Chapter 5

MAGNETOGASDYNAMIC SHOCK WAVES IN A NON-IDEAL GAS WITH HEAT CONDUCTION AND RADIATION HEAT-FLUX
5.1 INTRODUCTION

In aerodynamics, the analogy between the steady hypersonic flow past a slender blunted (planar or axisymmetric) power-law bodies and a one-dimensional unsteady self-similar flow behind a shock, driven by a piston moving with time according to a power-law, is well-known (see, for example, Grigoryan [1], Kochina and Melnikova [2], Wang [3], Helliwell [4], Sedov [5], Rosenau and Frankenthal [6], Steiner and Hirschler [7]). Wang [3] discussed the piston problem with radiative heat transfer in the thin and thick limits, as well as in the general case, within the idealized two direction approximation. Helliwell [4] studied the effects of radiative heat transfer for the classical plane, as well as for cylindrically or spherically symmetric one (Sedov [5]). He considered the power-law piston velocity, since in non-radiative hypersonic flow theory for slender bodies possessing power-law profiles, the flow in a shocked layer is given by the solution of the analogous unsteady piston problems with such a velocity (Mirelsh, [8]). Steiner and Hirschler [7] extended the self-similarity solution of the classical piston problem discussed by Sedov [5] to the dusty gas case.

Gretler and Wehle [9] studied the propagation of blast waves with an exponential heat release by taking internal heat conduction and thermal radiation in a detonating medium. Abdel-Raouf and Gretler [10] obtained a non-self-similar solution for the blast waves with internal heat transfer effects. Ghoniem et al. [11] obtained a self-similar solution for spherical explosions, taking into account the effects of both conduction and radiation in the limits of Rosseland radiative diffusion and Plank radiative emission. Recently,
Vishwakarma and Singh [12] obtained self-similar solutions for the propagation of a magnetogasdynamic shock wave in a non-uniform gas with heat conduction and radiation heat flux in the presence of an azimuthal magnetic field that was driven by a cylindrical or spherical piston moving with a velocity varying with time according to the power-law. In these works, the medium of the shock propagation was assumed to be a perfect gas.

The assumption that the gas is ideal is no longer valid when the flow takes place under extreme conditions. Anisimov and Spiner [13] studied the problem of point explosion in a low density non-ideal gas by taking the equation of state in a simplified form, which describes the behaviour of the medium satisfactorily. Roberts and Wu [14, 15] studied the problem of a spherical implosion by assuming that the gas obeys a simplified van der Waals equation of state.

In the present work, we therefore generalized the solution of Vishwakarma and Singh [12] for the perfect gas to the case of a flow of a non-ideal gas obeying the simplified van der Waals equation of state (Roberts and Wu [14, 15]). The piston velocity is assumed to vary as some power of time and the initial azimuthal magnetic field strength to vary as some power of distance. The heat transfer fluxes are expressed in terms of Fourier’s law of heat conduction and a diffusion radiation model for an optically thick grey gas, which is typical of large-scale explosions. The thermal conductivity and absorption coefficient of the gas are assumed to be proportional to appropriate powers of temperature and density (Ghoneim et al. [11]). It is assumed also that the gas is grey and opaque, and the shock is isothermal. The assumption that the shock is isothermal is a result of the mathematical approximation
in which the heat flux is taken to be proportional to the temperature gradient, this excludes the possibility of temperature jump (Zel’dovich and Raizer [16], Rosenau and Frankenthal [17, 18], Bhowmick [19], Singh and Srivastava [20]). The counter pressure (the pressure ahead of the shock) is taken into account. The radiation pressure and radiation energy are neglected (Wang [3], Abdel-Raouf and Gretler [10], Ghoniem et al. [11], Elliott [21]). The assumption of an optically thick grey gas is physically consistent with the neglect of radiation pressure and radiation energy (Nicastro [22]). The gas ahead of the shock is assumed to be at rest. In order to obtain similarity solutions the density of the undisturbed medium is assumed to be constant. The effects of viscosity and gravitation are not taken into account. Effects of a change in the value of the parameter of non-idealness of the gas \( \bar{b} \) and the Alfven-Mach number \( M_A \) on the strength of the shock and on the flow-field behind it are investigated.

