infinity.

From equations (29) and (30), it is observed that for \( m_2 = 0 \) the pressure and the energy density are always positive and we get \( p = \frac{1}{3} \rho \), which is the case of radiating model. In this case \( \phi = m_1^{-\frac{2}{3}}(c_3 + c_4) \) (constant).

### Physical behaviour of the models:

In case of a non-flat model when \( b \neq -1 \), the Ricci scalar becomes

\[
R = \frac{1}{m^2 t (1 + b)} - \frac{(1 - b)t}{(1 + b)}
\]

It is observed from equation (33) that when \( t \to 0 \); (i) \( R \to \infty \) if \( b = 0 \), (ii) \( R \to \infty \) if \( b \geq 1 \) and (iii) \( R \to \infty \) if \( b \leq -2 \). The equation (33) also suggests that when \( t \to \infty \); (i) \( R \to 0 \) if \( b \geq 0 \) and (ii) \( R \to \infty \) if \( b \leq -2 \).

The scalars of the expansion and shear are given by

\[
\theta = \frac{3}{(1 + b)t}, \quad \sigma = 0
\]

The model has singularity at \( t = 0 \). At \( t \to \infty \), the expansion ceases. Here \( \frac{\theta}{\sigma} = 0 \), which confirms the isotropic nature of the space-time which we have already obtained in equation (32).

Case (ii): \( b = -1 \). In this case using (23) in equations (26), (24) and (25) give

\[
\phi = m_1^{-\frac{3}{2}}e^{-\frac{3m_2^2}{2}}[c_3e^{\sqrt{k_2}t} + c_4e^{-\sqrt{k_2}t}]
\]

\[
8\pi p = \phi \left[ \frac{1}{m_1^2e^{2m_2t} - 3m_2^2} \right]
\]
where

\[ 8\pi \rho = 3\phi \left[ \frac{1}{m_1^2 e^{2m_2 t}} - m_2^3 \right] \quad (5.37) \]

and \( c_3 \) and \( c_4 \) are arbitrary constants of integration.

Therefore the geometry of the universe, in this case, is described by the line-element

\[ ds^2 = dt^2 - m_2^2 e^{2m_2 t} [dX^2 + e^{2X} (dy^2 + dz^2)] \quad (5.39) \]

where the Barber's scalar function \( \phi \) is given by the equation (35) and the corresponding physical parameters \( p \) and \( \rho \) are given by equations (36) and (37) respectively. For \( m_2 = 0 \), the model reduces to a static radiating model with constant density and pressure.

**Physical behaviour of the model:**

The Ricci scalar \( R \) is

\[ R = 2m_2^2 - \frac{1}{m_1^2 e^{2m_2 t}} \quad (5.40) \]

It is easily observed from equation (40) that (i) when \( t \to 0 \), \( R \to (2m_2^2 - \frac{1}{m_1^2}) \), and (ii) when \( t \to \infty \), \( R \to 2m_2^2 \). The expansion and shear scalars are

\[ \theta = 3m_2, \sigma = 0 \quad (5.41) \]

The model represents an uniform expansion as can be seen from equation (41). The flow of the fluid is geodetic as the acceleration vector \( f_i = (0, 0, 0, 0) \).
5.3.2 Flat Models

The condition for the flat model is obtained as

\[ \frac{1}{s^2} = (1 - b)H^2 \]  

(5.42)

Using equation (42), equation (24) and (25) reduce to

\[ 8\pi p = \phi b H^2 \]  

(5.43)

\[ \rho = -3\phi b H^2, \text{ where } b < 0 \]  

(5.44)

In this case the model has \( p < 0 \) which might describe the very early epoch of galaxy formation from the process of matter condensation.

Case (i): \( b \neq -1 \). Using equation (27) in equations (43) and (44) yield

\[ 8\pi p = \frac{\phi b}{(1 + b)^2 s^2} \]  

(5.45)

\[ \rho = -\frac{3\phi b}{(1 + b)^2 s^2}, \text{ with } b < 0 \]  

(5.46)

where \( \phi \) is already given by equation (28). Here the model describes the early phase of evolution as mentioned earlier.

It can be seen from equations (45) and (46) or otherwise also that the expressions for \( p \) and \( \rho \) will not be valid for the empty universe (i.e. \( p = \rho = 0 \)) and the stiff matter (i.e. \( p = \rho \)) models.

Case (ii): \( b = -1 \). Using equation (23), equations (43) and (44) reduce to

\[ 8\pi p = -\phi m_2^2 \]  

(5.47)

\[ \rho = 3\phi m_2^2 \]  

(5.48)
where $\phi$ is already given by equation (35). Here $\rho > 0$ for all times as $\phi > 0$ and the model describes the early phase of evolution as mentioned earlier.

