Chapter 2

PROPAGATION OF A CYLINDRICAL SHOCK WAVE GENERATED BY A MOVING PISTON IN A ROTATIONAL AXISYMMETRIC NON-IDEAL GAS
2.1 INTRODUCTION

The formulation of self-similar problems and examples describing the adiabatic motion of non-rotating gas models of stars, are considered by Sedov [1], Zel’dovich and Raizer [2], Lee and Chen [3] and Summers [4]. The experimental studies and astrophysical observations show that the outer atmosphere of the planets rotates due to rotation of the planets. Macroscopic motion with supersonic speed occurs in an interplanetary atmosphere and shock waves are generated. Thus the rotation of planets or stars significantly affects the process taking place in their outer layers. Therefore, questions connected with the explosions in rotating gas atmospheres are of definite astrophysical interest. Chaturani [5] studied the propagation of cylindrical shock waves through a gas having solid body rotation and obtained the solutions by a similarity method adopted by Sakurai [6]. Nath et al. [7] obtained the similarity solutions for the flow behind spherical shock waves propagating in a non-uniform rotating interplanetary atmosphere with increasing energy. Vishwakarma and Vishwakarma [8] and Vishwakarma et al. [9] obtained the similarity solutions for magnetogasdynamic cylindrical shock waves propagating in a rotating medium which is a perfect gas with variable density or a non-ideal gas with constant density. In all of the works, mentioned above, the ambient medium is supposed to have only one component of velocity that is azimuthal component.

At extreme conditions that prevail in most of the problems associated with shock waves, the assumption that the gas is ideal is no longer valid. The popular alternative to the ideal gas is a simplified van der Waals model.
Roberts and Wu [10, 11] studied the problem of a spherical implosion by assuming that the gas obeys a simplified van der Waals equation of state. Nath [12] obtained similarity solutions for the flow of a non-ideal gas behind a cylindrical (or spherical) shock wave, in the presence of a spatially decreasing azimuthal magnetic field, driven out by a piston moving with time according to power law by taking the equation of state used by Roberts and Wu [10, 11]. Vishwakarma et al. [13] investigated one-dimensional unsteady self-similar flow behind a strong shock driven out by a cylindrical or (spherical) piston moving with time according to power law in a medium which is assumed to be a non-ideal gas obeying a simplified van der Waals equation of state (Roberts and Wu [10, 11]).

In the present work, we obtained the self-similar solutions for the flow behind a cylindrical shock wave generated by a piston moving with time according to power law in a rotational axisymmetric non-ideal gas, which has a variable azimuthal fluid velocity together with a variable axial velocity (Levin and Skopina [14], Nath [15, 16]). The shock-Mach number is not infinite, but has a finite value. The fluid velocities in the ambient medium are assumed to obey power laws and the density of the ambient medium is assumed to be constant. Also, the angular velocity of rotation of the ambient medium is assumed to be obeying a power law and to be decreasing as the distance from the axis increases. It is expected that such an angular velocity may occur in the atmospheres of rotating planets and stars. The medium is assumed to be a non-ideal gas obeying a simplified van der Waals equation of state (Roberts and Wu [10, 11]). Effects of viscosity and heat conduction are not taken into account. Effects of a change in the parameter of non-idealness
of the gas and in the index for variation of piston velocity (or in the index for variation of initial velocity of the medium) are investigated.

2.2 BASIC EQUATIONS AND BOUNDARY CONDITIONS

The fundamental equations governing an unsteady, adiabatic axisymmetric rotational flow of a non-ideal gas in which heat conduction and viscous stress are negligible may, in Eulerian co-ordinates, be expressed as (Levin and Skopina [14], Nath [15, 16], Vishwakarma and Nath [17, 18])

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{\rho u}{r} &= 0 , \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{v^2}{r} &= 0 , \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{w}{r} &= 0 , \\
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} &= 0 , \\
\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) &= 0 ,
\end{align*}
\]

(2.2.1) - (2.2.5)

where \( r \) and \( t \) are independent space and time co-ordinates; \( u, v \) and \( w \) are the radial, azimuthal and axial components of the fluid velocity \( \vec{q} \) in the cylindrical co-ordinates \( (r, \theta, z) \); \( \rho, p \) and \( e \) are the density, the pressure and the internal energy per unit mass of the gas. Also,

\[
v = Ar ,
\]

(2.2.6)
where ‘\(A\)’ is the angular velocity of the medium at a radial distance \(r\) from the axis of symmetry.

