Chapter 5

SELF-SIMILAR FLOW OF A DUSTY GAS BEHIND A SHOCK WAVE WITH HEAT CONDUCTION AND RADIATION HEAT FLUX UNDER A GRAVITATIONAL FIELD
5.1 INTRODUCTION

The influence of radiation on the shock wave and on the flow-field behind the shock front has always been of great interest, for instance in the field of nuclear power and space research. Consequently, similarity solutions for shock waves in radiative gas dynamics have been given by Marshak [1], Elliott [2], Wang [3], Helliwell [4], Nicastro [5], Ghoniem et al [6], Vishwakarma and Nath [7] and many others. Marshak [1] studied the effect of radiation on the shock propagation by introducing the radiation diffusion approximation. He solved both the cases of constant density and constant pressure fields without invoking conditions of self-similarity. Elliott [2] considered the explosion problem by introducing the radiation flux in its diffusion approximation. Wang [3], Helliwell [4] and Nicastro [5] treated the problem of radiating walls, either stationary or moving, generating shocks at the head of self-similar flow-fields. Ghoniem et al [6] obtained a self-similar solution for spherical explosions taking into account the effects of both conduction and radiation in the two limits of Rosseland radiative diffusion and Plank radiative emission. Vishwakarma and Nath [7] obtained a non-self-similar solution for the shock propagation in an exponential medium with heat conduction and radiation heat flux.

The study of shock waves in a mixture of a gas and small solid particles is of great importance due to its applications to nozzle flow, lunar ash flow, bomb blast, coal mine blast, volcanic and cosmic explosions, supersonic flight in polluted air, collision of coma with a planet, shock in supernova explosions and many other engineering problems (see Pai et al [8], Higashino and Suzuki...
Miura and Glass [10], Gretler and Regenfelder [11], Popel and Gisko, Vishwakarma and Nath [13], Vishwakarma et al [14]). Miura and Glass [15] obtained an analytical solution of a planar dusty gas flow with constant velocities of the shock and the piston moving behind it. As they neglected the volume occupied by the solid particles mixed into the perfect gas, the dust virtually has a mass fraction but no volume fraction. Their results reflect the influence of the additional inertia of the dust upon the shock propagation. Pai et al [8] generalized the well known solution of a strong explosion due to an instantaneous release of energy in gas (Korobeinikov [16], Sedov [17]) to the case of two-phase flow of a mixture of perfect gas and small solid particles and brought out the essential effects due to presence of dusty particles on such a strong shock wave. As they considered non-zero volume fraction of solid particles in the mixture, their results reflect the influence of both the decrease of mixture’s compressibility and the increase of mixture’s inertia on the shock propagation (Steiner and Hirschler [18], Vishwakarma and Pandey [19]). In extreme conditions that prevail in most of the problems associated with shock waves, the assumption that the gas is ideal is no longer valid. Anisimov and Spiner [20] have taken an equation of state for non-ideal gases in a simplified form, and investigated the effect of parameter for non-idealness on the problem of a strong point explosion. Vishwakarma and Nath [13] obtained the similarity solution for the propagation of a strong shock wave in a mixture of a non-ideal gas and small solid particles driven out by a piston moving with time according to power law in both the cases, when the flow behind the shock was isothermal or adiabatic.

In all of the works mentioned above, the influence of gravitational field on
the medium is not considered. The gravitational force has considerable effect on many astrophysical problems. Carrus et al [21] have studied the propagation of shock waves in a gas under the gravitational attraction of central body of fixed mass (Roche model) and obtained the similarity solutions by numerical methods. Rogers [22] has discussed a method for obtaining analytical solution of the same problem. Singh [23] has studied the self-similar flow of a non-conducting perfect gas, moving under the gravitational attraction of a central body of fixed mass, behind a spherical shock wave assuming the total energy content between the expanding surface and the shock front to be increasing with time. Vishwakarma and Nath [24] have obtained similarity solutions for the propagation of a spherical shock wave in a mixture of a non-ideal gas and small solid particles driven out by a piston moving with time according to power law under the influence of gravitational field.

Therefore, in the present work, we investigated the self-similar flow behind a spherical shock wave propagating in a mixture of a non-ideal gas and small solid particles with heat conduction and radiation heat flux. The medium is assumed to be under a gravitational field due to heavy nucleus at the origin (Roche model). The unsteady model of Roche consists of a gas distributed with spherical symmetry around a nucleus having a large mass $m^*$. It is assumed that the gravitational effect of the mixture itself can be neglected compared with the attraction of the heavy nucleus. In order to get some essential features of shock propagation, the solid particles are considered as a pseudo-fluid and it is assumed that the equilibrium flow condition is maintained in the flow-field, and the viscous stress of the mixture is negligible (Pai et al [8], Higashino and Suzuki [9]). The heat transfer fluxes are expressed in
terms of Fourier’s law for heat conduction and a diffusion radiation model for an optically thick grey dusty gas, which is typical of large-scale explosions. The thermal conductivity and absorption coefficient of the medium are assumed to be proportional to powers of temperature and density (Ghoniem et al [6], Vishwakarma et al [14]). Also, it is assumed that the dusty gas is grey and opaque, and the shock is isothermal. The assumption that the shock is isothermal is a result of the mathematical approximation in which the heat flux is taken to be proportional to the temperature gradient; this excludes the possibility of temperature jump (Vishwakarma et al [14], Zel’dovich and Raizer [25], Rosenau and Frankenthal [26, 27], Vishwakarma and Nath [7, 28]). The counter-pressure (the pressure ahead of the shock) is taken into account. Radiation pressure and radiation energy are neglected (Elliot [2], Wang [3], Ghoniem et al [6]). The assumption of an optically thick grey gas is physically consistent with the neglect of radiation pressure and radiation energy (Nicastro [5]). Effects of an increase in the value of the parameter of non-idealness of the gas in the mixture \( \tilde{b} \), the mass concentration of solid particles in the mixture \( k_p \), the ratio of the density of solid particles to the initial density of the gas \( G_1 \) and the variation of the gravitational parameter \( G_0 \) on the flow variables behind the shock are investigated.
5.2 EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

