Chapter 2

The continuously development of superconducting wires / tapes shows promising results. From the viewpoint of practical applications, the most important characteristic of any superconductor is the maximum electrical transport current density that the superconductor is able to maintain without resistance. However, in practice the superconductor is subjected to heat input or disturbances. In cases when the heat input and resulting Joule heating irreversibly increases compared to cooling, the superconductor gets quenched. In this chapter, the mathematical formulation for the heat conduction, heat flow and temperature behavior of quenching HTS tape / wire for different experimental boundary conditions have also been discussed.

2.1 Heat conduction characteristics and thermal stabilization in YBCO tape

Yttrium Barium Copper Oxide (YBCO) based 2G/3G technical superconductors have emerged as one of the strongest candidates for several practical applications especially in the area of power utilities [2.1-2.5]. In most of these practical applications, the tape is either bath cooled or forced flow cooled [2.6-2.7] in nitrogen environment in the normal operational scenarios. In some of the cases, the superconductor winding pack is also conduction cooled with cryocoolers [2.8] or in case of cryogen free applications. In case off-normal scenarios, these superconductors do exceed in an irreversible fashion the critical temperature where usually the coolant is quickly expelled out and the superconductor becomes dry.

Such a case is usually referred as 'quench' of the superconductor. In such scenarios, the worst-case prediction is best described by carrying out a thermal conduction of the initial quench zone (IQZ) over the superconductor. Following a quench, the temperature of the winding pack at the region where the quench has initiated rises rapidly as a result of joule heating. This rise of temperature is uncontrolled which can induce unacceptable thermal stresses within the winding packs irrespective of whether the winding pack is actively or passively protected. Thus, it is imperative to learn the
maximum temperature as well as the resulting temperature distribution in the superconductor following a quench. This maximum temperature is also of engineering importance and is known as 'hot spot' temperature. The rate of rise of the temperature in addition to the hot spot temperature is very important since superconductor gets acted by proportional thermal stresses. This stress, in specific cases can also degrade the subsequent current carrying capability of the superconductor.

Under circumstances when cryostability is irreversibly violated following energy input exceeding the minimum quench energy, heating occurs. This heat pulse predominantly propagates along the length of superconductor. This heat propagation results in an evolving temperature profiles at different positions along the length and width of the tape, figure 2.1, as a function of time. The temperature $T(x, t)$ at position $x$ and time $t$ can be represented by 1-D thermal diffusion equation governed by

$$
\rho C_p(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( K(T) \frac{\partial T}{\partial x} \right)
$$

for $0 \leq x \leq L$ and $t \geq 0$ \hspace{1cm} (2.1)

where $\rho$ (kg/m$^3$) is the mass density of the tape, $C_p(T)$ (J/kg.K) is the specific heat, $K(T)$ (W/m.K) is the thermal conductivity and $L$ (m) is the length of tape under consideration.

![Figure 2.1: Schematic diagram of superconducting thin tape of finite length.](image)

For analysis, it is assumed that the temperature does not vary along thickness of the tape. This is an isotropic assumption justified in almost all practical cases since the
tape width is negligibly small compared to its length. The general solution of the equation (2.1) is obtained by applying the method of separation of variables as

\[ T(x, t) = e^{-\lambda t/\alpha^2} \left[ D\cos\left(\frac{x}{\alpha}\right) + E\sin\left(\frac{x}{\alpha}\right) \right] \]  

(2.2)

where \( \lambda = \frac{K(T)}{\rho C_p(T)} \) (m²/s) is the thermal diffusivity of the tape and \( \alpha \) is a negative separation constant.

Applying the thermally insulated boundary conditions \( T(0, t) = 0 \) and \( T(L, t) = 0 \), the equation (2.2) reduces to

\[ T_n(x, t) = E_n \sin\left(\frac{n\pi x}{L}\right) e^{-\lambda \left(\frac{n\pi}{L}\right)^2 t} \]  

(2.3)

where \( n \) is a +ve integer. Thus, the general equation for the temperature profile can be written as

\[ T(x, t) = \sum_{n=1}^{\infty} E_n \sin\left(\frac{n\pi x}{L}\right) e^{-\lambda \left(\frac{n\pi}{L}\right)^2 t} \]  

(2.4)

For \( t = 0 \), Eq (2.4) reduces to

\[ T(x, 0) = \sum_{n=1}^{\infty} E_n \sin\left(\frac{n\pi x}{L}\right) \]  