5.2 EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

The fundamental equations for one-dimensional unsteady flow of an electrically conducting and non-ideal gas with heat conduction and radiation heat flux taken into account in the presence of an azimuthal magnetic field may, in Eulerian co-ordinates, be expressed as (Gretler and Wehle[9], Abdel-Raouf
and Gretler[10], Ghoniem et al.[11], Christer and Helliwell[23], Summers[24])

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{j \rho u}{r} = 0 ,
\]

(5.2.1)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \left[ \frac{\partial p}{\partial r} + \mu h \frac{\partial h}{\partial r} + \frac{\mu h^2}{r} \right] = 0 ,
\]

(5.2.2)

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + (j - 1) \frac{hu}{r} = 0 ,
\]

(5.2.3)

\[
\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left[ \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right] + \frac{1}{pr^j} \frac{\partial}{\partial r} (r^j q) = 0 ,
\]

(5.2.4)

where \( r \) and \( t \) are independent space and time co-ordinates, \( \rho \) is the density, \( p \) the pressure, \( u \) the flow velocity, \( h \) the azimuthal magnetic field, \( e \) the internal energy per unit mass, \( q \) the heat flux, \( \mu \) the magnetic permeability, and \( j = 1 \) or \( 2 \) for line or point symmetry.

The total heat flux \( q \), which appears in the energy equation may be decomposed as

\[
q = q_C + q_R ,
\]

(5.2.5)

where \( q_C \) is the conduction heat flux, and \( q_R \) is the radiation heat flux.

According to Fourier’s law of heat conduction

\[
q_C = -K \frac{\partial T}{\partial r} ,
\]

(5.2.6)

where \( K \) is the coefficient of thermal conductivity and \( T \) is the absolute temperature of the medium.

Assuming local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas (Pomraning [25]), the radiative heat flux \( q_R \) may be obtained from the differential approximation of the
radiation-transport equation in the diffusion limit as
\[ q_r = -\frac{4}{3} \left( \frac{\sigma}{\alpha_R} \right) \frac{\partial T^4}{\partial r}, \]  
(5.2.7)
where \( \sigma \) is the Stefan-Boltzmann constant and \( \alpha_R \) is the Rosseland mean absorption coefficient.

The electrical conductivity of the gas is assumed to be infinite. Therefore the diffusion term from the magnetic field equation is omitted, and the electrical resistivity is ignored. Also, the effect of viscosity on the flow of the gas is assumed to be negligible.

The above system of equations should be supplemented with an equation of state. In most of the cases the propagation of shock waves arises in extreme conditions under which the assumption that the gas is ideal is not a sufficiently accurate description. To discover how deviations from the ideal gas can affect the solutions, we adopt a simple model. We assume that the gas obeys a simplified van der Waals equation of state of the form (Roberts and Wu [14, 15] and Vishwakarma et al. [26])
\[ p = \frac{\Gamma \rho T}{1 - b_1 \rho}, \]  
(5.2.8)
where \( \Gamma \) is the gas constant and the constant \( b_1 \) is the ‘van der Waals excluded volume’; it places a limit, \( \rho_{\text{max}} = 1/b_1 \), on the density of the gas.

The internal energy per unit mass of the non-ideal gas is given by
\[ e = C_v T = \frac{p(1 - b_1 \rho)}{\rho(\gamma - 1)}, \]  
(5.2.9)
where \( C_v \) is the specific heat of the gas at constant volume and \( \gamma \) is the ratio of specific heats.
The thermal conductivity $K$ and the absorption coefficient $\alpha_R$ are assumed to vary with temperature and density as (Ghoniem et al. [11])

$$K = K_0 \left( \frac{T}{T_0} \right)^{\beta_C} \left( \frac{\rho}{\rho_0} \right)^{\delta_C}, \quad \alpha_R = \alpha_{R_0} \left( \frac{T}{T_0} \right)^{\beta_R} \left( \frac{\rho}{\rho_0} \right)^{\delta_R},$$

where subscript ‘0’ denotes a reference state. The exponents in the above equations should satisfy the similarity requirements if a self-similar solution is sought.

A shock (cylindrical or spherical) is supposed to be propagating in the undisturbed non-ideal gas with constant density. Also, the azimuthal magnetic field in the undisturbed gas is assumed to vary as $h = A'R^{-\omega}$ (Rosenau [27]) where $A'$ and $\omega$ are constants.