We consider the medium to be a dusty gas (a mixture of small solid particles and a non-ideal gas), which is under the gravitational attraction of a heavy nucleus at the centre. The equation of state of the non-ideal gas in the mixture is taken to be (Vishwakarma and Nath [24]).

\[ p_g = R^* \bar{\rho}_g (1 + b \bar{\rho}_g) T, \quad (5.2.1) \]

where \( p_g \) and \( \bar{\rho}_g \) are the partial pressure and partial density of the gas in the mixture, \( T \) is the temperature of the gas (and of the solid particles as the equilibrium flow condition is maintained), \( R^* \) is the specific gas constant and ‘\( b \)’ is the internal volume of the molecules of the gas. In this equation, the deviations of an actual gas from the ideal state are taken into account, which result from the interaction between its component molecules. It is assumed that the gas is still so rarefied that triple, quadruple, etc. collisions between molecules are negligible, and their interaction is assumed to occur only through binary collisions.

The equation of state of the solid particles in the mixture is, simply,

\[ \rho_{sp} = \text{constant}, \quad (5.2.2) \]

where \( \rho_{sp} \) is the species density of the solid particles. Proceeding on the same lines as (Pai [29]), we obtain the equation of state of the mixture as

\[ p = \frac{(1 - k_p)}{(1 - Z)} [1 + b \rho(1 - k_p)] \rho R^* T, \quad (5.2.3) \]
where \( p \) and \( \rho \) are the pressure and density of the mixture, \( Z = \frac{V_{sp}}{V} \) is the volume fraction and \( k_p = \frac{m_{sp}}{m} \) is the mass fraction (concentration) of the solid particles in the mixture, \( m_{sp} \) and \( V_{sp} \) being, respectively, the total mass and the volumetric extension of the solid particles in a volume ‘\( V \)’ and mass ‘\( m \)’ of the mixture.

The relation between \( k_p \) and \( Z \) is given by (Pai [29])

\[
k_p = \frac{Z \rho_{sp}}{\rho} .
\]

(5.2.4)

In the equilibrium flow, \( k_p \) is constant in the whole flow-field. Therefore, from equation (5.2.4)

\[
\frac{Z}{\rho} = \text{constant} .
\]

(5.2.5)

Also, we have the relation (Pai [29])

\[
Z = \frac{k_p}{G(1 - k_p) + k_p} ,
\]

(5.2.6)

where \( G = \frac{\rho_{sp}}{\rho_g} \) is the ratio of density of solid particles to the species density of the gas.

The internal energy per unit mass of the mixture may be written as

\[
U_m = [k_p C_{sp} + (1 - k_p) C_v]T = C_{vm} T ,
\]

(5.2.7)

where \( C_{sp} \) is the specific heat of the solid particles, \( C_v \) is the specific heat of the gas at constant volume and \( C_{vm} \) is the specific heat of the mixture at constant volume.

The specific heat of the mixture at constant pressure is

\[
C_{pm} = k_p C_{sp} + (1 - k_p) C_p ,
\]

(5.2.8)
where \( C_p \) is the specific heat of the gas at constant pressure.

The ratio of the specific heats of the mixture is given by (Pai et al [8], Pai [29])

\[
\Gamma = \frac{C_{pm}}{C_{vm}} = \gamma \frac{1 + \delta \beta' / \gamma}{1 + \delta \beta'}, \tag{5.2.9}
\]

where

\[
\gamma = \frac{C_p}{C_v}, \quad \delta = \frac{k_p}{1 - k_p} \quad \text{and} \quad \beta' = \frac{C_{sp}}{C_v}.
\]

Now,

\[
C_{pm} - C_{vm} = (1 - k_p)(C_p - C_v) = (1 - k_p)R^*, \tag{5.2.10}
\]

neglecting the terms containing \( b^2 \rho^2 \) (Anisimov and Spiner [20], Vishwakarma and Nath [24]). The internal energy per unit mass of the mixture is, therefore, given by

\[
U_m = \frac{p(1 - Z)}{\rho(\Gamma - 1)[1 + b\rho(1 - k_p)]}. \tag{5.2.11}
\]

For an isentropic change of state of the mixture of non-ideal gas and small solid particles, under the thermodynamic equilibrium condition, we may calculate the equilibrium sound speed of the mixture, as follows

\[
a_{isen} = \left(\frac{dp}{d\rho}\right)_s^{\frac{1}{2}} = \left[\frac{(1 - k_p)(1 + b\rho)}{p(1 - Z)}\right]^\frac{1}{2}, \tag{5.2.12}
\]

neglecting \( b^2 \rho^2 \), where subscript ‘s’ refers to the process of constant entropy.

In addition, the isothermal speed of sound may also play a role, when thermal radiation is taken into account. The isothermal speed of sound in the mixture is

\[
a_{iso} = \left(\frac{dp}{d\rho}\right)_T^{\frac{1}{2}} = \left[\frac{1 + (2 - Z)b\rho(1 - k_p)}{(1 - Z)(1 + b\rho(1 - k_p))\rho}\right]^\frac{1}{2}, \tag{5.2.13}
\]
where subscript ‘\(T\)’ refers to the process of constant temperature.