(2.5)

Multiplying both sides of the equation (2.5) with \( \sin(m\pi x/L) \) and integrating over the length of the tape i.e. 0 to \( L \), one can obtain

\[ E_n = \frac{2}{L} \int_0^L \sin\left(\frac{n\pi x}{L}\right) T(x, 0) \, dx \]  

(2.6)

Considering the temperature to be uniform all along the length before cryo-instability i.e. at its beginning when \( t = 0 \), we have \( T(x, 0) = T_0 \), that is a constant throughout the tape. Thus,
\[ E_n = \frac{2T_o}{n\pi} \left[ 1 - \cos \left( n\pi \right) \right] \] (2.7)

The temperature at the different portion of a superconducting tape after the violation of cryostability without further cool down (adiabatic heating) can thus be represented by

\[ T(x, t) = T_0 \sum_{n=1}^{\infty} \frac{2}{n\pi} \left[ 1 - \cos \left( n\pi \right) \right] \sin \left( \frac{n\pi x}{L} \right) e^{-\left(\frac{n\pi}{L}\right)^2 t} \] (2.8)

This temperature profile \( T(x, t) \) represents the temperature variation along the entire lengths of the tape for a given time. Also it represents the temperature variation of a given co-ordinate with respect to time variation. As a case study, YBCO tape [2.9] of 10 cm length at an initial temperature of 80 K throughout its length is considered. Since the specific heat and thermal conductivity of this tape are temperature dependent, \( T(x, t) \) is plotted taking average of these variables using a MATLAB program with respect to time as a function of length ‘x’ as shown in the figure – 2.

Figure 2.2: Temperature with respect to different times at different positions of the HTS tape.

It demonstrates that the middle of the tape remains stable while the temperature increases in either direction as a standing wave. The temperature rise begins at both the
ends and rises very fast with time while the stability persists for longer duration at the center as shown in figure 2.3. It is also observed that as time span increases, the temperature also rise in a corresponding manner.

Figure 2.3: Temperature with respect to different coordinates at different times of the HTS tape.

The temperature at $x = 0.1$ & $9.9$ rises from 80.0 K to 85.5 K after 50 millisecond while the temperature at $x = 5$ rises from 80.0 K to 85.0 K after 97 sec. This shows that the temperature remains most stable at the center and very unstable at the edges as expected trying to equilibrate for a given heat input. This temperature instability moves towards the center as time increases which is also represented through a 3D view as shown in the figure 2.4.

The above numerical representation of the thermal stability of YBCO tape shows that the tape edges are necessarily to be adequately cooled so that hot spot developed locally by any non-uniformity of transport current can be prevented and hence quench can be avoided.
2.2 Theoretical prediction of heat flow in a conduction-cooled YBCO wire

Recently, high temperature superconductor (HTS) YBCO based strands and tapes have become attractive candidates for several practical applications such as in the areas of utilities, medical applications and in practical laboratory applications [2.10-2.14]. Usage of such HTS conductors are advantageous as a result of their reduced requirements of refrigeration especially, refrigeration at liquid nitrogen temperature of 80 K compared to the cost intensive refrigeration by liquid helium at 5 K. Refrigeration at 80 K with liquid nitrogen is significantly cheaper than helium cooling at 5 K technical YBCO based HTS materials are primarily available in the form of wires and tapes. These materials in various applications can be cooled in a LN$_2$ bath or can be conduction cooled with cold fingers in cryocoolers. In such scenarios, in off normal situations, the HTS material loses its superconductivity owing to temporal and spatial disturbances in the form of heat inputs either from external sources such as time varying magnetic fields or from internal sources such as flux jumping etc. It is these situations which are theoretically investigated.
considering a finite length cylindrical YBCO circular solid strand of finite length 'L' and radius 'b' as shown in the figure 2.5.

