CHAPTER-2
INVESTIGATION OF BLOOD FLOW THROUGH ARTERIES WITH TEMPERATURE AND CONCENTRATION DEPENDENT VISCOSITY

2.1 INTRODUCTION:

It has been observed that in patients with myocardial infarction— a kind of heart disease—diabetes, hypertension and renal diseases, viscosity of blood is appreciably higher than that in a normal individual Dintenfass (1967). Schmid-Schonbein and Volger (1976). It has been shown that the blood viscosity plays an important role in the development of many diseases [Dintenfass, (1965), Messmer et al (1972), Langsjoen and Inmon (1968)]. It is therefore of interest to find the physiological viscosity of blood. Unfortunately this has not yet been estimated because it varies with the flow conditions.

Interest in the fluid dynamics of blood flow through the arteries has led to the development of a number of theories related to the problem of viscosity variation in blood [Merrill (1969), Haynes (1960), Bayliss (1960)]. Also considering the importance of variation of viscosity for pathological point of view, several other investigators proposed the various models on taking the concentration, temperature or radial dependence of the viscosity [Bayliss (1952), Barbee (1973), Raymond (1976), Wall and Nagata (2000), Schafer and Herwig (1993)].

The present investigation is from the class of laminar forced convection problems and is concerned with the flow of study axisymmetric Newtonian viscous fluid through a circular tube under uniform axial pressure
gradient in the presence of uniform axial temperature and concentration gradients with simultaneous diffusion and convection of heat and mass. The primary aim of taking up this investigation is to decide the conditions under which the fluid flux through any cross-section in the Hagen-Poiseuille flow and Alfredson system [Schlichting (1960)] or plane poiseuille flow Elapson (1998), could be increased by suitably modifying the fluid mechanics inside the tube, which may be more helpful to understand the pathological situations, without altering the uniform axial pressure gradient which drives the flow. The analysis carried out takes into account the variation in the fluid viscosity due to variation in temperature and concentration. Consideration of equation of momentum, continuity, heat conduction and mass diffusion then leads to one single non-linear integro-differential equation containing two non-dimensional parameters which governs the axial velocity component. The non-linear integro-differential character of the above mathematical equation precludes its solution exactly and consequently the usual 'parameter perturbation technique' is utilized and the first four terms of a suitable perturbation expansion (in which first term represents the usual Hagen-Poiseuille solution) representing the solution of the problem are calculated. An important qualitative result derived on the basis of first approximate solution, is that the fluid flux through any cross-section of the tube decreases/increases while the wall shearing stress increases/decreases from their corresponding Hagen-Poiseuille values, provided the fluid viscosity increases/decreases along the flow direction every where in the domain. The above result is highly significant from the quantitative point of view, since it establishes a 17% flux increase over the Hagen-Poiseuille value. Second and Third approximate solutions are obtained and the axial velocity component at
various points at the tube cross-section and the fluid flux are numerically computed from these approximate solution for various values of the non-dimensional parameters, as shown in graphs. These results may be useful for the better understanding of pathological situations prevalent in the blood vessels.

2.2 THE MATHEMATICAL MODEL:

In this paper we mathematically analyse an incompressible viscous fluid model of steady blood flow through an artery under uniform axial pressure gradients in the presence of uniform axial temperature and concentration gradients and simultaneous diffusion and convection of heat and mass. The subsequent analysis carried out takes into account the variation in fluid viscosity due to variation in temperature and concentration.

Polar cylindrical co-ordinate system \((r, \theta, z)\) is considered with the \(z\)-axis coinciding with the central line of the artery, whose equation is given by

\[
r = R_0 \quad 0 \leq z \leq L
\]  

(2.2.1)

2.3 THE GOVERNING EQUATION:

The governing equation in cylindrical polar coordinates, which mathematically describe the laminar flow problem of an incompressible viscous fluid, are given by:

Continuity Equation:

\[
\frac{\partial v_z}{\partial z} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} = 0
\]  