In this case the vorticity vector \(\vec{\zeta} = \frac{1}{2} \text{curl} \vec{q}\), has the components

\[
\zeta_r = 0, \quad \zeta_\theta = -\frac{1}{2} \left( \frac{\partial \omega}{\partial r} \right), \quad \zeta_z = \frac{1}{2r} \frac{\partial}{\partial r} (rv). \quad (2.2.7)
\]

The above system of equations should be supplemented with an equation of state. We assume that the gas obeys a simplified van der Waals equation of state of the form (Vishwakarma et al.[9], Roberts and Wu [10, 11], Nath [12], Vishwakarma et al.[13])

\[
p = \frac{\Gamma \rho T}{1 - b_1 \rho}, \quad e = C_v T = \frac{p(1 - b_1 \rho)}{\rho (\gamma - 1)}, \quad (2.2.8)
\]

where \(\Gamma\) is the gas constant, \(C_v = \frac{\Gamma}{\gamma - 1}\) is the specific heat at constant volume and \(\gamma\) is the ratio of specific heats. The constant \(b_1\) is the ‘van der Waals excluded volume’; it places a limit, \(\rho_{\text{max}} = \frac{1}{b_1}\), on the density of the gas.

We assume that a cylindrical shock wave is propagated outwards from the axis of symmetry in the undisturbed medium with constant density, which has zero radial velocity, a variable azimuthal velocity and a variable axial velocity. The flow variables immediately ahead of the shock front are

\[
u = u_1 = 0, \quad (2.2.9)\\
\rho = \rho_1 = \text{constant}, \quad (2.2.10)\\
v = v_1 = v_0 R^{\lambda}, \quad (2.2.11)\\
w = w_1 = w_0 R^{\mu}, \quad (2.2.12)
\]
where $v_0$, $w_0$, $\lambda$ and $\mu$ are constants, $R$ is the shock radius, and the subscript ‘1’ refers to the conditions immediately ahead of the shock.

The momentum equation (2.2.2) in the undisturbed state of the gas, gives

$$p_1 = \frac{\rho_1 v_0^2 R^{2\lambda}}{2\lambda}, \quad \lambda > 0.$$  \hspace{1cm} (2.2.13)

Ahead of the shock, the components of the vorticity vector, therefore vary as

$$\zeta_{r1} = 0,$$  \hspace{1cm} (2.2.14)

$$\zeta_{\theta 1} = -\frac{1}{2} \mu w_0 R^{\mu-1},$$  \hspace{1cm} (2.2.15)

$$\zeta_{z1} = \frac{1}{2} v_0 (\lambda + 1) R^{\lambda-1}.$$  \hspace{1cm} (2.2.16)

From equations (2.2.6) and (2.2.11), we find that the initial angular velocity varies as

$$A_1 = v_0 R^{\lambda-1},$$  \hspace{1cm} (2.2.17)

it decreases as the distance from the axis increases, if $\lambda - 1 < 0$ i.e. $\lambda < 1$.

The jump conditions at the shock wave are given by the principle of conservation of mass, momentum and energy across the shock, namely

$$\rho_2 (V - u_2) = \rho_1 V,$$  \hspace{1cm} (2.2.18a)

$$p_2 + \rho_2 (V - u_2)^2 = p_1 + \rho_1 V^2,$$  \hspace{1cm} (2.2.18b)

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} (V - u_2)^2 = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2} V^2,$$  \hspace{1cm} (2.2.18c)

$$v_2 = v_2,$$  \hspace{1cm} (2.2.18d)

$$w_2 = w_1,$$  \hspace{1cm} (2.2.18e)
where the subscript ‘2’ denotes the conditions immediately behind the shock front and \( V \left( = \frac{dR}{dt} \right) \) denotes the velocity of the shock front.

From equations (2.2.18), we obtain

\[
\begin{align*}
    u_2 &= (1 - \beta)V, \quad (2.2.19a) \\
    \rho_2 &= \frac{\rho_1}{\beta}, \quad (2.2.19b) \\
    p_2 &= \left[ \frac{1}{\gamma M^2} + (1 - \beta) \right] \rho_1 V^2, \quad (2.2.19c) \\
    v_2 &= v_1, \quad (2.2.19d) \\
    w_2 &= w_1, \quad (2.2.19e)
\end{align*}
\]

where \( M = \left( \frac{\rho_1 V^2}{\gamma p_1} \right)^{\frac{1}{2}} \) is the shock-Mach number referred to the frozen speed of sound \( \left( \frac{\gamma p_1}{\rho_1} \right)^{\frac{1}{2}} \). The quantity \( \beta (0 < \beta < 1) \) is obtained by the relation

\[
    \beta = \frac{(\gamma - 1) + 2\bar{b} + 2M^{-2}}{\gamma + 1}, \quad (2.2.20)
\]

where \( \bar{b} = b_1 \rho_1 \) is the non-idealness parameter.