The fundamental equations for one-dimensional, unsteady, adiabatic and spherically symmetric flow of a mixture of non-ideal gas and small solid particles with heat conduction and radiation heat flux taken into account, in the presence of a gravitational field, may be expressed as (Vishwakarma and Nath [24, 28])

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0,
\]

\(\text{(5.2.14)}\)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{G^* m^*}{r^2} = 0,
\]

\(\text{(5.2.15)}\)

\[
\frac{\partial U_m}{\partial t} + u \frac{\partial U_m}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) + \frac{1}{\rho r^2} \frac{\partial}{\partial r} (r^2 q) = 0,
\]

\(\text{(5.2.16)}\)

where \(r\) and \(t\) are independent space and time coordinates, \(u\) the flow velocity, \(G^*\) the gravitational constant, \(m^*\) the mass of the heavy nucleus at the centre and \(q\) the heat flux. Here it is assumed that the gravitational effect of the medium itself is negligible in comparison with the attraction of the heavy nucleus.

The total heat flux \(q\), which appears in the energy equation, may be decomposed as

\[
q = q_C + q_R,
\]

\(\text{(5.2.17)}\)

where \(q_C\) is the conduction heat flux, and \(q_R\) the radiation heat flux.

According to the Fourier’s law of heat conduction,

\[
q_C = -k \frac{\partial T}{\partial r},
\]

\(\text{(5.2.18)}\)

where \(k\) is the coefficient of thermal conductivity of the gas.

Assuming local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas (Pomraning [30]), the radiative
heat flux $q_R$ may be obtained from the differential approximation of the radiation-transport equation in the diffusion limit as

$$q_R = -\frac{4}{3} \left( \frac{\sigma}{\alpha_R} \right) \frac{\partial T^4}{\partial r},$$

where $\sigma$ is the Stefan-Boltzmann constant and $\alpha_R$ is the Rosseland mean absorption coefficient.

The thermal conductivity $k$ and the absorption coefficient $\alpha_R$ of the medium are assumed to vary with temperature and density. These can be written in the form of power laws, namely (Ghoniem et al [6], Vishwakarma and Nath [28])

$$k = k_0 \left( \frac{T}{T_0} \right)^{\beta_C} \left( \frac{\rho}{\rho_0} \right)^{\delta_C}, \quad \alpha_R = \alpha_{R_0} \left( \frac{T}{T_0} \right)^{\beta_R} \left( \frac{\rho}{\rho_0} \right)^{\delta_R},$$

where subscript ‘0’ denotes a reference state. The exponents in the above equations should satisfy the similarity requirements, if a self-similar solution is sought.

We assume that a spherical shock wave is propagating outwards from the centre of symmetry in the undisturbed medium (mixture of a non-ideal gas and small solid particles) with constant density, under the gravitational force. The flow variables immediately ahead of the shock front are

$$u = 0,$$

$$\rho = \rho_1 = \text{constant},$$

$$p = p_1 = \frac{\rho_1 m^* G^*}{R},$$

$$q = q_1 = 0 \text{ (Laumbach and Probstein [31])},$$
where \( R \) is the shock radius and the subscript ‘1’ denotes the conditions immediately ahead of the shock.

The disturbance is headed by an isothermal shock (the formation of isothermal shock is a result of the mathematical approximation in which the flux is taken to be proportional to the temperature gradient; this excludes the possibility of a temperature jump, see for example Zel’dovich and Raizer [25], Vishwakarma and Nath [28]) and hence, the conditions across it are

\[
\rho_2 (\dot{R} - u_2) = \rho_1 \dot{R},
\]
\[
p_2 + \rho_2 (\dot{R} - u_2)^2 = p_1 + \rho_1 \dot{R}^2,
\]
\[
U_{m_2} + \frac{p_2}{\rho_2} + \frac{1}{2}(\dot{R} - u_2)^2 - \frac{q_2}{\rho_1 \dot{R}} = U_{m_1} + \frac{p_1}{\rho_1} + \frac{1}{2} \dot{R}^2,
\]
\[
Z_2 = \rho_2 \rho_1
\]
\[
T_2 = T_1,
\]

where subscript ‘2’ denotes the conditions immediately behind the shock front and \( \dot{R} = \frac{dR}{dt} \) denotes the velocity of the shock front. From equations (5.2.22), we obtain

\[
u_2 = (1 - \beta) \dot{R},
\]
\[
\rho_2 = \frac{\rho_1}{\beta},
\]
\[
Z_2 = \frac{Z_1}{\beta},
\]
\[
p_2 = \left[ (1 - \beta) + \frac{1}{\gamma M^2} \right] \rho_1 \dot{R}^2,
\]
\[
q_2 = (1 - \beta) \left[ \frac{Z_1 + b(1 - k_p)}{\gamma M^2 (\beta - Z_1) \{1 + b(1 - k_p)\}} - \frac{1}{2} (1 + \beta) \right] \rho_1 \dot{R}^3,
\]
where 

\[ M = \left( \frac{\rho_1 \hat{R}^2}{\gamma p_1} \right)^{\frac{1}{2}} \]

is the shock - Mach number referred to the frozen speed of sound \( \left( \frac{\gamma p_1}{\rho_1} \right)^{\frac{1}{2}} \), and \( \bar{b} = b \rho_1 \) is the parameter of non-idealness of the gas. The quantity \( \beta (0 < \beta < 1) \) is obtained by the relation

\[
\beta^3 - \left[ Z_1 + 1 + \frac{1}{\gamma M^2} \right] \beta^2 + \left[ \frac{Z_1 \bar{b}(1 - k_p)(1 + \gamma M^2) + Z_1 \gamma M^2 + 1}{\gamma M^2 \{1 + b(1 - k_p)\}} \right] \beta \\
+ \frac{(1 - Z_1) \bar{b}(1 - k_p)}{\gamma M^2 \{1 + b(1 - k_p)\}} = 0.
\]  