5.4 Some particular cases

In this section, we intend to solve the field equations explicitly considering different equations of the state. As it is difficult to solve the field equations (7) - (11) for complete non-static case where both the metric potentials depend on $x$ and $t$ co-ordinate, we consider the case where $B = B(t)$ and $A = A(x)$. In this case the field equations (7) - (11) reduce to

$$2 \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -8\pi \phi^{-1} \rho$$  \hspace{1cm} (5.49)

$$\frac{\ddot{B}}{B} = -8\pi \phi^{-1} \rho$$  \hspace{1cm} (5.50)

$$\frac{\dot{B}^2}{B^2} = -8\pi \phi^{-1} \rho$$  \hspace{1cm} (5.51)

$$\dot{\phi} + 2 \frac{\dot{B}}{B} \phi + \frac{A'}{A^2} \phi' - \frac{\phi''}{A^2} = (\rho - 3p)(\frac{8\pi \lambda}{3})$$  \hspace{1cm} (5.52)

This is a system of four equations with five unknowns. In order to make the system a complete determinable system, we choose an equation of state for solving the equations.

5.4.1 Vacuum Universe ($p = \rho = 0$)

For this case, equations (49) - (52) reduce to

$$2 \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = 0$$  \hspace{1cm} (5.53)

$$\frac{\ddot{B}}{B} = 0$$  \hspace{1cm} (5.54)
\[
\frac{B^2}{B^2} = 0
\]  
(5.55)

\[
\dot{\phi} + 2\frac{B}{B} \dot{\phi} + \frac{A'}{A^3} \phi' - \frac{\phi''}{A^2} = 0
\]  
(5.56)

From equation (55), we get

\[B = \text{constant}\]

Then equation (56) reduces to

\[
\dot{\phi} + \frac{A'}{A^3} \phi' - \frac{\phi''}{A^2} = 0
\]  
(5.57)

If \(\phi\) is function of \(t\) only, then

\[\phi = l_1 t + l_2\]  
(5.58)

where \(l_1\) and \(l_2\) are constants of integration.

Also when \(\phi\) is a function of \(x\) only, we obtain

\[\phi = l_3 \int A(x)dx + l_4\]  
(5.59)

where \(l_3\) and \(l_4\) are constants of integration and \(A\) is an arbitrary function of \(x\).

If \(\phi\) is a separable function of \(x\) and \(t\) in the form \(f(x) + g(t)\) with zero separable constant then we get the Barber's scaler form

\[\phi = l_1 t + l_3 \int A(x)dx + l_5\]  
(5.60)

where \(l_5\) is a constant of integration.

If \(\phi\) is a separable function of \(x\) and \(t\) in the form \(f(x).g(t)\) with zero separable constant, equation (57) yields

\[\phi = (l_1 t + l_2)(\int A(x)dx + l_3)\]  
(5.61)

Thus the vacuum cosmological model in Barber's second self creation theory can be completely determined for any arbitrary metric potential \(A = A(x)\).
5.4.2 Zel'dovich universe \( (\rho = \rho) \)

For this case, equating equations (49) and (50), we obtain

\[
\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} = 0 \tag{5.62}
\]

which, after integration, yields

\[
B = \sqrt{l_6 t + l_7} \tag{5.63}
\]

where \(l_6\) and \(l_7\) are constants of integration.

With the help of (63), both the equations (49) and (50) lead to

\[
\rho(=\rho) = \left[ \frac{l_6^2}{32\pi(l_6 t + l_7)^2} \right] \phi \tag{5.64}
\]

Using equation (64) in (52), we obtain

\[
\ddot{\phi} + 2\frac{\dot{B}}{B} \dot{\phi} + \frac{A'}{A^3} \phi' - \phi'' + \left[ \frac{\lambda l_6^2}{6(l_6 t + l_7)^2} \right] \phi = 0 \tag{5.65}
\]

For the sake of simplicity, we assume \(\phi\) is a function of \(t\) only, then equation (65) reduces to

\[
\ddot{\phi} + \left[ \frac{l_6}{(l_6 t + l_7)} \right] \phi + \left[ \frac{\lambda l_6^2}{6(l_6 t + l_7)^2} \right] \phi = 0 \tag{5.66}
\]

With a suitable transformation \(T \rightarrow t\), equations (64) and (65) reduce to

\[
p = \rho = \frac{\phi}{32\pi T^2} \tag{5.67}
\]

\[
T^2 \ddot{\phi} + T \dot{\phi} + \left( \frac{\lambda}{6} \right) \phi = 0 \tag{5.68}
\]

In view of this rescaled time we find the general solution of (68) in the form

\[
\phi = \phi_1 + \phi_2 \tag{5.69}
\]

where
\[ \phi_1 = aT\sqrt{\left(\frac{a}{\beta}\right)} \]  
\[ \phi_2 = bT\sqrt{\left(\frac{a}{\beta}\right)} \]  
\[ (5.70) \]
\[ (5.71) \]

where \( a \) and \( b \) are two arbitrary constants and for a physically realistic solution \( \lambda < 0 \). Now equation (64) gives the physical parameter pressure cum density as

\[ p = \rho = \left(\frac{a}{4}\right) \left[ T\left\{\sqrt{\left(\frac{a}{\beta}\right)^2}\right\} - 2\right] \]  
\[ (5.72) \]
or

\[ p = \rho = \left(\frac{b}{4}\right) \left[ T\left\{-\sqrt{\left(\frac{a}{\beta}\right)^2}\right\} - 2\right] \]  
\[ (5.73) \]

In this case for both the models of cosmological model for Zel’dovich universe in Barber’s self-creationn theory can be given by

\[ ds^2 = dT^2 - dx^2 - T(dy^2 + dz^2) \]  
\[ (5.74) \]

where the Barber’s scaler is given by equations (70) and (71) and the corresponding physical parameters \( p \) and \( \rho \) are given by equations (72) and (73) respectively.