Following Levin and Skopina [14] and Nath [15,16], we obtain the jump conditions for the components of vorticity vector across the shock front as

\[
\begin{align*}
    \zeta_\theta_2 &= \frac{\zeta_\theta_1}{\beta}, \quad (2.2.21) \\
    \zeta_z_2 &= \frac{\zeta_z_1}{\beta}. \quad (2.2.22)
\end{align*}
\]
2.3 SELF-SIMILARITY TRANSFORMATIONS

The inner boundary of the flow-field behind the shock is assumed to be an expanding piston. In framework of self-similarity Sedov [1] the velocity \( u_p = \frac{dr_p}{dt} \) of the piston is assumed to obey a power law which results in (Vishwakarma and Nath [12, 16], Nath [17, 18], Steiner and Hirschler [19])

\[
\frac{dr_p}{dt} = U_0 \left( \frac{t}{t_0} \right)^n,
\]

where \( r_p \) is the radius of the piston and \( t_0 \) denotes the time at a reference state, \( U_0 \) is the piston velocity at \( t = t_0 \) and \( n \) is a constant. The consideration of ambient pressure and ambient angular velocity imposes a restriction on \( n \) as \( 0 < n < \infty \) (see equations (2.3.5) and (2.3.6)). Thus the piston is continuously accelerated, and a shock is formed which arrives at the strong shock limit at large times. Concerning the shock boundary conditions, self-similarity requires that the velocity of the shock \( V = \frac{dR}{dt} \) is proportional to the velocity of the piston, that is

\[
V = \frac{dR}{dt} = CU_0 \left( \frac{t}{t_0} \right)^n,
\]

where \( C \) is a constant.

Using equation (2.3.2), the time and space co-ordinate can be changed into a dimensionless self-similarity variable \( \eta \) as

\[
\eta = \frac{r}{R} = \left[ (n+1) t_0 \right] \left( \frac{r}{U_0} \right)^{\frac{n}{n+1}}.
\]

Evidently, \( \eta = \eta_p = \frac{r_p}{R} \) at the piston and \( \eta = 1 \) at the shock front. To obtain the similarity solutions, we write the unknown variables in the following form (Nath [12, 16], Vishwakarma and Nath [17], Steiner and Hirschler
\[
\begin{align*}
\frac{r}{t} U(\eta), \quad \rho = \rho_1 D(\eta), \quad v = \frac{r}{t} \phi(\eta), \quad w = \frac{r}{t} W(\eta), \quad p = \frac{r^2}{t^2} \rho_1 P(\eta),
\end{align*}
\]
where \( U, D, \phi, W \) and \( P \) are functions of \( \eta \) only.

For existence of similarity solutions, \( M \) should be constant, therefore

\[
\lambda = \frac{n}{n + 1}.
\]

Thus,

\[
M^2 = \frac{2n(n + 1) \pi^{n+1}}{\gamma v_0^2} \left[ \frac{CU_0}{t_0^n} \right]^{\frac{2}{n+1}} = \text{constant},
\]
where \( 0 < n < \infty \). Equation (2.3.6) shows that the solutions of the present problem cannot be reduced to the case in which the ambient medium is non-rotating (i.e. the case in which \( v_0 = 0 \)).

Also, the total energy of the disturbance is given by

\[
E = 2\pi \int_{r_p}^R \rho \left[ e + \frac{1}{2} (u^2 + v^2 + w^2) \right] r dr.
\]

Applying the similarity transformations (2.3.4) in (2.3.7), we obtain

\[
E = 2\pi \rho_1 R^{2(n+1)/(n+1)} \left[ \frac{CU_0}{t_0^n(n + 1)} \right]^{\frac{2}{n+1}} \int_{\eta_p}^1 \left[ \frac{P(1 - \bar{b}D)}{(\gamma - 1)} + \frac{1}{2} D(U^2 + V^2 + W^2) \right] \eta^3 d\eta.
\]

