APPENDICES

COMPARISON BETWEEN EXISTING AND THE PROPOSED DFL CONTROL STRATEGIES IN ANALYTICAL TERMS

APPENDIX – I

The Case of SMIB Power System:

In the existing DFL compensated model (equations 3.10 – 3.12)

$$\Delta\dot{P}_e(t) = -\frac{1}{T'_d}\Delta P_e(t) + \frac{1}{T'_d}v_f(t)$$

which can be written in the following way:

$$\Delta\dot{P}_e(t) = -\frac{1}{T'_{do}}\Delta P_e(t) + \left[\left(\frac{1}{T'_{do}} - \frac{1}{T'_d}\right)\right]\Delta P_e(t) + \frac{1}{T'_d}v_f(t)$$

$$= -\frac{1}{T'_{do}}\Delta P_e(t) + \frac{1}{T'_{do}}\left[\left(1 - \frac{T'_{do}}{T'_d}\right)\Delta P_e(t) + \frac{T'_{do}}{T'_d}v_f(t)\right]$$

$$= -\frac{1}{T'_{do}}\Delta P_e(t) + \frac{1}{T'_{do}}\left[\left(1 - \frac{x_{ds}}{x'_{ds}}\right)\Delta P_e(t) + \frac{x_{ds}}{x'_{ds}}v_f(t)\right] \quad \text{(A - I.1)}$$

Comparing equation (3.24) with equation (A-I.1) it can be concluded that

$$\hat{v}_f(t) = \left[\left(1 - \frac{x_{ds}}{x'_{ds}}\right)\Delta P_e(t) + \frac{x_{ds}}{x'_{ds}}v_f(t)\right] \quad \text{(A - I.2)}$$

The following derivation verifies it.

Putting the expression for $v_f(t)$ in the R.H.S of equation (A-I.2)

$$\left(1 - \frac{x_{ds}}{x'_{ds}}\right)\Delta P_e(t) + \frac{x_{ds}}{x'_{ds}}\Big\{k_e u_f(t) I_q(t) + T'_{do}(x_d - x'_d) I_q^2(t)\Delta\omega(t)$$

$$+ \frac{x'_{ds}}{x_{ds}}T'_{do}\left(Q_e(t) + \frac{V_s^2}{x_{ds}}\right)\Delta\omega(t) - P_m\Big\}$$

$$= -\frac{(x_d - x'_d)}{x'_{ds}}\Delta P_e(t) + k_e u_f(t) I_q(t) + \left(\frac{x_{ds}}{x'_{ds}} - 1\right) k_e u_f(t) I_q(t) + \frac{x_{ds}}{x'_{ds}} T'_{do}(x_d - x'_d) I_q^2(t)\Delta\omega(t)$$

$$+ T'_{do}\left(Q_e(t) + \frac{V_s^2}{x_{ds}}\right)\Delta\omega(t) - \frac{x_{ds}}{x'_{ds}} P_m$$

$$= k_e u_f(t) I_q(t) + \left(\frac{x_d - x'_d}{x'_{ds}}\right)\left(E_f(t) I_q(t) - \Delta P_e(t)\right) + \frac{x_{ds}}{x'_{ds}} \cdot T'_{do}(x_d - x'_d) I_q^2(t)\Delta\omega(t)$$

$$+ T'_{do}\left(Q_e(t) + \frac{V_s^2}{x_{ds}}\right)\Delta\omega(t) - \frac{x_{ds}}{x'_{ds}} P_m$$

$$= k_e u_f(t) I_q(t) + \left(\frac{x_d - x'_d}{x'_{ds}}\right)\left(E_f(t) I_q(t) - P_e(t)\right) + \frac{x_{ds}}{x'_{ds}} \cdot T'_{do}(x_d - x'_d) I_q^2(t)\Delta\omega(t)$$