![Figure 2.5: Schematic diagram of YBCO wire of finite length.](image)

The entire surface along with the core of the wire is initially kept at 80 K in LN$_2$ temperature. For time $t > 0$, following an instantaneous thermal disturbance, the cryostability has been irreversibly violated. Consequently, the strand surface assumes a constant temperature surface to begin with and the volumetric heating occurs following the normal zone propagation. Normal zones are the resistive zones inside the superconductor, where the thermal inputs exceed the stability margins. Thus, a heating occurs. Due to this, the heat pulse predominantly propagates along ($r, z$) direction of the superconductor strand / wire. The azimuthal symmetricity has been assumed in this problem. Thus, considering the radial and axial heat flow in a finite circular cylinder bounded by the surfaces $z = 0, z = L$ and $r = b$ and initially being held at a temperature at $T_0$, as follows,

$$\frac{\partial^2 T}{\partial t^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\lambda} \frac{\partial T}{\partial t} \quad \text{for } 0 < r < b, 0 < z < L \text{ and } t > 0$$

(2.9)
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where \( \lambda = \frac{K(T)}{\rho C_p(T)} \) is the thermal diffusivity of the wire, \( \rho \) is the mass density of the wire, \( \rho C_p(T) \) is the specific heat and \( K(T) \) is the thermal conductivity. The boundary conditions which this strand follow are \( T(r, z, 0) = T_0, T(r, 0, t) = 0, T(b, z, t) = 0 \) and \( T(r, L, t) = 0 \). Equation (2.9) is solved by applying the method of separation of variables as

\[
T(r, z, t) = \psi(r) R(z) \Gamma(t)
\]

so that

\[
\frac{1}{R} \left( \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) + \frac{1}{Z} \frac{d^2 Z}{dz^2} = - \frac{1}{\lambda T} \frac{dT}{dt}
\]

(2.11)

To make some sense to the solutions of the above equation (2.11), both sides of this equation is equated with a negative separation constant \((- \alpha^2)\). Further equating \( Z \)-dependent part of this equation to a -ve constant \((- \gamma^2)\), the solutions can be written as

\[
\Gamma(t) = A e^{-\alpha^2 t}
\]

(2.12)

\[
Z(z) = B \sin(\gamma z) + C \cos(\gamma z)
\]

(2.13)

\[
R(r) = D J_0(\beta r) + E Y_0(\beta r)
\]

(2.14)

where \( \beta^2 = \alpha^2 - \gamma^2 \). Since the region in \( r \)- and \( z \)-directions is finite and the temperature is bounded at \( r = 0 \), the physical meaningful solutions for equation (2.13) and equation (2.14) are possible only when the boundary conditions are satisfied which lead to \( C = 0 \) and \( E = 0 \). Thus we could get

\[
\gamma = \frac{m \pi}{L} \quad \text{for} \quad m = 1, 2, 3, \ldots
\]

(2.15)

and

\[
R(r) = D J_0(\beta r)
\]

(2.16)

where \( J_0(\beta r) \) is the Bessel function of first kind with the zero order. Applying the boundary condition to the equation (2.16) results that \( J_0(\beta b) = 0 \) which yield the eigenvalues as \( \beta_1 b = 2.404826, \beta_2 b = 5.520078, \beta_3 b = 8.653728, \beta_4 b = 11.791534 \) and so on [2.15-2.17]. Thus, a particular solution of the eq (2.1) can be written as
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\[ T(r,z,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F_{m,n} \sin\left(\frac{m\pi}{L} z\right) J_{n}(\beta_{n} r) e^{-\left(\beta_{n}^{2} + r^{2}\right)\lambda t} \]  \hspace{1cm} (2.17)

Further considering the initial condition i.e. \( T_{0} \) to be constant, from equation (2.17), we have

\[ T_{0} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F_{m,n} \sin\left(\frac{m\pi}{L} z\right) J_{n}(\beta_{n} r) \]  \hspace{1cm} (2.18)

Multiplying equation (2.18) with the operators \( \int_{0}^{L} \sin\left(\frac{n\pi z}{L}\right) dz \) and \( \int_{0}^{b} J_{0}(\beta_{n} r) r \, dr \), the temperature at the different portion of YBCO wire after the violation of cryostability without further cool down (adiabatic heating) can be represented as

\[ T(r,z,t) = T_{0} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} 4 \frac{1 - \cos (m\pi)}{b\beta_{n} J_{1}(\beta_{n} b)} \sin\left(\frac{m\pi}{L} z\right) e^{-\left(\beta_{n}^{2} + \frac{r^{2}}{L^{2}}\right)\lambda t} \]  \hspace{1cm} (2.19)

This temperature profile \( T(r, z, t) \) represents the temperature variation along the entire lengths as well as along the radial direction of the wire with respect to a given time. A YBCO wire of a finite length of 10.0 cm and radius of 1.0 mm at an initial temperature of 80 K throughout its surface is considered for analysis. Since the specific heat and thermal conductivity of this wire are temperature dependent, the averages of these variables with respect to time are taken for the analyses. Since \( T(r, z, t) \) is an infinite series of \( 'm' \) and \( 'n' \), the convergence of this series has been done using MATLAB program. The temperature is plotted with respect to time for different axial lengths \( 'z' \) and radial distances \( 'r' \) as shown in the figure 2.6.
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Figure 2.6: Temperature with respect to time at different positions and radius of HTS wire.