(5.2.24)

The expression for the initial volume fraction of the solid particles \( Z_1 \) is given by, from equation (5.2.6),

\[
Z_1 = \frac{V_{sp}}{V_1} = \frac{k_p}{(1 - k_p) G_1 + k_p},
\]

(5.2.25)

where \( G_1 = \frac{\rho_{sp}}{\rho_{g1}} \) is the ratio of the species density of solid particles to the initial species density of the gas in the mixture.

### 5.3 SIMILARITY TRANSFORMATIONS

The inner boundary of the flow-field behind the shock is assumed to be an expanding piston. In the framework of self-similarity (Sedov [17]) the velocity \( u_p = \frac{dr_p}{dt} \) of the piston is assumed to obey a power law, which results in (Steiner and Hirschler [18], Vishwakarma and Nath [28])

\[
u_p = \frac{dr_p}{dt} = U_0 \left( \frac{t}{t_0} \right)^n,
\]

(5.3.1)

where \( r_p \) is the radius of the piston, \( t_0 \) denotes a reference time, \( U_0 \) is the piston velocity at \( t = t_0 \) and \( n \) is a constant. The consideration of ambient
Pressure $p_1$ sets a value of $n$ as $n = -\frac{1}{3}$ (see equation (5.3.5)). Thus the piston velocity jumps, almost instantaneously, from zero to infinity leading to the formation of a shock of high strength in the initial phase. The piston is then decelerated. Concerning the shock boundary condition, self-similarity requires that the velocity of the shock $\dot{R} = \frac{dR}{dt}$ is proportional to the velocity of the piston, that is

$$\dot{R} = \frac{dR}{dt} = C U_0 \left( \frac{t}{t_0} \right)^n,$$

(5.3.2)

where $C$ is a constant. Using equation (5.3.2), the time and space coordinate can be changed into a dimensionless similarity variable $\eta$ as

$$\eta = \frac{r}{R} = \left[ \frac{(n+1)t_0^n}{CU_0} \right] \left( \frac{r}{t^{n+1}} \right).$$

(5.3.3)

Evidently, $\eta = \eta_p = \frac{r_p}{R}$ at the piston and $\eta = 1$ at the shock. To obtain the similarity solutions, we write the unknown variables in the following form (Vishwakarma and Nath [13], Steiner and Hirschler [18])

$$u = \frac{r}{t} U(\eta), \quad \rho = \rho_1 D(\eta),$$

$$p = \frac{r^2}{t^2} \rho_1 P(\eta), \quad Z = Z_1 D(\eta), \quad q = \frac{r^3}{t^3} \rho_1 Q(\eta),$$

(5.3.4)

where $U$, $D$, $P$, and $Q$ are functions of $\eta$ only.

For existence of similarity solutions, $M$ should be a constant, therefore

$$n = -\frac{1}{3}.$$

(5.3.5)

Thus

$$M^2 = \frac{(n + 1)^{\frac{2n}{n+1}}}{\gamma G_0} = \text{constant},$$

(5.3.6)
where \( G_0 = m^* G^* \left( \frac{t_0^n}{CU_0} \right)^3 \) is the gravitational parameter. Equation (5.3.6) shows that the solutions of the present problem can be reduced to the non-gravitational case (i.e. the case in which \( G_0 = 0 \)) only when the shock is infinitely strong. In this case the initial pressure is neglected and the shock Mach-number \( M \to \infty \).

Using the similarity transformations (5.3.4), the equations of motion (5.2.14)-(5.2.16) are transformed into

\[
[U - (n+1)] \frac{dD}{d\eta} + D \frac{dU}{d\eta} + \frac{3DU}{\eta} = 0, 
\]

\[
[U - (n+1)] \frac{dU}{d\eta} + \frac{U(U-1)}{\eta} + \frac{2P}{D} \frac{dP}{d\eta} + \frac{1}{\eta^4} \frac{G_0(n+1)\eta^{\frac{n+1}{2}}}{\eta^4} = 0, 
\]

\[
\frac{dP}{d\eta} + \frac{2(U-1)P}{[U-(n+1)]\eta} + J \frac{dD}{d\eta} + \left[ (\Gamma - 1)\{1 + \bar{b}D(1 - k_p)\} \left( 5Q + \eta \frac{dQ}{d\eta} \right) \right] = 0, 
\]

where

\[
J = \frac{P[-(\Gamma - 1)\{1 + \bar{b}(1 - k_p)D\}^2 - 1 + \bar{b}D(1 - k_p)(Z_1 D - 2)]}{D(1 - Z_1 D)(1 + b(1 - k_p)D)}. 
\]