$$+ T'_{do}\left(Q_e(t) + \frac{V_s^2}{x_{ds}}\right)\Delta\omega(t) - \frac{x_{ds}}{x'_{ds}} P_m + \frac{(x_d - x'_d)}{x'_{ds}} P_m$$

$$= k_e u_f(t) I_q(t) + \frac{(x_d - x'_d)}{x'_{ds}}\left(E_f(t) - E_q(t)\right) I_q(t) + \frac{x_{ds}}{x'_{ds}} \cdot T'_{do}(x_d - x'_d) I_q^2(t)\Delta\omega(t)$$

$$+ T'_{do}\left(Q_e(t) + \frac{V_s^2}{x_{ds}}\right)\Delta\omega(t) - P_m$$

$$= k_e u_f(t) I_q(t) + \frac{(x_d - x'_d)}{x'_{ds}} T'_{do}\dot{E}_q(t) I_q(t) + \frac{x_{ds}}{x'_{ds}} \cdot T'_{do}(x_d - x'_d) I_q^2(t)\Delta\omega(t)$$

$$+ T'_{do}\left(Q_e(t) + \frac{V_s^2}{x_{ds}}\right)\Delta\omega(t) - P_m$$

$$= k_e u_f(t) I_q(t) + T'_{do}\left[\frac{(x_d - x'_d)}{x'_{ds}}\dot{E}_q(t) I_q(t) + \frac{x_{ds}}{x'_{ds}}(x_d - x'_d) I_q^2(t)\Delta\omega(t) + \left(Q_e(t) + \frac{V_s^2}{x_{ds}}\right)\Delta\omega(t)\right] - P_m$$

(A - I.3)

It is to be noted that all the quantities possessing uncertainties (i.e. x_{ds}, x'_{ds} and also V_s if the infinite bus voltage is assumed to have uncertain variations) are associated with the terms within the third bracket in equation (A-I.3).

Using equations (3.6) and (3.7),

$$\left(Q_e(t)+\frac{V_s^2}{x_{ds}}\right)\Delta\omega(t)=E_q(t)\dot{I}_q(t) \qquad \text{(A - I.4)}$$

Again,

$$\frac{(x_d-x_d')}{x_{ds}'}\dot{E}_q(t)I_q(t)+\frac{x_{ds}}{x_{ds}'}(x_d-x_d')I_q^2(t)\Delta\omega(t)$$

$$=(x_d-x_d')I_q(t)\left[\frac{\dot{E}_q(t)}{x_{ds}'}+\frac{x_{ds}}{x_{ds}'}I_q(t)\Delta\omega(t)\right]$$

Now, recognizing the fact that for the power system model (equations (3.1) – (3.9)) $I_d(t)$ is given by:

$$I_d=\frac{E_q'(t)-V_s\cos\delta(t)}{x_{ds}'}$$

and then differentiating the above expression of I_d and using equation (3.6),

$$\dot{I}_d(t)=\frac{\dot{E}_q(t)}{x_{ds}'}+\frac{x_{ds}}{x_{ds}'}I_q(t)\Delta\omega(t) \qquad \text{(A - I.5)}$$

Now, utilising equations (A-I.4) and (A-I.5) to replace the terms within the third bracket in equation (A-I.3) we have:

$$\hat{v}_f(t)=k_e u_f(t)I_q(t)+T_{do}'\left[(x_d-x_d')\dot{I}_d(t)I_q(t)+E_q(t)\dot{I}_q(t)\right]-P_m$$

which is identical to the equation (3.20) as expected can sequently one can say that all the uncertainties in the model given by equations (3.10) – (3.12) are taken into account by the new input $\tilde{v}_f(t)$. The first part of the R.H.S of equation (A-I.2) contributes to the uncertainty in the system matrix (**A**+Δ**A**) of equation (3.17) and the second part includes the uncertainty in the input matrix (**B** + Δ**B**) along with the uncertainties in input $v_f(t)$.