These results demonstrate that the edges of the wire quickly attain constant temperature environment while the temperature increases in either direction slowly. Also it can be observed that the temperature at the center remains more stable and increases in the radial outward direction. The temperature at the center of the wire for different position coordinates with respect to time is represented through 3D view as shown the figure 2.7.

Figure 2.7: 3D representation of YBCO wire temperature distribution with respect to position coordinates and times at \( r = 0 \).
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The temperature at the center for the positions $z = 0.1$ & 9.9 rises from 80.0 K to 85.0 K after 2.5 millisecond while the temperature at these two positions rises very fast in radially outward directions. Similarly at the center for the positions $z = 0.2$ to 9.8, the temperature rises from 80.0 K to 85.0 K after 3.0 millisecond and the behavior remains identical to that for radial outward direction. The temperature rise up to the edge temperature environment for the entire wire surface is as shown in figure 2.8.

![Figure 2.8: Temperature with respect to time for complete cycle of HTS wire.](image)

The above figures also reveal the following results as:

- The lowest temperature is centered along the $z$-axis, while the surface temperature at $r = b$ is zero which meets the boundary condition at the surface.
- The isothermal temperature planes are increasing as a function of time according to exponential part of equation (2.19).
- During irreversible cryostability, the temperature varies according to the roots of the zeroth order Bessel's function.

These results show that the temperature rise of YBCO wire exhibited rapid increase at the edges and at the outer surface and propagates towards the core. In order to maintain the superconductivity behavior in the strand, more efficient cooling at the edges
is mandatory without any interruptions. Carefully designed experiments on conduction cooled YBCO strands can verify these predictions.

2.3 Temperature instability in high-$T_c$ superconducting wire exposed to thermal disturbance

Bismuth strontium calcium copper oxide (BSCCO) and Yttrium barium copper oxide (YBCO) based smart high-$T_c$ superconductors are currently being produced commercially as long piece lengths [2.18-2.22]. Thus high temperature superconductors have emerged alternative candidates for several applications such as in utility applications and in current leads in large high field magnet systems [2.23-2.35].

Since these conductors are either forced flow cooled or cooled in a bath of liquid nitrogen (LN$_2$), the operator of such high temperature superconductor based devices are cheaper and attractive. However, in reality there are several off-normal operation scenarios also, where the conductor in the device / magnet / current leads etc. are to be protected. In the event of a resistive (normal) operation regime, local hot spots may arise depending on the temperature and spatial characteristics of the normal zone propagation. In all cases the conductor gets heated up and a temperature distribution occurs across the conductor cross-section. From design and successful operation point of view, an estimation of the temperature profile across the conductor in response to an irreversible transition to normal zone is extremely useful. This prior information not only leads to a sound design but also quantitatively predicts the operation margins.

Superconductors are available in both cylindrical strands form as well as in the tape geometry [2.36-2.41]. An event where the operating cooling scenario is abruptly changed (such as loss of coolant flow) has been considered as the reason leading to rapid transition to the normal state of the conductor. For the mathematical formulation, a long cylindrical wire of radius $b$ at a constant temperature $T_i$ exposed to a surrounding environment at a temperature $T_o$ is considered such that its surface exchange heat uniformly by convection as shown in the figure 2.9.
Figure 2.9: Schematic diagram of a long cylindrical wire.