(2.3.1)
Momentum equations

\[ \rho \left( \frac{Dv_r}{Dt} - \frac{v_r^2}{r} \right) = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial r} \left( 2\mu \frac{\partial v_r}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left[ \mu \left( \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_r}{r} \right) \right] \]
\[ + \frac{\partial}{\partial z} \left[ \mu \left( \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right) \right] + \frac{2\mu}{r} \left( \frac{\partial v_r - \frac{1}{r} \frac{\partial v_\theta}{\partial r}}{\partial r} - \frac{v_r}{r} \right) \]  \( (2.3.2) \)

\[ \rho \left( \frac{Dv_\theta}{Dt} - \frac{v_r v_\theta}{r} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \frac{1}{r} \frac{\partial}{\partial \theta} \left[ \mu \left( \frac{2}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{2v_r}{r} \right) \right] + \frac{\partial}{\partial r} \left[ \mu \left( \frac{1}{r} \frac{\partial v_\theta}{\partial r} + \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} \right) \right] \]
\[ + \frac{\partial}{\partial z} \left[ \mu \left( \frac{1}{r} \frac{\partial v_\theta}{\partial z} + \frac{\partial v_z}{\partial \theta} \right) \right] + \frac{2\mu}{r} \left( \frac{\partial v_\theta - \frac{1}{r} \frac{\partial v_r}{\partial \theta}}{\partial \theta} + \frac{v_\theta}{r} \right) \]  \( (2.3.3) \)

\[ \rho \left( \frac{Dv_z}{Dt} \right) = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial z} \left( \frac{2\mu}{r} \frac{\partial v_z}{\partial z} \right) + \frac{\partial}{\partial r} \left[ \mu \left( \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial \theta} \right) \right] \]
\[ + \frac{\partial}{\partial \theta} \left[ \mu \left( \frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial \theta} \right) \right] + \frac{\mu}{r} \left( \frac{\partial v_r}{\partial z} + \frac{\partial v_\theta}{\partial r} \right) \]  \( (2.3.4) \)

Energy Equation

\[ K_1 \left[ \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) \right] + \phi = \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_\theta \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \]  \( (2.3.5) \)

Concentration equation

\[ K_2 \left[ \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial c}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 c}{\partial \theta^2} + \frac{\partial^2 c}{\partial z^2} \right) \right] = \frac{\partial c}{\partial t} + v_r \frac{\partial c}{\partial r} + \frac{v_\theta}{r} \frac{\partial c}{\partial \theta} + v_z \frac{\partial c}{\partial z} \]  \( (2.3.6) \)

The viscosity is assumed to vary with temperature and concentration.

\[ \mu = \mu_0 \left[ 1 + \alpha_2 (T - T_0) + \beta_2 (c - c_0) \right] \]  \( (2.3.7) \)

Where
\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r} + v_\theta \frac{\partial}{\partial \theta} + v_z \frac{\partial}{\partial z} \]

and the coefficient of dissipation

\[ \phi_i = \frac{2\mu}{\rho C_T} \left[ \frac{(\partial v_r)^2}{\partial r} + \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right)^2 + \left( \frac{\partial v_z}{\partial z} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} \right)^2 + \frac{1}{2} \left( \frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right)^2 \right] \]

(2.3.8)

The above equations are to be considered with the no slip conditions on the velocity components at the boundary. For steady axisymmetric axial flow through a circular artery in the presence of uniform axial temperature and concentration-gradients, we take.

\[ \frac{\partial}{\partial t} = 0, \quad \frac{\partial}{\partial \theta} = 0, \]
\[ v_r = v_\theta = 0 \]
\[ v_z = v_z(r), \]
\[ \frac{\partial p}{\partial z} = \text{constant} \]

(2.3.9)

It is further assumed [Goldstein (1965)]

\[ T = A z + f(r), \]
\[ C = B z + g(r). \]

We wish to solve the flow problem represented by the above mathematical equations.
2.4 THE MATHEMATICAL ANALYSIS:

Making use to the equation (2.3.9), we find that the equations (2.3.1) and (2.3.3) are trivially satisfied and the equations (2.3.2) and (2.3.4) respectively reduce to the following equations.

\[ O = \frac{\partial p}{\partial r} + \frac{\partial \mu}{\partial z} \frac{dv_z}{dr}, \]  
(2.4.1)

and

\[ O = \frac{\partial p}{\partial r} + \mu \left( \frac{d^2 v_z}{dr^2} + \frac{1}{r} \frac{dv_z}{dr} \right) + \frac{\partial \mu}{\partial r} \frac{dv_z}{dr}, \]  
(2.4.2)