By using equations (5.2.18)-(5.2.20) in equation (5.2.17), we obtain

\[
q = - \left[ \frac{k_0}{T_0 \beta c \rho_0 \delta c} T^{\beta c} \rho^{\delta c} + \frac{16\sigma T_0^{\beta c} \rho_0 \delta R}{3\delta R_0} T^{3-\beta R} \rho^{-\delta R} \right] \frac{\partial T}{\partial r}. 
\]
Using equations (5.2.3) and (5.3.4) in equation (5.3.10), we obtain

\[ Q = - \left[ \frac{k_0 \left( \frac{C U_0}{T_0} \right)^{\frac{1}{n}} \rho_1^{\delta_C - 1} \eta^{2\beta_C - 1} \dot{R}^{2\beta_C - 2 - \frac{1}{n}} P^{\beta_C} (1 - Z_1 D)^{\beta_C} D^{\delta_C - \beta_C}}{T_0^{\beta_C} \rho_0^{\delta_C} (1 - k_p)^{1 + \beta_C} R_0^{1 + \beta_C} (n + 1)^{2\beta_C - 2} \left\{ 1 + \bar{b}(1 - k_p) D \right\}^{\beta_C}} \right. 
\]

\[ + \frac{16 \sigma T_0^{\beta_R} \rho_0^{\delta_R} \rho_1^{\delta_R - 1} \left( \frac{C U_0}{T_0} \right)^{\frac{1}{n}} \eta^{5 - 2\beta_R} \dot{R}^{4 - 2\beta_R - \frac{1}{n}} P^{3 - \beta_R} (1 - Z_1 D)^{3 - \beta_R} D^{\delta_R - \delta_R - 3}}{3 \alpha_R (1 - k_p)^{4 - \beta_R} R_0^{4 - \beta_R} (n + 1)^{4 - 2\beta_R} \left\{ 1 + \bar{b}(1 - k_p) D \right\}^{3 - \beta_R}} 
\]

\[ \times \left\{ \frac{\eta P(1 - Z_1 D)}{\eta D \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} + \frac{(1 - Z_1 D)}{D \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} \frac{dP}{d\eta} \right\} 
\]

\[ \left\{ D + \bar{b}(1 - k_p) D^2 \right\}^2 \frac{dD}{d\eta} \right] \left\{ \frac{PZ \{ D + \bar{b}D^2(1 - k_p) \} + \left( 1 + 2\bar{b}(1 - k_p) D \right) P(1 - Z_1 D) \frac{dD}{d\eta}}{D^2 \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} \right\} 
\]

Equation (5.3.11) shows that the similarity solution of the present problem exists only when

\[ \beta_C = 1 + \frac{1}{2n} \quad \text{and} \quad \beta_R = 2 - \frac{1}{2n}. \]

Therefore equation (5.3.11) becomes

\[ Q = - X \left[ \frac{2P(1 - Z_1 D)}{\eta D \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} + \frac{(1 - Z_1 D)}{D \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} \frac{dP}{d\eta} - \right. 
\]

\[ \left. \frac{P \left\{ 1 + \bar{b}(1 - k_p) D(2 - Z_1 D) \right\}}{D^2 \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} \frac{dD}{d\eta} \right] \]

Equation (5.3.11) becomes

\[ Q = - X \left[ \frac{2P(1 - Z_1 D)}{\eta D \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} + \frac{(1 - Z_1 D)}{D \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} \frac{dP}{d\eta} - \right. 
\]

\[ \left. \frac{P \left\{ 1 + \bar{b}(1 - k_p) D(2 - Z_1 D) \right\}}{D^2 \left\{ 1 + \bar{b}(1 - k_p) D \right\}^2} \frac{dD}{d\eta} \right] \]

\[ (5.3.12) \]

where

\[ X = (n + 1)^{-\frac{1}{n}} \left[ \Gamma_C D^{\delta_C - 1 - \frac{1}{2n}} + \Gamma_R D^{-\delta_R - 1 - \frac{1}{2n}} \right] (1 - k_p)^{-\left( 2 + \frac{1}{2n} \right) \eta(1 + \frac{1}{n})} \left[ \frac{P(1 - Z_1 D)}{1 + \bar{b}(1 - k_p) D} \right]^{1 + \frac{1}{2n}} \]

and \( \Gamma_C \) and \( \Gamma_R \) are the conductive and radiative non-dimensional heat transfer parameters, respectively. The parameters \( \Gamma_C \) and \( \Gamma_R \) depend on the
thermal conductivity $k$ and the mean free path of radiation $\frac{1}{\alpha_R}$, respectively, and also on the exponent $n$, and they are given by

$$
\Gamma_C = \frac{k_0 \rho_0 \delta_C R^{2}}{t_0 T_0^{\frac{1}{2}} \left( \frac{C U_0}{\sqrt{R^{2}T_0}} \right)^{\frac{1}{2}}} \quad \text{and} \quad \Gamma_R = \frac{16 \sigma_0 \delta_R T_0^{2}}{3 \alpha_R t_0 R^{2} \rho_1^{\delta_R+1}} \left( \frac{C U_0}{\sqrt{R^{2}T_0}} \right)^{\frac{1}{2}}.
$$

By solving equations (5.3.7)-(5.3.9) and (5.3.12) for $\frac{dU}{d\eta}$, $\frac{dP}{d\eta}$, $\frac{dQ}{d\eta}$, and $\frac{dD}{d\eta}$, we have

$$
\frac{dU}{d\eta} = - \left[ \frac{U - (n + 1)}{D} \frac{dD}{d\eta} + \frac{3U}{\eta} \right],
$$

(5.3.13)

$$
\frac{dP}{d\eta} = \{U - (n + 1)\}^2 \frac{dD}{d\eta} + \frac{UD}{\eta} (2U - 3n - 2) - \frac{2P}{\eta} - \frac{DG_0(n + 1)^{\frac{2}{\eta+1}}}{\eta^{4}},
$$

(5.3.14)

$$
\frac{dQ}{d\eta} = - \frac{[U - (n + 1)](1 - z_1 D)}{(\Gamma - 1)[1 + bD(1 - k_p)]} \left[ \{U - (n + 1)\}^2 \frac{dD}{d\eta} + \frac{UD}{\eta} (2U - 3n - 2)
\right.