The function $T(r, t)$ represents the temperature distribution for times $t > 0$. The diffusion equation for such a system can be represented as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\lambda} \frac{\partial T}{\partial t}$$
for $0 < r < b$ and $t > 0$ \hspace{1cm} (2.20)

with boundary conditions

\[ \left. \right|_{r=b} \frac{\partial T}{\partial r} + HT = 0 \] at \hspace{1cm} (r = b, t > 0 and \hspace{1cm} H = h/k \hspace{1cm} (2.21)

\[ \left. T = T_i \right|_{t=0} \] for \hspace{1cm} $t = 0$ in $0 < r < b$ \hspace{1cm} (2.22)

where $\lambda = \frac{K(T)}{\rho C_p(T)}$ is the thermal diffusivity of the wire (m²/s), \( h \) is the heat transfer coefficient (W m⁻² K⁻¹), \( \rho \) is the mass density of the wire (kg/m³), \( C_p(T) \) is the specific heat (J kg⁻¹ K⁻¹) and \( K(T) \) is the thermal conductivity (W m⁻¹ K⁻¹). In order to convert the convectional boundary condition to a homogeneous quantity, a new variable \( \theta \) has been defined as

$$\theta = T - T_o$$ \hspace{1cm} (2.23)

Now the equation (2.20) can be written as

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} = \frac{1}{\lambda} \frac{\partial \theta}{\partial t}$$ \hspace{1cm} (2.24)

Equation (2.24) is solved by applying the method of separation of variables as

$$\theta (r, t) = R(r) \Gamma(t)$$ \hspace{1cm} (2.25)

so that

$$\frac{1}{R} \left( \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) = \frac{1}{\lambda \Gamma} \frac{d\Gamma}{dt}$$ \hspace{1cm} (2.26)
Equating the both sides of this equation (2.26) with a negative separation constant \((-\alpha^2)\) will make sense to the solutions. Thus the general solutions of equation (2.26) will be

\[
\Gamma(t) = Ae^{-\alpha^2 \mu t}
\]

and

\[
R(r) = BJ_0(\alpha r) + CY_0(\alpha r)
\]

The physically meaningful solutions for the equation (2.28) is possible only when \(C = 0\). Thus the equation (2.28) reduces to

\[
R(r) = BJ_0(\alpha r)
\]

where \(J_0(\alpha)\) is the Bessel function of first kind of zero order. By applying the boundary condition equation (2.21) to equation (2.29), we have

\[
H J_0(\alpha b) = \alpha J_1(\alpha b)
\]

where \(J_1(\alpha)\) is the Bessel function of first kind of first order. The eigenvalues \(\alpha\) can be found out from the equation (2.30). Thus, the general solution can be written as

\[
\theta(r, t) = \sum_{n=1}^{\infty} C_n J_0(\alpha_n r) e^{-\alpha_n^2 \mu t}
\]

Again applying the boundary condition equation (2.22) to equation (2.31) and multiplying with the operator \(\int_0^b J_0(\alpha_n r) r dr\) from both sides, we have

\[
\theta(r, t) = \sum_{n=1}^{\infty} \frac{2}{b(\alpha_n^2 + H^2) J_0(\alpha_n b)} H \theta J_0(\alpha_n r) e^{-\alpha_n^2 \mu t}
\]

The general equation governing the temperature profile for a long cylindrical wire can thus be represented as

\[
T(r, t) = T_0 + \frac{2H}{b(T_i - T_0)} \sum_{n=1}^{\infty} \frac{J_0(\alpha_n r)}{\alpha_n^2 + H^2} J_0(\alpha_n b) e^{-\frac{\lambda \alpha_n^2}{b} t}
\]

This temperature profile \(T(r, t)\) represents the temperature variation along the radial direction of the wire with respect to a given time.
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For analysis, YBCO based HTS round shaped cylindrical strand of 1.0 mm radius is considered at an initial temperature of 80 K throughout its length. Due to cooling failure HTS superconductor strand surrounding temperature raised to 110 K. Since the specific heat and thermal conductivity of this strand are temperature dependent, the averages of these variables with respect to time are taken for the analyses. For analysis, three cases of the heat transfer are considered such as (i) steady state heat transfer where 'h' can be considered to 500 W m$^{-2}$ K$^{-1}$, (ii) moderate heat transfer where 'h' can be considered to 1000 W m$^{-2}$ K$^{-1}$ and (iii) transient high heat transfer where 'h' can be considered to 5000 W m$^{-2}$ K$^{-1}$. For these heat transfer cases, the eigen values are calculated using equation (13) and are tabulated in the table - 1 for few values. Since $T(r, t)$ is an infinite series of 'n', the convergence of this series has been done using MATLAB program.