Further, using the equation (2.3.9) and neglecting the \( \bar{\phi} \), equations (2.3.5) and (2.3.6) respectively simplify to

\[ K_1 \left( \frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} \right) = Av_z, \]  
(2.4.3)

\[ K_2 \left( \frac{d^2 q}{dr^2} + \frac{1}{r} \frac{dq}{dr} \right) = Bv_z. \]  
(2.4.4)

The bounded solutions for the above equations are respectively, given by

\[ f(r) = \frac{A}{K_1} \left[ \frac{1}{r} \left( \int rv_z dr \right) \right] dr + C_1 \]  
(2.4.5)

and
\( g(r) = \frac{B}{K_2} \left[ \frac{1}{r} \left( \int \nu_z \, dr \right) \right] + C_2 \)  

(2.4.6)

Where \( C_1 \) and \( C_2 \) are arbitrary constants of integration which may be determined by the condition that \( f(r) \) and \( g(r) \) vanish at the wall of the artery.

We replace \( \mu \) by \( \mu_0 \) in the term \( \mu \left( \frac{d^2 \nu_z}{dr^2} + \frac{1}{r} \frac{d \nu_z}{dr} \right) \) in the equation (2.4.2).

This is valid because the radius of the artery is small and this makes the contribution arising from the variability in viscosity in the above term negligible as compared to the corresponding contributions in the term \( \frac{\partial \mu}{\partial r}, \frac{\partial \nu_z}{\partial r} \) in the equation (2.4.2). The second assumption that the dissipation term \( \tilde{\phi} \) is negligible in equation (2.3.5) is obvious.

Hence, equation (2.4.2) is modified as

\[ O = -\frac{\partial p}{\partial z} + \mu_0 \left( \frac{d^2 \nu_z}{dr^2} + \frac{1}{r} \frac{d \nu_z}{dr} \right) + \frac{\partial \mu}{\partial r} \frac{\partial \nu_z}{\partial r} \]  

(2.4.7)

The value of \( \frac{\partial \mu}{\partial r} \) is obtained from the equations (2.3.7), (2.4.5) and (2.4.6), substituting in the equation (2.4.7) we obtain,

\[ \frac{d^2 \nu_z}{dr^2} + \frac{1}{r} \frac{d \nu_z}{dr} \frac{1}{\mu_0} - \frac{1}{\partial z} - \left( \frac{\alpha_z A}{K_1} + \frac{\beta_z B}{K_2} \right) \frac{1}{r} \int \nu_z \, dr \]  

(2.4.8)

We can put equation (2.4.8) in its non-dimensional form as

\[ \frac{d^2 u}{d\eta^2} + \frac{1}{\eta} \frac{du}{d\eta} + p_i = -\frac{\beta_z du}{\eta} \int \rho u d\eta \]  

(2.4.9)
When \( \beta_i = \left( \frac{\alpha_i A}{K_1} + \frac{\beta_i B}{K_2} \right) \)

The boundary conditions are given by

\[
\begin{align*}
(1) & \; \text{u is finite at } \eta = 0 \\
(2) & \; \text{u} = 0 \quad \text{at } \eta = 1
\end{align*}
\]

The solution of the equation (2.4.9) can be obtained for small values of \( \beta_1 \) by the perturbation method as

\[
u(\eta, \beta_i) = u_0(\eta) + \beta_1 u_1(\eta) + \beta_1^2 u_2(\eta) + \ldots + \beta_1^n u_n(\eta)
\]

(2.4.11)

Hence, an approximate solution of the equation (2.4.9) can be written as

\[
u^*(\eta) = \sum_{m=0}^{n} \beta_1^m u_m(\eta), \quad m = 0,1,2,3,\ldots,n
\]

(2.4.12)

Where \( u_0^*, u_1^*, u_2^*, \text{ and } u_3^* \) are the zeroth, first, second and third approximate solutions of \( u \), substituting (2.4.12) for \( u \) in equation 2.4.9), we obtain.