Hence, the total energy of the shock wave is non-constant and varies as \( R^{2(2n+1)/(n+1)} \). The increase of total energy may be achieved by the pressure exerted on the fluid by the inner expanding surface (a contact surface or a piston).
By use of equations (2.3.4), equations (2.2.1) to (2.2.5) can be transformed and simplified to

\[
[U - (n + 1)] \frac{dD}{d\eta} + D \frac{dU}{d\eta} + \frac{2DU}{\eta} = 0, \tag{2.3.9}
\]

\[
[U - (n + 1)] \frac{dU}{d\eta} + \frac{U(U - 1)}{\eta} + \frac{1}{D} \frac{dP}{d\eta} + \frac{2P}{D} - \frac{\phi^2}{\eta} = 0, \tag{2.3.10}
\]

\[
[U - (n + 1)] \frac{d\phi}{d\eta} + \frac{(2U - 1)\phi}{\eta} = 0, \tag{2.3.11}
\]

\[
[U - (n + 1)] \frac{dW}{d\eta} + \frac{(U - 1)W}{\eta} = 0, \tag{2.3.12}
\]

\[
\frac{dP}{d\eta} - \frac{\gamma P}{D} \frac{dD}{d\eta} + \frac{2P(U - 1)}{[U - (n + 1)]\eta} = 0. \tag{2.3.13}
\]

Solving the equations (2.3.9) to (2.3.13) for \( \frac{dU}{d\eta}, \frac{d\phi}{d\eta}, \frac{dW}{d\eta}, \frac{dP}{d\eta} \) and \( \frac{dD}{d\eta} \), we have

\[
\frac{dU}{d\eta} = - \frac{[U - (n + 1)] dD}{D} - \frac{2U}{\eta}, \tag{2.3.14}
\]

\[
\frac{d\phi}{d\eta} = - \frac{(2U - 1)\phi}{[U - (n + 1)]\eta}, \tag{2.3.15}
\]

\[
\frac{dW}{d\eta} = - \frac{(U - 1)W}{[U - (n + 1)]\eta}, \tag{2.3.16}
\]

\[
\frac{dP}{d\eta} = \frac{\gamma P}{D} \frac{dD}{d\eta} - \frac{2P(U - 1)}{[U - (n + 1)]\eta}, \tag{2.3.17}
\]

\[
\frac{dD}{d\eta} = \frac{D(1 - bD)}{\eta \gamma P - D \{U - (n + 1)\}} \left[ 2UD\{U - (n + 1)\} - UD(U - 1) - 2P + \phi^2 D + \frac{2P(U - 1)}{\{U - (n + 1)\}} \right]. \tag{2.3.18}
\]

Applying the similarity transformations on equation (2.2.7), we obtain the non-dimensional components of the vorticity vector \( l_r = \frac{\zeta_r}{V/R}, l_\theta = \frac{\zeta_\theta}{V/R} \).
\[ l_z = \frac{\zeta}{\sqrt{R}} \] in the flow-field behind the shock as

\[ l_r = 0 \] \hspace{1cm} (2.3.19)

\[ l_\theta = \frac{nW}{2(n + 1)[U - (n + 1)]} \] \hspace{1cm} (2.3.20)

\[ l_z = -\frac{(2n + 1)\phi}{2(n + 1)[U - (n + 1)]} \] \hspace{1cm} (2.3.21)

Using the self-similarity transformations (2.3.4) and equation (2.3.2), the shock conditions (2.2.19) can be rewritten as

\[ U(1) = (1 - \beta)(n + 1) \] \hspace{1cm} (2.3.22a)

\[ D(1) = \frac{1}{\beta} \] \hspace{1cm} (2.3.22b)

\[ P(1) = \frac{1}{\gamma M^2} + (1 - \beta) \left( n + 1 \right)^2 \] \hspace{1cm} (2.3.22c)

\[ \phi(1) = (n + 1)^{\frac{1}{n + 1}} \sqrt{\frac{2n(n + 1)^{\frac{n-1}{n+1}}}{\gamma M^2}} \] \hspace{1cm} (2.3.22d)

\[ W(1) = \frac{w_0}{v_0} \left( n + 1 \right)^{\frac{1}{n + 1}} \sqrt{\frac{2n(n + 1)^{\frac{n-1}{n+1}}}{\gamma M^2}} \] \hspace{1cm} (2.3.22e)

where \( \lambda = \mu \) is taken in order to make the shock conditions consistent for similarity solutions.