### 5.4.3 Radiation universe ( \( p = \frac{1}{3} \rho \) )

In this case, equations (49) - (52) reduce to

\[ 2\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} = -\frac{8\pi \phi^{-1} \rho}{3} \]  
\[ (5.75) \]

\[ \frac{\dot{B}}{B} = -\frac{8\pi \phi^{-1} \rho}{3} \]  
\[ (5.76) \]

\[ \frac{\dot{B}^2}{B^2} = -8\pi \phi^{-1} \rho \]  
\[ (5.77) \]
\[ \ddot{\phi} + 2\frac{\dot{B}}{B} \dot{\phi} + \frac{A'}{A^2} \phi' - \frac{\phi''}{A^2} = 0 \]  

(5.78)

From equations (75) and (76), we have

\[ \frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} = 0 \]  

(5.79)

which, on integration, yields

\[ B = \sqrt{a_1 t + a_2} \]  

(5.80)

where \( a_1 \) and \( a_2 \) are integrating constants.

Now the pressure cum density can be obtained as

\[ p(= \frac{1}{3} \rho) = \left[ \frac{a_1^2}{32\pi(a_1 t + a_2)^2} \right] \phi \]  

(5.81)

If \( \phi \) is function of \( t \) only, then

\[ \phi = a_3 \ln(a_1 t + a_2) \]  

(5.82)

where \( a_3 \) is constant of integration.

If \( \phi \) is function of \( x \) only, we get

\[ \phi = a_4 \int A(x)dx + a_5 \]  

(5.83)

where \( a_4 \) and \( a_5 \) are integrating constants.

If \( \phi \) is a separable function of \( x \) and \( t \) in the form \( f(x) + g(t) \) with zero separable constants, we obtain the Barber’s scalar as

\[ \phi = a_3 \ln(a_1 t + a_2) + a_4 \int A(x)dx + a_5 \]  

(5.84)

When \( \phi \) is a separable function of \( x \) and \( t \) in the form \( f(x),g(t) \) with zero separable constant, we obtain
\[
\phi = \left\{ a_5(a_1 t + a_2)^{1/2} + a_6 \right\} \left\{ a_7 \int A(x) dx + a_8 \right\} 
\]

(5.85)

where \( a's \) are constants of integration.

With a suitable \( t \)-transformation the radiation model in Barber's second self creation theory can be given by metric

\[
ds^2 = dT^2 - dx^2 - T(dy^2 + dz^2)
\]

(5.86)

**Physical behaviour of the models:**

It is observed that even though the metric for Zel'dovich and radiation models are same but the Barber's scalar functions are different. The scalar expansion \( \theta \) for both the models are

\[
\theta = \frac{1}{T}
\]

from which it is evident that \( \theta \to 0 \) as \( T \to \infty \) i.e. the universe is expanding with increase of time and the rate of expansion is slow with increase of time.

The shear scalar \( \sigma^2 \) for the model is

\[
\sigma^2 = \frac{1}{6T^2}
\]

Since \( \sigma^2 \to \infty \) as \( T \to 0 \) and \( \sigma^2 \to 0 \) as \( T \to \infty \), the shape of the model change uniformly in \( x \) and \( y \)-direction only and the rate of change of the universe becomes slow with increase of time.

It has also been observed that

\[
\lim_{T \to \infty} \left( \frac{\sigma}{\theta} \right) \approx \frac{1}{\sqrt{6}}
\]

which indicates that the universe remains anisotropic throughout the evolution. The present upper limit for \( \frac{\sigma}{\theta} \) is \( 10^{-3} \), as obtained from indirect arguments con-
cerning the isotropy of primordial black body radiation by Collins et al [16]. Thus our model can apply to all stages of the evolution of the universe.

Also the rotation $\omega$ and the acceleration vector $f_i$ come out to be zero. Thus the flow of the fluid is geodetic in both the cases.

It is obvious from the Zel’dovich and radiation models that all the physical and kinematical parameters, the evolution of the universe starts with singularity at $T = 0$. This may correspond to a Bigbang singularity.

5.5 Conclusion

In this paper we have obtained exact solutions of Barber’s self creation theory for constant deceleration parameter. The nature of the Barber’s scalar function $\phi$ and the energy density $\rho$ have been examined for both the (i) power law and (ii) exponential expansion of both the non-flat and flat universe. In particular, a simple method has been developed for constructing vacuum model, Zel’dovich model and radiation model.
Bibliography