\[
\left( \frac{d^2}{d\eta^2} + \frac{1}{\eta} \frac{d}{d\eta} \right) \sum_{m=0}^{n} \beta_1^m u_m(\eta), + \rho = -\frac{\beta_1}{\eta} \frac{d}{d\eta} \sum_{m=0}^{n} \beta_1^m u_m(\eta) d\eta \times \int \eta \sum_{m=0}^{n} \beta_1^m u_m(\eta) d\eta,
\]

(2.4.13)

on equating the various powers of \( \beta_1 (i.e., \beta_0, \beta_1, \beta_1^2, \beta_1^3, \ldots) \), the governing equations for \( u_0, \ u_1, \ u_2 \) and \( u_3 \) are given respectively by the following equations
The boundary conditions for these perturbation terms are,

(1) \( u_0, u_1, u_2 \) and \( u_3 \) are finite at \( \eta = 0 \)

(2) \( u_0 = u_1 = u_2 = u_3 = 0 \) at \( \eta = 1 \)  

(i) The solution of \( u_0 \)

Equation (2.4.14) can be rewritten as follows

\[
\frac{d}{d\eta} \left( \eta \frac{du_0}{d\eta} \right) = -P_i \eta \tag{2.4.19}
\]

Integrating this equation with respect to \( \eta \), we get

\[
u_0 = -\frac{P_i \eta^2}{4} + C_1 \log \eta + C_2 \tag{2.4.20}
\]

Where \( C_1 \) and \( C_2 \) are the arbitrary constants using the boundary conditions given by the equation (2.4.18), we obtain
\[ u_0 = -\frac{P_1(1-\eta^2)}{2^3} \]  \hspace{1cm} (2.4.21)

(ii) The solution of \( u_1 \)

Substituting the value of \( u_0 \) from the equation (2.4.21) in equation (2.4.15), we get.

\[ \frac{d^2 u_i}{d\eta^2} + \frac{1}{\eta} \frac{du_i}{d\eta} = -\frac{P_1^2}{2^3} \int (\eta^3 - \eta) d\eta \]  \hspace{1cm} (2.4.22)

Integrating the above equation with respect to \( \eta \), we get

\[ u_i = -\frac{P_1^2}{2^3} \left( \frac{\eta^6}{2^4,3^2} - \frac{\eta^4}{2^2} \right) + c_1 \log \eta + c_2 \]  \hspace{1cm} (2.4.23)

Where \( c_1 \) and \( c_2 \) are arbitrary constant. Using the boundary conditions given by the equation (2.4.18). We get

\[ u_i = P_1^2 \left[ -\frac{1}{2^7,3^2} (1-\eta^6) - \frac{1}{2^8} (1-\eta^4) \right] \]  \hspace{1cm} (2.4.24)

(iii) The solution of \( u_2 \)

Substituting the values of \( u_0 \) and \( u_1 \) from the equations (2.4.21) and (2.4.24) respectively, in the equation (2.4.16), we get

\[ \frac{d^2 u_2}{d\eta^2} + \frac{1}{\eta} \frac{du_2}{d\eta} = P_1^3 \left[ -\frac{7}{2^{10},3^2} \eta^3 - \frac{\eta^5}{2^9} + \frac{\eta^7}{2^9} - \frac{7\eta^9}{2^{11},3^2} \right] \]  \hspace{1cm} (2.4.25)

Integrating the above equation with respect to \( \eta \), we get
where \( c_1 \) and \( c_2 \) are arbitrary constants. Using the boundary conditions given by the equation (2.4.18), we get

\[
\begin{align*}
\frac{d^2 u_2}{d\eta^2} + \frac{1}{\eta} \frac{du_2}{d\eta} &= P_1^3 \left[ \frac{-7\eta^4}{2^{14}, 3^2} - \frac{\eta^6}{2^{11}, 3^2} + \frac{\eta^8}{2^{15}} - \frac{7\eta^{10}}{2^{13}, 3^2, 5^2} + c_1 \log \eta + c_2 \right] \\
\end{align*}
\]

(2.4.26)

(iv) The solution of \( u_3 \)

Substituting the values of \( u_0, u_1 \) and \( u_2 \) from the equations (2.4.21), (2.4.24) and (2.4.27) respectively, in equation (2.4.17), we obtain.