Table 2.1: Twenty five Roots of $H Jo(ab) = \alpha J1(ab)$

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Roots of Bessel functions (ab)</th>
<th>For h = 500 W m$^{-2}$ K$^{-1}$</th>
<th>For h = 1000 W m$^{-2}$ K$^{-1}$</th>
<th>For h = 5000 W m$^{-2}$ K$^{-1}$</th>
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<td>73.04179233</td>
<td>73.06137945</td>
</tr>
<tr>
<td>25</td>
<td>$\alpha_{25}b$</td>
<td>76.18104785</td>
<td>76.18339602</td>
<td>76.20217471</td>
</tr>
</tbody>
</table>
The temperature is plotted with respect to time for different radial distances ‘r’ for a given heat transfer co-efficient and is shown in the figure 2.10.

This figure demonstrates that the entire conductor reaches ~ 110 K after 10 second. The temperature along the radial distance at different time periods is plotted in the figure 2.11. It shows that there is little variation in the temperature profile along the radial direction for a given time.
Similarly, the temperature profiles along the conductor’s radial axis are calculated for the moderate and transient high heat transfer coefficients. The temperature of the conductor for these three heat transfers at the center of is plotted in the figure 2.12, which shows that the temperature of the conductor raises sharply irrespective of the heat transfer co-efficient. Also it can be seen that with higher heat transfer, the wire quickly attains the surrounding temperature. This indicates that the better heat transfer coefficient leads to the conductor reaching quickly to the environment temperature.

Figure 2.11: Temperature of YBCO conductor with respect to the radial distance for different times.
Figure 2.12: Temperature of YBCO wire at the center \((r = 0)\) with respect to time for the different heat transfer coefficients.

The transient behavior of YBCO conductor subjected to an environment at higher temperature is numerically formulated by solving one-dimensional heat conduction equation in a cylindrical geometry. The results show that the temperature rise of HTS superconductor could be subsided by adequate cooling. In order to maintain the superconductivity behavior, the proper measures should be added so that in case of cooling disruption, the conductor can be protected from quench.
2.4 Temperature profile evolution in quenching high-\(T_c\) Superconducting Composite Tape

High temperature superconductors (HTS) have become attractive candidates for several of the practical applications such as in fault current limiters, high voltage DC transmission lines, superconducting magnet storage devices etc. [2.42-2.50]. Also these high temperature superconductors have advantage of operational conditions of 80 K i.e. liquid nitrogen (LN\(_2\)) environment which is commercially beneficial. Further handling of LN\(_2\) is very easy compared to other cryogens. However, high temperature superconductors have inevitable off-normal operating scenarios. In such off-normal scenarios, the superconductivity is temporarily lost and the superconductor is call to be “quenched”. The quench of the superconductor generally happens because of exceeding one of its intrinsic parameters such as transport current, total perpendicular magnetic field, operating temperature or operating maximum strain level. In practice, such consequences may arise for example in case if the transport current is accidentally increased beyond the critical value or the background field exceeds the critical value or the coolant temperature exceeds the critical value as a result of temporary loss of flow or the electromagnetic strain resulting from the interaction of the transport current and the field exceeding the critical strain value.

All these can happen in both temporally or spatially either localized or distributed. As a result, in the following quench, the transport current gets shared or completely diverted to the stabilizer matrix. This leads to Joule’s heating of the superconductor. The resistive region grows and is known as ‘normal zone growth’. Associated with the normal zone growth, heating occurs till the transport current is not diverted through an external circuit and the load is bypassed from the feeding power supply. The uncontrolled heating can lead to serious thermal stresses within the winding pack of the high-\(T_c\) superconductor. Thus the temperature rise is the most significant observable effect in a quench event. In practical applications, the limitations of the thermal stress by quickly spreading the temperature uniformly all over the superconductor are always significant criteria. Thus, the information on the temperature profile evolution across the ends of the superconductor is always an important parameter. In fact in practical applications, since
the temperature on the superconductor is always measured and monitored so all important
temperature profile evolutions must be predicted and determined in high-\( T_c \) based
practical applications. These predictions can be used as important design tool towards
conceiving high-\( T_c \) based applications. With the prior knowledge on the temperature
characteristics of the high-\( T_c \) based applications, both the cryogenic stability as well as
the “protection” aspects of the applications can be appropriately enhanced.