- \frac{2P}{\eta} - \frac{DG_0(n + 1)^{\frac{2}{\eta+1}}}{\eta^{4}}
\left. + \frac{(\Gamma - 1)[1 + bD(1 - k_p)]5Q}{[U - (n + 1)]\eta(1 - Z_1 D)} \right].
$$

(5.3.15)

$$
\frac{dD}{d\eta} = \frac{D^2 \{1 + \tilde{b}(1 - k_p)D\}^2}{P\{1 + bD(1 - k_p)(2 - Z_1 D)\} - (1 - Z_1 D)[U - (n + 1)]^2 D\{1 + bD(1 - k_p)\}^2}
\times

\left[ \frac{Q}{X} + \frac{2P(1 - Z_1 D)}{\eta D\{1 + bD(1 - k_p)\}} + \frac{(1 - Z_1 D)}{D\{1 + bD(1 - k_p)\}} \times \left\{ \frac{UD}{\eta} (2U - 3n - 2) - \frac{2P}{\eta} - \frac{DG_0(n + 1)^{\frac{2}{\eta+1}}}{\eta^{4}} \right\} \right].
$$

(5.3.16)

Using the self-similarity transformations (5.3.4) and equation (5.3.2), equations (5.2.23) can be rewritten as

$$
U(1) = (n + 1)(1 - \beta),
$$

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\[ D(1) = \frac{1}{\beta}, \]
\[ P(1) = (n + 1)^2 \left[ (1 - \beta) + \frac{1}{\gamma M^2} \right], \quad (5.3.17) \]
\[ Q(1) = (n + 1)^3(1 - \beta) \left[ \frac{Z_1 + \beta(1 - k_p)}{\gamma M^2(\beta - Z_1)} \left\{ 1 + b(1 - k_p) \right\} - \frac{1}{2}(1 + \beta) \right]. \]

The piston path coincides at \( \eta_p = \frac{r_p}{R} \) with a particle path. Using equations (5.3.1) and (5.3.4), the relation
\[ U(\eta_p) = n + 1 \quad (5.3.18) \]
can be derived. In addition to shock conditions (5.3.17), the kinematic condition (5.3.18) at the piston surface must be satisfied.

The adiabatic compressibility of the mixture of non-ideal gas and small solid particles may be calculated as (c.f. Moelwyn-Hughes [32])
\[ C_{\text{adi}} = \frac{1}{\rho} \left( \frac{dp}{dp} \right)_s = \frac{1}{\rho a^2} = \frac{(1 - Z)[1 + b\rho(1 - k_p)]}{[\Gamma + (2\Gamma - Z)b\rho(1 - k_p)]\rho}. \quad (5.3.19) \]
By using equations (5.3.4) in (5.3.19), we get the expression for adiabatic compressibility in the non-dimensional form as
\[ (C_{\text{adi}})_{p_1} = \frac{(1 - Z_1 D)[1 + \beta D(1 - k_p)](n + 1)^2}{[\Gamma + (2\Gamma - Z_1 D)bD(1 - k_p)]\eta^2 \gamma M^2 P}. \quad (5.3.20) \]

Using equations (5.3.4) in (5.2.13), we get the expression for reduced isothermal speed of sound as
\[ a_{\text{iso}} = \frac{1}{R} \left[ \frac{(1 + (2 - Z_1 D)\beta D(1 - k_p))P}{(1 - Z_1 D)D\{1 + \beta D(1 - k_p)\}} \right]^{\frac{1}{2}} \frac{\eta}{(n + 1)}. \quad (5.3.21) \]
Normalizing the variables \( u, p, \rho \) and \( q \) with their respective values at the shock, we obtain
\[ \frac{u}{u_2} = \frac{U(\eta)}{U(1)^{\eta}}, \quad \frac{p}{p_2} = \frac{P(\eta)}{P(1)^{\eta^2}}, \]

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\[ \frac{\rho}{\rho_2} = \frac{D(\eta)}{D(1)}, \quad \frac{q}{q_2} = \frac{Q(\eta)}{Q(1)} \eta^3. \]  

(5.3.22)

The ordinary differential equations (5.3.13) to (5.3.16) with the boundary conditions (5.3.17) can now be numerically integrated to obtain the solution for the flow behind the shock surface.

5.4 RESULTS AND DISCUSSION

The distribution of the flow variables between the shock front \((\eta = 1)\) and the piston \((\eta = \eta_p)\) is obtained by the numerical integration of equations (5.3.13) to (5.3.16) with the boundary conditions (5.3.17) by the Runge-Kutta method of fourth order. For the purpose of numerical integration, the values of the constant parameters are taken to be (Pai et al [8], Vishwakarma and Nath [13, 24, 33]) \(\gamma = 1.4; \beta' = 1; \quad k_p = 0, 0.1, 0.3; \quad G_1 = 10, 100; \quad \bar{b} = 0, 0.05, 0.1; \quad G_0 = 0.01, 0.05, 0.1; \quad n = -\frac{1}{3}; \quad \delta_C = 1; \quad \delta_R = 2; \quad \Gamma_C = 0.1; \quad \Gamma_R = 10.\) The values \(\gamma = 1.4; \beta' = 1,\) may correspond to the mixture of air and glass particles (Miura and Glass, [15]). The value \(k_p = 0\) corresponds to the dust-free case and \(k_p = 0, \bar{b} = 0\) to the perfect gas case. Also, \(n = -\frac{1}{3}\) corresponds to a decelerated piston. The set of values \(\delta_C = 1, \delta_R = 2\) is the representative of the case of a high temperature, low-density medium (Ghoniem et al [6]). Also, the set of values \(\Gamma_C = 0.1, \Gamma_R = 10\) is the representative of the case in which there is heat transfer by both conduction and radiative diffusion.