\[
\begin{align*}
\frac{d^2 u_3}{d\eta^2} + \frac{1}{\eta} \frac{du_3}{d\eta} &= P_1^3 \left[ \frac{553\eta^3}{2^{17}, 3^2, 5^2} + \frac{7\eta^5}{2^{14}, 3^2} + \frac{5\eta^7}{2^{15}, 3^2} - \frac{19\eta^9}{2^{17}, 3^2, 5^2} + \frac{157\eta^{11}}{2^{17}, 3^2, 5} - \frac{137\eta^{13}}{2^{16}, 3^2, 5^2} \right] \\
\end{align*}
\]

(2.4.28)

Integrating the above equating with respect to \( \eta \), we get

\[
\begin{align*}
\begin{bmatrix}
\frac{553}{2^{21}, 3^2, 5^2} \eta^4 + \frac{7\eta^6}{2^{16}, 3^4} + \frac{5\eta^8}{2^{21}, 3^2} - \frac{19\eta^{10}}{2^{17}, 3^2, 5^2} + \frac{157}{2^{21}, 3^2, 5} \\
-\frac{137}{2^{18}, 3^2, 5^2, 7^2} + C_1 \log \eta + C_2
\end{bmatrix}
\end{align*}
\]

(2.4.29)

where \( C_1 \) and \( C_2 \) are arbitrary constants. Using the boundary conditions given by the equation (2.4.18), we get

\[
\begin{align*}
\begin{bmatrix}
\frac{-553}{2^{21}, 3^2, 5^2} (1-\eta^4) - \frac{7}{2^{16}, 3^4} (1-\eta^6) - \frac{5}{2^{21}, 3^2} (1-\eta^8) + \frac{19}{2^{17}, 3^2, 5^2} (1-\eta^{10}) \\
-\frac{157}{2^{21}, 3^2, 5} (1-\eta^{12}) + \frac{137}{2^{18}, 3^2, 5^2, 7^2} (1-\eta^{14})
\end{bmatrix}
\end{align*}
\]

(2.4.30)
Now, using the equation (2.4.12) we obtain the expression of zeroth, first, second and third approximate solutions as follows:

\[ u_0^* = P_1 \left( \frac{1 - \eta^2}{2^2} \right) \]  
\[ (2.4.31) \]

\[ u_1^* = u_0^* (\eta) + \beta_1 P_1^2 \left[ \frac{1}{2^7,3^2} (1 - \eta^6) - \frac{1}{2^8} (1 - \eta^4) \right] \]  
\[ (2.4.32) \]

\[ u_2^* = u_1^* (\eta) + \beta_2 P_1^3 \left[ \frac{7}{2^{24},3^2} (1 - \eta^6) + \frac{1}{2^{21},3^2} (1 - \eta^6) - \frac{1}{2^{25}} (1 - \eta^8) + \frac{7}{2^{23},3^2,5^2} (1 - \eta^{10}) \right] \]  
\[ (2.4.33) \]

\[ u_3^* (\eta) = u_2^* (\eta) + \beta_3 P_1^4 \begin{bmatrix} -553 & 7 \frac{2^{21},3^2,5^2}{2^{21},3^2,5^2} (1 - \eta^6) - \frac{5}{2^{21},3^2} (1 - \eta^8) & \frac{157}{2^{21},3^2,5^2} (1 - \eta^{12}) + \frac{137}{2^{18},3^2,5^2,7^2} (1 - \eta^{14}) \end{bmatrix} \]  
\[ (2.4.34) \]

2.5 **VOLUME FLOW RATE:**

The zeroth order non-dimensional volume flow rate can be obtained as follows:

\[ Q_0 = 2 \int_0^1 \eta u_0 d\eta \]  
\[ (2.5.1) \]

Substituting the value of \( u_0 \) from equation (2.4.21) in equating (2.5.1) and then evaluating the integral. We get,

\[ Q_0 = P_1/8 \]  
\[ (2.5.2) \]

The first order non-dimensional volume flow rate can be obtained as follows:

\[ Q_1 = 2 \int_0^1 \eta u_1 d\eta. \]  
\[ (2.5.3) \]
Substituting the value of $u_1$ from the equation (2.4.14) in equation (2.4.27) and then evaluating the integral. We get,

$$Q_1 = -\frac{p^2}{2\gamma}.$$  \hspace{1cm} (2.5.4)