For this purpose, a thin high temperature superconducting (HTS) tape of finite
length ‘\( L \)’ is considered for analysis, figure 2.13. Due to very slow local normal zone
propagation of the order of mm/sec in case of HTS [2.51], the heat generated along the
length of superconductor results in a temperature gradient at different positions with
respect to time.

![Figure 2.13: Schematic diagram of YBCO thin tape of finite length.](image)

The normal-zone propagation process in a superconductor is governed by a heat
balance one dimensional equation which can be written as

\[
\rho C_p(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( K(T) \frac{\partial T}{\partial x} \right) + q(T) - w(T) \tag{2.34}
\]

where ‘\( \rho \)’ is the mass density of the tape, ‘\( C_p(T) \)’ is the specific heat, ‘\( K(T) \)’ is the thermal
conductivity, ‘\( q(T) \)’ is the heat generation due to Joule’s heating and ‘\( w(T) \)’ is the heat
transfer. In our case, the boundary conditions \( T(0, t) = T_i, T(L, t) = T_i \) and \( T(x, 0) = T_i \)
will govern the above heat balance equation (2.34). This equation can be solved by
applying the method of separation of variables as

\[
T(x, t) = \Psi(x, t) + \phi(x) \tag{2.35}
\]
so that
\[
\frac{d^2 \Psi}{dx^2} + \frac{d^2 \phi}{dx^2} + \frac{q'}{\lambda} = \frac{d^2 \Psi}{dt}
\] (2.36)

where \( \lambda = \frac{K(T)}{\rho C_p(T)} \) is the thermal diffusivity of the tape and \( q = \frac{q(T) - u(T)}{\rho C_p(T)} \).

Equation (2.36) can be split into two parts such that
\[
\frac{d^2 \Psi}{dx^2} = \frac{1}{\lambda} \frac{d^2 \Psi}{dt}
\] (2.37)

and
\[
\frac{d^2 \phi}{dx^2} + \frac{q'}{\lambda} = 0
\] (2.38)

The equations (2.37) and (2.38) can be solved in a similar way as above by applying the method of separation of variables and equating with a reciprocal of negative separation constant \((-1/\alpha^2)\) in order to have a finite solutions for all the values of time, we have the general solutions as
\[
\Psi(x, t) = e^{-\frac{1}{\lambda} \frac{t}{\alpha^2}} \left[ D \cos \left( \frac{x}{\alpha} \right) + E \sin \left( \frac{x}{\alpha} \right) \right]
\] (2.39)

and
\[
\phi(x) = -\frac{q'}{\lambda} x^2 + F_1 x + F_2
\] (2.40)

where \( D, E, F_1 \) and \( F_2 \) are unknown constants. On substituting the initial condition to the equation (2.35), we obtain
\[
\Psi(0, t) + \phi(0) = T_i \text{ and } \Psi(L, t) + \phi(L) = T_i
\] (2.41)

Let \( \Psi(0, t) = 0 \text{ and } \Psi(L, t) = 0 \) (2.42)

so that \( \phi(0) = T_i \text{ and } \phi(L) = T_i \) (2.43)

and \( \Psi(x, 0) = T_i - \phi(x) \) (2.44)

Under the boundary conditions as stated above, the equation (2.39) will yield
\[
D = 0 \text{ and } \sin \left( \frac{L}{\alpha} \right) = 0
\]
\[\Rightarrow \alpha = L / n \pi, \quad n = 1, 2, 3, \ldots
\]
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Thus the equation (2.39) reduces to a general equation given by

$$\Psi(x, t) = \sum_{n=1}^{\infty} E_n \sin \left( \frac{n\pi x}{L} \right) e^{-\lambda \left( \frac{nx}{L} \right)^2}$$

(2.45)

where $n$ is a +ve integer.