The piston path coincides with a particle path at \( \eta_p = \frac{r_p}{R} \). Using (2.3.1) and (2.3.4), the relation

\[ U(\eta_p) = n + 1 \] \hspace{1cm} (2.3.23)

can be derived. In addition to the shock conditions (2.3.22), the kinematic condition (2.3.23) at the piston surface must be satisfied.
Normalizing the flow variables \( u, v, w, p, \) and \( \rho \) with their respective values at the shock, we obtain,

\[
\frac{u}{u_2} = \frac{U(\eta)}{U(1)\eta}, \quad \frac{v}{v_2} = \frac{\phi(\eta)}{\phi(1)\eta}, \quad \frac{w}{w_2} = \frac{W(\eta)}{W(1)\eta}, \quad \frac{p}{p_2} = \frac{P(\eta)}{P(1)\eta^2}, \quad \frac{\rho}{\rho_2} = \beta D(\eta).
\]

(2.3.24)

### 2.4 RESULTS AND DISCUSSION

Distribution of the flow variables between the shock front \( (\eta = 1) \) and the piston \( (\eta = \eta_p) \) is obtained by numerical integration of equations (2.3.14) to (2.3.18) with boundary conditions (2.3.22) by the Runge-Kutta method of the fourth order. For the purpose of numerical integration, the values of the constant parameters are taken to be (Vishwakarma and Nath [17, 18], Steiner and Hirschler [19]) \( \gamma = 1.4; M^2 = 25; \frac{u_0}{v_0} = 0.5; \bar{b} = 0, 0.01, 0.02 \) and \( n = 2, 3, 4 \). The value \( \bar{b} = 0 \) corresponds to the perfect gas case and the values \( n = 2, 3, 4 \) correspond to an accelerated piston.

Values of the density ratio across the shock front \( \beta \) and the piston position \( \eta_p \) are tabulated in table 1 for different values of \( \bar{b} \) and \( n \) with \( \gamma = 1.4, M^2 = 25 \) and \( \frac{u_0}{v_0} = 0.5 \).
Table 1. Variation of the density ratio $\beta \left( = \frac{\rho_1}{\rho_2} \right)$ across the shock front and the position of the piston $\eta_p$ for different values of $\bar{b}$ and $n$ with $\gamma = 1.4$, $M^2 = 25$ and $\frac{w_0}{v_0} = 0.5$.

<table>
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<th>$\bar{b}$</th>
<th>$\beta$</th>
<th>$n$</th>
<th>Position of the piston $\eta_p$</th>
</tr>
</thead>
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</table>
Fig. 1. Variation of the reduced radial component of fluid velocity $\frac{u}{u_2}$ in the flow-field behind the shock front
Fig. 2. Variation of the reduced azimuthal component of fluid velocity $\frac{V}{V_2}$ in the flow-field behind the shock front.
Fig. 3. Variation of the reduced axial component of fluid velocity $\frac{W}{W_0}$ in the flow-field behind the shock front.
Fig. 4. Variation of the reduced pressure $\frac{p}{p_2}$ in the flow-field behind the shock front.
Fig. 5. Variation of the reduced density $\frac{\rho}{\rho_2}$ in the flow-field behind the shock front
Fig. 6. Variation of the non-dimensional azimuthal component of vorticity vector $l_0$ in the flow-field behind the shock front
Fig. 7. Variation of the non-dimensional axial component of vorticity vector $l_z$ in the flow-field behind the shock front.
Fig. 8. Variation of the dimensionless compressibility $J$ in the flow-field behind the shock front for different values of $\beta$ with $n=2$. 
Figs 1-7 show the variation of the flow variables $\frac{u}{u_2}$, $\frac{v}{v_2}$, $\frac{w}{w_2}$, $\frac{p}{p_2}$, $\frac{\rho}{\rho_2}$, the non-dimensional azimuthal component of vorticity vector $l_\theta$ and the non-dimensional axial component of vorticity vector $l_z$, with $\eta$ at various values of the parameters $\bar{b}$ and $n$. It is shown that, as we move inward from the shock front towards the inner contact surface (piston), the reduced radial component of the fluid velocity $\frac{u}{u_2}$, the reduced pressure $\frac{p}{p_2}$ and the reduced density $\frac{\rho}{\rho_2}$ increase. These flow variables have higher values at the piston than at the shock front. It is also shown that the reduced azimuthal component of fluid velocity $\frac{v}{v_2}$ and the reduced axial component of fluid velocity $\frac{w}{w_2}$ decrease, as we move inward from the shock front. These flow variables have lower values at the piston than at the shock front. Fig. 6 shows that the non-dimensional azimuthal component of the vorticity vector $l_\theta$ decreases behind the shock front and tends to negative infinity at the inner contact surface (piston), whereas Fig. 7 shows that the non-dimensional axial component of the vorticity vector $l_z$ increases behind the shock front and tends to infinity at the inner contact surface (piston).