Table 1 shows the variation of the density ratio \(\beta\) across the shock front and the position of the piston \(\eta_p\) for different values of \(k_p, G_1\) and \(\bar{b}\) with
\( \beta' = 1, \gamma = 1.4, n = -\frac{1}{3}, G_0 = 0.05, \delta_C = 1, \delta_R = 2, \Gamma_C = 0.1 \text{ and } \Gamma_R = 10. \)

Also, table 2 shows the variation of the density ratio \( \beta \) across the shock front and the position of the piston \( \eta_p \) for different values of \( G_0 \) with \( \beta' = 1, \gamma = 1.4, n = -\frac{1}{3}, k_p = 0.1, G_1 = 10, \bar{b} = 0.05, \delta_C = 1, \delta_R = 2, \Gamma_C = 0.1 \text{ and } \Gamma_R = 10. \)

Figs. 1-12 show the variation of the flow variables \( \frac{u}{u_2}, \frac{\rho}{\rho_2}, \frac{p}{p_2}, \frac{q}{q_2}, \frac{\dot{a}_{iso}}{R} \) and the adiabatic compressibility \( (C_{ad})p_1 \) with \( \eta \) at various values of the parameters \( k_p, G_1, \bar{b}; \text{ and } G_0. \) It is shown that, as we move inward from the shock front towards the inner contact surface (piston), the reduced fluid velocity \( \frac{u}{u_2} \), the reduced total heat flux \( \frac{q}{q_2} \) and the adiabatic compressibility \( (C_{ad})p_1 \) increase whereas the reduced density \( \frac{\rho}{\rho_2} \), the reduced pressure \( \frac{p}{p_2} \) and the isothermal speed of sound \( \frac{\dot{a}_{iso}}{R} \) decrease.
Table 1. Variation of the density ratio $\beta(\rho_1/\rho_2)$ across the shock front and the position of the piston surface $\eta_p$ for different values of $k_p$, $G_1$ and $\bar{b}$ with $\beta' = 1$, $G_0 = 0.05$, $n = -\frac{1}{3}$, $\gamma = 1.4$, $\delta_c = 1$, $\delta_R = 2$, $\Gamma_C = 0.1$, $\Gamma_R = 10$

<table>
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<th>$Z_1$</th>
<th>$\bar{b}$</th>
<th>$\beta$</th>
<th>$\eta_p$</th>
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Table 2. Variation of the density ratio $\beta \left(\frac{\rho_1}{\rho_2}\right)$ across the shock front and the position of the piston surface $\eta_p$ for different values of $G_0$ with $k_p = 0.1$, $G_1 = 10$, $\bar{b} = 0.05$, $\beta' = 1$, $n = -\frac{1}{3}$, $\gamma = 1.4$, $\delta_C = 1$, $\delta_R = 2$, $\Gamma_C = 0.1$, $\Gamma_R = 10$

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It is found that the effects of an increase in the value of the parameter of non-idealness $\bar{b}$ of the gas are:

(i) to increase the value of $\beta$ (i.e. to decrease the shock strength, see Table 1);

(ii) to increase the distance of the piston $(1 - \eta_p)$ from the shock front (see Table 1), i.e. the flow-field behind the shock becomes somewhat rarefied. This shows the same result as in (i), i.e. there is a decrease in the shock strength;

(iii) to increase the flow variables $\frac{\dot{u}}{u_2}$, $\frac{\dot{\rho}}{\rho_2}$, $\frac{\dot{p}}{p_2}$ and isothermal speed of sound $\frac{\dot{a}_R}{R}$ at any point in the flow-field behind the shock front (see Figs. 1-3 and 5);

(iv) to decrease the total heat flux $\frac{\dot{q}}{q_2}$ and the adiabatic compressibility $(C_{adi})p_1$ (see Figs. 4 and 6).

The effects of an increase in the ratio of the density of solid particles to the initial density of gas $G_1$ are:
Fig. 1. Variation of reduced density $\frac{\rho}{\rho_2}$ in the flow-field behind the shock front for different values of $k_0$, $\gamma$, and $\delta$. 

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### Table

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### Figure 2

Variation of reduced pressure $\frac{p}{p_2}$ in the flow-field behind the shock front for different values of $k_p$, $c_i$, and $\hat{v}$.

- **Dust free case**
- **Mixture of a perfect gas and small solid particles**
- **Mixture of a non-ideal gas and small solid particles**
<table>
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<th>$k_0$</th>
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<th>$\tilde{b}$</th>
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Fig. 3. Variation of reduced velocity $\frac{u}{u_2}$ in the flow-field behind the shock front for different values of $k_0, c_i$ and $\tilde{b}$.
Fig. 4. Variation of reduced total heat flux $\frac{q}{q_2}$ in the flow-field behind the shock front for different values of $k_0$, $\sigma$, and $\bar{b}$.
Fig. 5. Variation of reduced isothermal speed of sound $\frac{a_0}{R}$ in the flow-field behind the shock front for different values of $k_0, G_j$, and $b$.  