The second order non-dimensional volume flow rate can be obtained as follows:

$$Q_2 = 2\int_0^1 \eta u_2 d\eta.$$ \hspace{1cm} (2.5.5)

Substituting the value of $u_2$ from the equation (2.4.27) in equation (2.5.5) and then evaluating the integral. We get,

$$Q_2 = \frac{113p_1^3}{2^{14}, 3^3, 5}.$$ \hspace{1cm} (2.5.6)

The third order non-dimensional volume flow rate can be obtained as follows:

$$Q_3 = 2\int_0^1 \eta u_3 d\eta.$$ \hspace{1cm} (2.5.7)

Substituting the value of $u_3$ from the equation (2.4.20) in equation (2.4.31) and then evaluating the integral. We get,

$$Q_3 = -\frac{3151}{2^{31}, 3^2, 5, 7} \frac{p_1^4}{2^{11}, 3^3, 5, 7}.$$ \hspace{1cm} (2.5.8)

The zeroth, first, second and third order approximate solutions for the non-dimensional volume flow rate can be evaluated in a way similar to the one adopted in the previous section.
They are respectively given by the following:

\[ Q_0^* = \frac{P_1}{8} \]  
(2.5.9)

\[ Q_1^* = Q_0^* - \frac{\beta_1 P_1^2}{2^9}. \]  
(2.5.10)

\[ Q_2^* = Q_1^* + \frac{113\beta_1^2 P_1^3}{2^{14}, 3^2, 5}. \]  
(2.5.11)

\[ Q_3^* = Q_2^* - \frac{3151\beta_1^3 P_1^4}{2^{21}, 3^2, 5}. \]  
(2.5.12)

### 2.6 SHEAR STRESS AT THE WALL

The non-dimensional shear stress at the wall is given by:

\[ \tau_w^* = -\frac{du}{d\eta}_{|_{n=1}} \]  
(2.6.1)

The zeroth, first, second and third order approximate solutions for the non-dimensional wall shear stress can be evaluated in a way similar to the one adopted in the previous section.

Hence, they are respectively given by the following

\[ \tau_{n=0}^* = \frac{P_1}{2} \]  
(2.6.2)

\[ \tau_{n=1}^* = -\frac{P_1^2 \beta_1}{2^2, 3} + \tau_{n=0}^* \]  
(2.6.3)

\[ \tau_{n=2}^* = \frac{19 P_1^3 \beta_1^2}{2^{12}, 3, 5} + \tau_{n=1}^* \]  
(2.6.4)
\[ \tau_{w_0} = \frac{-509 P^4 \beta^3}{2^{18}, 3^{1}, 7} + \tau_{r_i} \]  

(2.6.5)

### 2.7 Temperature Distribution:

The non-dimensional temperature distribution is given by

\[ F(\eta) = K^{-1} \int_{\eta}^{1} \left[ \left( \eta \mu d\eta \right) \right] d\eta + C_1 \]  

(2.7.1)

Where \( C_1 \) is an arbitrary constant.

**Boundary conditions:**

\[ F_0 = F_1 = F_2 = F_3 = 0 \text{ at } \eta = 1 \]  

(2.7.2)

Where \( F_0, F_1, F_2 \) and \( F_3 \) are the zeroth, first, second and third order perturbation terms of \( F \).

The zeroth, first, second and third order approximate solutions for non-dimensional temperature distribution which satisfy the above boundary conditions, can be evaluated in way similar to one adopted in previous section.

Hence, they are respectively given by the following

\[ F^*_0 = \frac{P_1 K_{1}^{-2}}{2^2} \left[ \frac{1}{2^4} (1-\eta^4) - \frac{1}{2^2} (1-\eta^2) \right] \]  

(2.7.3)

\[ F^*_1 = F^*_0 + \frac{K_{1}^{-2} \beta_{1}^2}{2^3} \left[ \frac{1}{2^{10}, 3^2} (1-\eta^8) - \frac{1}{2^7, 3^2} (1-\eta^6) + \frac{2}{2^7, 3^2} (1-\eta^2) \right] \]  

(2.7.4)
From the equation (2.4.6), the non-dimensional concentration distribution is given by

\[ G(r) = K^2 \int \frac{1}{r^2} \left( \int \eta d\eta \right) d\eta + C, \]  

(2.8.1)

where \( C \) is an arbitrary constant, the boundary conditions are given by

\[ G_0 = G_1 = G_2 = G_3 = 0 \quad \text{at} \quad \eta = 1 \]  

(2.8.2)

Also, we can obtain the zeroth, first, second and third order approximate solutions for non-dimensional concentration distribution, which satisfy the above boundary condition, by method similar to one given earlier.