The flow variables immediately ahead of the shock front are

$$u = u_1 = 0,$$  \hspace{1cm} (5.2.11)

$$\rho = \rho_1 = \text{constant},$$  \hspace{1cm} (5.2.12)

$$h = h_1 = A'R^{-\omega},$$  \hspace{1cm} (5.2.13)

$$p = p_1 = \frac{(1 - \omega) \mu A'^2}{2\omega} \frac{\rho^2}{R^2\omega} \quad (0 < \omega < 1),$$  \hspace{1cm} (5.2.14)

$$q = q_1 = 0, \quad (\text{Laumbach and Probstein [28]})$$  \hspace{1cm} (5.2.15)

where $R$ is the shock radius and the subscript ‘1’ denotes the conditions immediately ahead of the shock.

For an isentropic change of state of the non-ideal gas, we may calculate the so-called speed of sound in non-ideal gas as follows:

$$a = \left( \frac{dp}{d\rho} \right)_s^{1/2} = \left[ \frac{\gamma p}{\rho(1 - b_1\rho)} \right]^{1/2},$$  \hspace{1cm} (5.2.16)
where the subscript ‘s’ refers to the process of constant entropy. In addition, the isothermal speed of sound may also play a role when thermal radiation is taken into account. The isothermal speed of sound in non-ideal gas is

\[ a_{\text{isoth}} = \left( \frac{dp}{d\rho} \right)_T^{1/2} = \left[ \frac{p}{\rho(1 - b_1 \rho)} \right]^{1/2}. \quad (5.2.17) \]

The shock is assumed to be isothermal (the formation of isothermal shock is a result of the mathematical approximation in which the heat flux is taken to be proportional to the temperature gradient; this excludes the possibility of a temperature jump; see for example, Zel’dovich and Raizer [16], Rosenau and Frankenthal [17, 18] and hence the conditions across it are

\[ \rho_2 (V - u_2) = \rho_1 V, \quad (5.2.18a) \]
\[ h_2 (V - u_2) = h_1 V, \quad (5.2.18b) \]
\[ p_2 + \frac{1}{2} \mu h_2^2 + \rho_2 (V - u_2)^2 = p_1 + \frac{1}{2} \mu h_1^2 + \rho_1 V^2, \quad (5.2.18c) \]
\[ e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} (V - u_2)^2 + \frac{\mu h_2^2}{\rho_2} = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2} V^2 + \frac{\mu h_1^2}{\rho_1} + \frac{q_2}{\rho_1 V}, \quad (5.2.18d) \]
\[ T_2 = T_1, \quad (5.2.18e) \]

where the subscript ‘2’ denotes the conditions immediately behind the shock front, and \( V = dR/dt \) denotes the velocity of the shock front.

From equations (5.2.18), we get

\[ u_2 = (1 - \beta) V, \quad (5.2.19a) \]
\[ \rho_2 = \frac{\rho_1}{\beta}, \quad (5.2.19b) \]
\[ h_2 = \frac{h_1}{\beta}, \quad (5.2.19c) \]

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\[ p_2 = \left[ \frac{1}{\gamma M^2} + (1 - \beta) + \frac{M_A^{-2}}{2} \left( 1 - \frac{1}{\beta^2} \right) \right] \rho_1 V^2, \quad (5.2.19d) \]

\[ q_2 = (1 - \beta) \left[ \left( \frac{\bar{b}}{\beta - \bar{b}} \right) \frac{1}{\gamma M^2} - \frac{1}{2} (1 + \beta) + \frac{M_A^{-2}}{\beta} \right] \rho_1 V^3, \quad (5.2.19e) \]

where \( \bar{b} = b_1 \rho_1 \) is the parameter of non-idealness of the gas, \( M = (\rho_1 V^2/\gamma p_1)^{\frac{1}{2}} \) is the shock-Mach number referred to the frozen speed of sound \((\gamma p_1/\rho_1)^{\frac{1}{2}}\) and \( M_A = (\rho_1 V^2/\mu h_1)^{\frac{1}{2}} \) is the Alfven-Mach number.