APPENDIX – II

The Case of Multi-machine System:

Applying the two-axis model (equations (3.1) – (3.9)) of synchronous generators in multimachine system the set of equations (4.5) – (4.8) are transformed into the followings:

$$P_{ei}(t) = \sum_{j=1}^{n} E_{qi}(t) E_{qj}(t) Y_{ij} \sin\left(\delta_{ij(t)}\right) \tag{A - II.1}$$

$$Q_{ei}(t) = -\sum_{j=1}^{n} E_{qi}(t) E_{qj}(t) Y_{ij} \cos\left(\delta_{ij(t)}\right) \tag{A - II.2}$$

$$I_{di}(t) = \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \cos\left(\delta_{ij(t)}\right) = -\frac{Q_{ei}(t)}{E_{qi}(t)} \tag{A - II.3}$$

$$I_{qi}(t) = \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \sin\left(\delta_{ij(t)}\right) = \frac{P_{ei}(t)}{E_{qi}(t)} \tag{A - II.4}$$

while, equations (4.1) – (4.4) and equations (4.9), (4.10) are still valid for the two-axis model.

Differentiating equation (A-II.1)

$$\dot{P}_{ei}(t) = \dot{E}_{qi}(t) \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \sin\left(\delta_{ij}(t)\right) + E_{qi}(t) \sum_{j=1}^{n} \dot{E}_{qj}(t) Y_{ij} \sin\left(\delta_{ij}(t)\right) + E_{qi}(t) \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \cos\left(\delta_{ij}(t)\right)\left(\dot{\delta}_i(t) - \dot{\delta}_j(t)\right)$$

$$= \dot{E}_{qi}(t) I_{qi}(t) + E_{qi}(t) \sum_{j=1}^{n} \dot{E}_{qj}(t) Y_{ij} \sin(\delta_{ij}(t))$$

$$+ E_{qi}(t) \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \cos(\delta_{ij}(t))(\Delta\omega_i(t) - \Delta\omega_j(t))$$

$$= \left[\dot{E}_{qi}(t) + (x_{di} - x'_{di}) \dot{I}_{di}(t)\right] I_{qi}(t) + E_{qi}(t) \sum_{j=1}^{n} \dot{E}_{qj}(t) Y_{ij} \sin(\delta_{ij}(t)) - Q_{ei}(t) \Delta\omega_i(t)$$

$$- E_{qi}(t) \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \cos(\delta_{ij}(t)) \Delta\omega_j(t)$$

$$= \frac{1}{T'_{do_i}} \left[E_{fi}(t) - E_{qi}(t)\right] I_{qi}(t) + (x_{di} - x'_{di}) \dot{I}_{di}(t) I_{qi}(t) - Q_{ei}(t) \Delta\omega_i(t)$$

$$+ E_{qi}(t) \sum_{j=1}^{n} \dot{E}_{qj}(t) Y_{ij} \sin(\delta_{ij}(t)) - E_{qi}(t) \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \cos(\delta_{ij}(t)) \Delta\omega_j(t)$$

$$= \frac{1}{T'_{do_i}} \left[k_{ei} u_{fi}(t) I_{qi}(t) - E_{qi}(t) I_{qi}(t)\right] + (x_{di} - x'_{di}) \dot{I}_{di}(t) I_{qi}(t) - Q_{ei}(t) \Delta\omega_i(t)$$

$$+ E_{qi}(t) \sum_{j=1}^{n} \dot{E}_{qj}(t) Y_{ij} \sin(\delta_{ij}(t)) - E_{qi}(t) \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \cos(\delta_{ij}(t)) \Delta\omega_j(t)$$

$$= -\frac{1}{T'_{do_i}} P_{ei}(t) + \frac{1}{T'_{do_i}} \left[k_{ei} u_{fi}(t) I_{qi}(t) + T'_{do_i} (x_{di} - x'_{di}) \dot{I}_{di}(t) I_{qi}(t)\right]$$

$$+ \left\{ -Q_{ei}(t) \Delta\omega_i(t) + E_{qi}(t) \sum_{j=1}^{n} \dot{E}_{qj}(t) Y_{ij} \sin(\delta_{ij}(t)) - E_{qi}(t) \sum_{j=1}^{n} E_{qj}(t) Y_{ij} \cos(\delta_{ij}(t)) \Delta\omega_j(t) \right\}$$

(A - II.5)

The last two terms written the 2nd bracket represent the interactions effects of the remote machines dynamics. All other terms in equation (A-II.5) are locally available to the ith machine.