Hence, they are respectively given by the following.

\[ G_0 = \frac{P}{2^2 K^2} \left[ \frac{1}{2^2} (1-\eta^2) - \frac{1}{2^2} (1-\eta^4) \right] \]  

(2.8.3)

\[ G_1 = G_0 + \frac{K^2 \beta r^2}{2^3} \left[ \frac{1}{2^3} (1-\eta^8) \right. \]  

(2.8.4)
\[ G'_2 = G'_1 + \beta^2 P_1^2 K^{-2}_2 \left[ \frac{7}{2^{17}, 3^4, 5^2} (1-\eta^{12}) - \frac{1}{2^{17}, 3^2} (1-\eta^{10}) + \frac{1}{2^{17}, 3^2} (1-\eta^8) \right] \]  

\[ G'_3 = G'_2 + K_1^{-2} \beta_1^4 P_1^4 \left[ \frac{137}{2^{16}, 3^2, 5^2, 7^2} (1-\eta^{16}) - \frac{157}{2^{23}, 3^4, 5, 7^2} (1-\eta^{14}) + \frac{19}{2^{21}, 3^4, 5^3} (1-\eta^{12}) \right] \]  

2.9 RESULTS AND DISCUSSIONS:

The role played by the variation of the viscosity in the blood vessels is of crucial importance in order to understand the fluid mechanics, and also due to growing interest in medical literature for diagnostic purpose. The essential points in this regard can be broadly described as follows.

The variation of viscosity in the axial and radial directions due to variations in temperature and concentration facilitates or reduces the flow by diminishing or increasing the shearing stress at the pipe wall. As a consequence the modifications in the axial velocity, fluid flux etc. are introduced.

Figure 2.1: Shows the variations in the axial velocity versus radial coordinate for various values of \( \beta_1 \) and a fixed value of \( P_1 \). For the decreasing value of \( \beta_1 \) from position to negative, the axial velocity increases at every point and this is expected also on the basis of the fundamental mechanism enunciated above. The negative value of \( \beta_1 \) indicates a decrease in the viscosity and hence decreases the wall shear. Opposite effect is visible when \( \beta_1 \) takes positive values.
Figure 2.2: gives the variations in the axial velocity versus the radial coordinate for various values of $P_1$ and a fixed value of $\beta_1$ and shows some interesting conclusions.

The wall shear-stress increases with increase in $P_1$ and this reduces the axial velocity at every point. Thus for a fixed positive value of $\beta_1$, an increase in the axial pressure gradient is met with an increased wall shear stress and thus reducing the flow.

Figures 2.3: Shows the variation in the fluid flux across any cross-section of the artery versus $\beta_1$ with a fixed value of $P_1$. It is seen that the fluid flux increases by over 17% when compared to the Hagen-Poiseuille value ($\beta_1=0$) for $\beta_1=-0.1$ and $P_1=70$, when which is highly significant. This flux increase diminishes with increasing values of $\beta_1$ and for $\beta_1 = \pm 0.1$ and $P_1 = 70$, we obtain that fluid flux decreases by over 6% when compared to the Hagen Poiseuille value.

Finally we remark that the double diffusive mechanism introduced has much relevance in the present investigation and cannot be replaced by a single diffusive mechanism and yet retain the same degree of generality. Since it is possible that the temperature and concentration effects on the viscosity variation oppose each other. These results may be helpful in a better visualization of the pathological situations in the blood vessels.
Fig 2.1: Variation of axial velocity with $\beta_1$ for fixed value of $P_1 = 70$
Fig 2.2: Variation of axial velocity with $P_1$ for fixed value of $\beta_1 = 0.1$
Fig 2.3: Variation of fluid flux with $\beta_1$ for fixed value of $P_1 = 70$. 