Similarly, the other boundary conditions will give $F_2 = T_i$ and $F_1 = \frac{q'}{2\lambda}$ so that the equation (2.40) can be written as

$$\phi(x) = T_i + \frac{q'}{2\lambda} (L-x)x$$

(2.46)

Again from the equations (2.44) and (2.45), we obtain

$$T_i - \phi(x) = \sum_{n=1}^{\infty} E_n \sin \left( \frac{n\pi x}{L} \right)$$

(2.47)

Multiplying both sides of the equation (2.47) with $sin \left( \frac{m\pi x}{L} \right)$ and integrating from 0 to $L$ and then substituting $\phi(x)$ value, the recurrence constant can be obtained as

$$E_n = -\frac{2q' L^2}{\lambda} \left[ \frac{1-(-1)^n}{(n\pi)^3} \right]$$

(2.48)

Thus the temperature at the different portion of HTS tape for given heat load can be represented as

$$T(x,t) = T_i + \frac{q'}{2\lambda} (L-x)x + \frac{2q' L^2}{\lambda} \sum_{n=1}^{\infty} \left[ \frac{1-(-1)^n}{(n\pi)^3} \right] \sin \left( \frac{n\pi x}{L} \right) e^{-\lambda \left( \frac{nx}{L} \right)^2}$$

(2.49)

This temperature profile $T(x, t)$ represents the temperature variation along the entire lengths of the tape for a given time. Also it represents the temperature variation of a given co-ordinate with respect to time variation.

For analysis, YBCO tape [11] with an overall dimension $(100 \times 4 \times 0.2 \text{ mm}^3)$ with superconducting film thickness of $0.8 \mu m$ at an initial temperature of $80$ K throughout has been considered. Since the specific heat and thermal conductivity of this tape are temperature dependent, the averages of these variables with respect to time are considered
for the analyses. From the temperature profile expression, it is evident that over the temperature range of interest, the averaged value is a good approximation. The averaged values of thermal conductivity $<K(T)>$ and specific heat $<C_p(T)>$ of the YBCO tape has been taken as of $2.929135 \text{ W/m.K}$ and $191.83 \text{ J/kg.K}$ respectively. The composite density of the tape has been taken as $6300 \text{ kg/m}^3$. A heat generation of $10 \text{ W/cm}^2$ has been considered and $T(x, t)$ is plotted with respect to time as a function of axial length ‘$x$’ using MATLAB program. The results are shown in figure 2.14.

![Figure 2.14: Temperature of YBCO tape with respect to the position co-ordinates for different times.](image)

From the results above, it is evident that both edges of the tape get saturated rapidly for each of the time period. The temperature remains steady throughout the length of the tape. Also it is observed that as the time increases, the HTS temperature also increases. All these observations are in accordance with physical observations of edge cooled high temperature superconductors. The temperature plateau indicates the fact that the hot spot is centered and an efficient heat extraction of heat from the interior of the
tape is required as a necessary cryostability criterion. This also supports the fact that more
the wetted perimeter better is the heat transfer and more settable becomes the conductor
in application.

Next for a given time period, different heat generation is applied to the tape. The
rise in the temperature of the tape at a fixed range of locations was observed. The
optimum amount of energy required to just raise the temperature of the high temperature
superconductor above its critical temperature was then determined. This energy input is
often referred as the 'minimum quench energy' of the superconductor. In all practical
application, the disturbance energy must be less than this energy else irreversible normal
zone would appear and the tape would be quenched. The figure 2.15 shows that the
temperatures rise against given input energies. It is illustrated that for given fixed
boundary conditions and for a fixed heat transfer coefficient, the rise in temperature is
proportional to the heat input. The critical temperature 93 K is reached when the heat
generation is ~ 8.0 W/cm³ for the time period of 2.0 sec. This critical disturbance for this
particular tape is roughly 12.8 J/m while for its superconducting section it is 0.0512 J/m
of the tape.

![Figure 2.15: Temperature along the length of YBCO tape for different heat generation for
a given time period of t = 2.0 sec, excluding the both edges.](image)

Since in practical applications, spatial and temporal disturbances are inevitable,
the magnitude of the disturbance that leads to the quench of edge cooled high temperature

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superconductor has been investigated and computed to have threshold quench energy of 12.8 J/m for overall tape while for its superconducting section is 0.0512 J/m. Thus, in all applications, care must be taken to limit the disturbance energy to a value lesser than 12.8 J/m for stable operation of the superconductor. Following a quench, the superconductor is also required to be protected from overheating that ultimately leads to thermal stress. The maximum hot spot temperature for heat inputs in excess of the minimum quench energy has been analyzed from first principles. In such cases, it is observed that the center of the tape attains the hot spot temperature over a broad width. The maximum temperature increases with the heat inputs and the irreversible break-down of cooling against the joule heat generation occurs. For a cryostable operation of the high temperature superconductors, the wetted perimeter of the tape may be increased enabling enhanced extraction of heat.

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