Now, comparing equation (A-II.5) with the expression of $\ddot{\upsilon}_f(t)$ it can be concluded that if the 2nd bracketed term of equation (A-II.5) is equal to $E_{qi}(t)\,\dot{I}_{qi}(t)$ the equation (A-II.5) takes the form

$$\Delta\dot{P}_{ei}(t) = -\frac{1}{T'_{do_i}}\Delta P_{ei}(t) + \frac{1}{T'_{do_i}}\hat{v}_{fi}(t)$$

and the proposed DFL control law is applicable for the multi-machine power system to obtain totally interaction free linear model for the ith (i = 1,2,…….n) machine.

The verification is given below:

Using equation (A-II.4) for $I_{qi}(t)$

$$E_{qi}(t)\dot{I}_{qi}(t) = E_{qi}(t)\left[\sum_{j=1}^{n}\dot{E}_{qj}(t)Y_{ij}\sin(\delta_{ij}(t)) + \sum_{j=1}^{n}E_{qj}(t)Y_{ij}\cos(\delta_{ij}(t))(\Delta\omega_i(t) - \Delta\omega_j(t))\right]$$

$$= E_{qi}(t)\sum_{j=1}^{n}\dot{E}_{qj}(t)Y_{ij}\sin(\delta_{ij}(t)) + E_{qi}(t)\sum_{j=1}^{n}E_{qj}(t)Y_{ij}\cos(\delta_{ij}(t))\Delta\omega_i(t) - E_{qi}(t)\sum_{j=1}^{n}E_{qj}(t)Y_{ij}\cos(\delta_{ij}(t))\Delta\omega_j(t)$$

$$= E_{qi}(t)\sum_{j=1}^{n}\dot{E}_{qj}(t)Y_{ij}\sin(\delta_{ij}(t)) - Q_{ei}(t)\Delta\omega_i(t) - E_{qi}(t)\sum_{j=1}^{n}E_{qj}(t)Y_{ij}\cos(\delta_{ij}(t))\Delta\omega_j(t)$$

Hence, equation (A-II.5) boils down to

$$\dot{P}_{ei}(t) = -\frac{1}{T'_{do_i}} P_{ei}(t) + \frac{1}{T'_{do_i}} \left[k_{ei} u_{fi}(t) I_{qi}(t) + T'_{do_i} \left\{ (x_{di} - x'_{di}) \dot{I}_{di}(t) I_{qi}(t) + E_{qi}(t) \dot{I}_{qi}(t) \right\} \right]$$

$$\therefore \Delta\dot{P}_{ei}(t) = -\frac{1}{T'_{do_i}} \Delta P_{ei}(t)$$

$$+ \frac{1}{T'_{do_i}} \left[k_{ei} u_{fi}(t) I_{qi}(t) + T'_{do_i} \left\{ (x_{di} - x'_{di}) \dot{I}_{di}(t) I_{qi}(t) + E_{qi}(t) \dot{I}_{qi}(t) \right\} - P_{m_i} \right]$$

or, $\Delta\dot{P}_{ei}(t) - \frac{1}{T'_{do_i}} \Delta P_{ei}(t) + \frac{1}{T'_{do_i}} \hat{v}_{fi}(t)$

So, it can be concluded that $\hat{v}_{fi}$ (t) incorporates all the effects of nonlinearities and the dynamic interactions with other machines in the system with the ith machine.