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Fig. 6. Variation of adiabatic compressibility $(C_{\text{ad}})\rho_1$ in the flow-field behind the shock front for different values of $k_p$, $c_i$, and $\beta$.
Fig. 7. Variation of reduced density $\frac{\rho}{\rho_2}$ in the flow-field behind the shock front for different values of $G_0$.
Fig. 8. Variation of reduced pressure $\frac{p}{p_2}$ in the flow-field behind the shock front for different values of $G_0$.
Fig. 9: Variation of reduced velocity $\frac{u}{u_s}$ in the flow-field behind the shock front for different values of $G_o$. 

1. $G_o = 0.01$
2. $G_o = 0.05$
3. $G_o = 0.1$
Fig. 10. Variation of reduced total heat-flux $\frac{q}{q_2}$ in the flow-field behind the shock front for different values of $G_0$. 

1. $G_0 = 0.01$
2. $G_0 = 0.05$
3. $G_0 = 0.1$
Fig. 11. Variation of reduced isothermal speed of sound $\frac{a_{\infty}}{R}$ in the flow-field behind the shock front for different values of $G_0$. 

1. $G_0 = 0.01$
2. $G_0 = 0.05$
3. $G_0 = 0.1$
Fig. 12. Variation of adiabatic compressibility \((C_{ad})_1\) in the flow-field behind the shock front for different values of \(G_0\).
(i) to decrease \( \beta \) (i.e. to increase the shock strength, see Table 1);

(ii) to decrease the distance between the piston and the shock front (see Table 1). This means that an increase in the ratio of the density of solid particles to the initial density of the gas has an effect of increasing the shock strength, which is same as indicated in (i) above;

(iii) to decrease the flow variables \( \frac{u}{u_2}, \frac{\rho}{\rho_2}, \frac{p}{p_2} \) and the isothermal speed of sound \( \frac{a_{iso}}{R} \) but to increase the total heat flux \( \frac{q}{q_2} \) and the adiabatic compressibility \( (C_{adi})p_1 \) (see Figs. 1-6).

The above effects are more impressive at higher values of \( k_p \). These effects may be physically interpreted as follows:

By an increase in \( G_1 \) (at constant \( k_p \)), there is a high decrease in \( Z_1 \), i.e. the volume fraction of solid particles in the undisturbed medium becomes, comparatively, very small. This causes comparatively more compression of the mixture in the region between the shock and the piston, which displays the above effects.

The effects of an increase in the mass concentration of the solid particles \( k_p \) are:

(i) to increase the value of \( \beta \) (i.e. to decrease the shock strength) when \( G_1 = 10 \) and to decrease it, when \( G_1 = 100, \bar{b} \neq 0 \) (see Table 1);

(ii) to increase the distance of the piston from the shock front, when \( G_1 = 10 \), the effect is small and of opposite nature, when \( G_1 = 100, \bar{b} \neq 0 \) (see Table 1);
(iii) to increase the density $\frac{\rho}{p^2}$, pressure $\frac{P}{p^2}$ and isothermal speed of sound $u_{iso}$, when $G_1 = 10$ and to decrease it, when $G_1 = 100, \bar{b} \neq 0$. The velocity $\frac{u}{u_2}$ is almost unchanged for $G_1 = 10$ or $100$ (see Figs. 1-3 and 5);

(iv) to increase the total heat flux $\frac{q}{q^2}$ in general (see Fig. 4);

(v) to decrease the adiabatic compressibility $(C_{adi})_1$, when $G_1 = 10$ and to increase it when $G_1 = 100, \bar{b} \neq 0$ (see Fig. 6).

Physical interpretations of these effects are as follows:

In case of $G_1 = 10$, small solid particles of density equal to ten times that of the perfect gas in the mixture occupy a significant portion of the volume which lowers the compressibility of the medium remarkably. Then an increase in $k_p$ further reduces the compressibility which causes an increase in the distance between the shock front and the piston, a decrease in the shock strength, and the above behaviour of the flow variables. In case of $G_1 = 100$, small solid particles of density equal to hundred times that of the perfect gas in the mixture occupy a very small portion of the volume, and therefore compressibility is not lowered much; but the inertia of the mixture is increased significantly due to the particle load. An increase in $k_p$ from 0.1 to 0.3 for $G_1 = 100$, means that the perfect gas in the mixture constituting 90% of the total mass and occupying 99.88% of the total volume now constitutes 70% of the total mass and occupies 99.57% of the total volume. Due to this fact, the density of the non-ideal gas in the mixture is highly decreased which overcomes the effect of incompressibility of the mixture and ultimately causes
a small decrease in the distance between the inner expanding surface (piston) and the shock front, an increase in the shock strength, and the above nature of the flow variables.

The effects of an increase in the value of the gravitational parameter $G_0$ are:

(i) to increase the value of $\beta$ (i.e. to decrease the shock strength; see Table 2);

(ii) to increase the distance of the piston from the shock front (see Table 2);

(iii) to increase the flow variables $\frac{\rho}{\rho_2}$, $\frac{p}{p_2}$, $\frac{u}{u_2}$, $\frac{q}{q_2}$, the isothermal speed of sound $\frac{a_{iso}}{R}$ and the adiabatic compressibility $(C_{ad})p_1$ at any point in the flow field behind the shock (see Figs. 7-12).
REFERENCES