As can be seen from equation (2.3.18) for non-dimensional density $D$, there is a singularity at the piston where $U = n + 1$, because this equation becomes singular there. In case of accelerated piston ($0 < n < \infty$), this singularity is non-removable, and the derivative of the density tends to infinity, as shown in Fig. 5. This singularity can be physically interpreted as follows (Steiner and Hirschler [19]): the path of the accelerated piston converges with the path of the particle immediately ahead condensing the gas to infinity.

It is found that the effects of an increase in the value of the parameter of
non-idealness $\bar{b}$ of the gas are:

(i) to decrease $\eta_p$, i.e. to increase the distance of the piston from the shock front. Physically it means that the gas behind the shock is less compressed, i.e. the shock strength is decreased (see Table 1);

(ii) to increase the value of $\beta$ (i.e. to decrease the shock strength), which is same as in (i) above (see Table 1);

(iii) to decrease the radial component of velocity $\frac{u}{u_2}$, the pressure $\frac{p}{p_2}$ and the density $\frac{\rho}{\rho_2}$ at any point in the flow-field behind the shock (see Figs. 1, 4 and 5);

(iv) to increase the azimuthal component of velocity $\frac{v}{v_2}$ and the axial component of velocity $\frac{w}{w_2}$ at any point in the flow-field behind the shock (see Figs. 2 and 3); and

(v) to increase the non-dimensional azimuthal component of vorticity vector $l_\theta$ and to decrease the non-dimensional axial component of vorticity vector $l_z$ at any point in the flow-field behind the shock.

To interpret the decrease of the shock strength due to non-idealness of the gas, a dimensionless compressibility of the gas $J = \frac{\gamma p_2}{\rho} \left( \frac{\partial \rho}{\partial p} \right)_s$ is considered, where $\left( \frac{\partial \rho}{\partial p} \right)_s$ denotes derivative of $\rho$ with respect to $p$ at constant entropy (Vishwakarma et al. 2007). The dimensionless compressibility $J$ can be expressed as

$$J = \frac{p_2}{p} (1 - b \rho) = \left[ 1 - \frac{\bar{b}}{\beta} \left( \frac{\rho}{\rho_2} \right) \right] \left( \frac{p}{p_2} \right)^{-1}.$$
It is plotted against \( \eta \left( = \frac{r}{R} \right) \) in Fig. 8. This figure shows a significant decrease in compressibility by an increase in the parameter of non-idealness of the gas \( \bar{b} \). This decrease in the compressibility causes less compression of the gas behind the shock and hence a decrease in the shock strength.

The effects of an increase in the value of the index for variation of piston velocity \( n \) (i.e. the effects of an increase in the value of the index for variation of initial velocity of the medium \( \lambda \) or \( \mu \)) are

(i) to increase \( \eta_p \), i.e. to decrease the distance of the piston from the shock front. Physically it means that the gas behind the shock is more compressed, i.e. the shock strength is increased (see Table 1);

(ii) to increase the radial component of velocity \( \frac{u}{u_2} \), the pressure \( \frac{p}{p_2} \) and the density \( \frac{\rho}{\rho_2} \) at any point in the flow-field behind the shock (see Figs. 1, 4 and 5);

(iii) to decrease the azimuthal component of velocity \( \frac{v}{v_2} \) and the axial component of velocity \( \frac{w}{w_2} \) at any point in the flow-field behind the shock (see Figs. 2 and 3); and

(iv) to decrease the non-dimensional azimuthal component of vorticity vector \( l_\theta \) and to increase the non-dimensional axial component of vorticity vector \( l_z \) at any point in the flow-field behind the shock (see Figs. 6 and 7).

The present self-similar model may be used to describe some of the overall features of a “driven” shock wave produced by a flare energy release \( E \).
(c.f. equation (2.3.8)) that is time dependent. The energy ‘$E$’ increases with time and the solutions then correspond to a blast wave produced by intense, prolonged flare activity in a rotating star when the wave is driven by fresh erupting plasma for some-time and its energy tends to increase as it propagates from the sun.
REFERENCES