The quantity \( \beta (0 < \beta < 1) \) is obtained by the relation

\[ \beta^3 - \beta^2 \left( \frac{1}{\gamma M^2} + \frac{M_A^{-2}}{2} + \bar{b} \right) - \beta \left( \frac{1 - \bar{b}}{2} \right) M_A^{-2} + \frac{\bar{b} M_A^{-2}}{2} = 0. \quad (5.2.20) \]

### 5.3 SIMILARITY TRANSFORMATIONS

The inner boundary of the flow-field behind the shock is assumed to be an expanding surface (piston). In the framework of self-similarity (Sedov [5]) the velocity \( u_p = \frac{dr_p}{dt} \) of the piston is assumed to follow the power law which reads (Steiner and Hirchler [7])

\[ u_p = \frac{dr_p}{dt} = U_0 \left( \frac{t}{t_0} \right)^n, \quad (5.3.1) \]

where \( r_p \) is the radius of the piston, \( t_0 \) denotes a reference time, \( U_0 \) is the piston velocity at \( t = t_0 \) and \( n \) is a constant. The consideration of ambient pressure \( p_1 \) and ambient magnetic field \( h_1 \) imposes a restriction on ‘\( n \)’ as \(-1/2 < n < 0\) (see equations (5.3.5) and (5.3.6)). Thus, the piston velocity jumps at \( t = 0 \) from zero to infinite velocity leading to the formation of a
shock of high strength in the initial phase. In regard to the shock boundary conditions, self-similarity requires that the velocity of the shock $V = \frac{dR}{dt}$ is proportional to the velocity of the piston, that is

$$V = \frac{dR}{dt} = CU_0 \left( \frac{t}{t_0} \right)^n,$$

(5.3.2)

where $C$ is a constant. The time and space co-ordinate can be transformed into a dimensionless self-similarity variable as follows:

$$\eta = \frac{r}{R} = \left[ \frac{(n + 1)t_0^n}{U_0 C} \right] \left( \frac{r}{l(n+1)} \right).$$

(5.3.3)

Evidently, the variable $\eta$ assumes the value ‘1’ at the shock front and $\eta_p = \frac{r_p}{R}$ at the piston. To obtain the similarity solutions, we write the flow velocity $u$, density $\rho$, pressure $p$, azimuthal magnetic field $h$ and the total heat flux $q$ as (Abdel-Rauf and Gretler[10], Ghoniem et al.[11], Vishwakarma and Singh[12], Vishwakarma and Yadav[29])

$$u = VU(\eta), \quad \rho = \rho_1 D(\eta), \quad p = \rho_1 V^2 P(\eta),$$

$$\sqrt{\mu h} = \sqrt{\rho_1} V H(\eta), \quad q = \rho_1 V^3 Q(\eta),$$

(5.3.4)

where $U$, $D$, $P$, $H$ and $Q$ are functions of $\eta$ only.

For the existence of similarity solutions, $M$ and $M_A$ should be constants, therefore

$$\omega = -\left( \frac{n}{n + 1} \right).$$

(5.3.5)

Thus,

$$M^2 = -\frac{2\rho_1 (n + 1)^{2n+1}}{\gamma(2n + 1)\mu A^2 \left( \frac{t_0^n}{C U_0} \right)^{\frac{2n}{2n+1}}} = -\frac{2n}{\gamma(2n + 1)} M_A^2,$$

(5.3.6)
where \(-1/2 < n < 0\). With the help of equations (5.3.4), the conservation equations (5.2.1) to (5.2.4) can be transformed into a system of ordinary differential equations with respect to \(\eta\):

\[
(U - \eta) \frac{dD}{d\eta} + D \frac{dU}{d\eta} + \frac{jU}{\eta} D = 0, \quad (5.3.7)
\]

\[
(U - \eta) \frac{dU}{d\eta} + \left(\frac{n}{n+1}\right) U + \frac{1}{D} \frac{dP}{d\eta} + \frac{H}{D} \frac{dH}{d\eta} + \frac{H^2}{\eta D} = 0, \quad (5.3.8)
\]

\[
(U - \eta) \frac{dH}{d\eta} + \left(\frac{n}{n+1}\right) H + H \frac{dP}{d\eta} + (j-1) \frac{HU}{\eta} = 0, \quad (5.3.9)
\]

\[
(U - \eta) \frac{dP}{d\eta} - \gamma P(U - \eta) \frac{dD}{d\eta} + \frac{(\gamma - 1)}{(1 - bD)} \frac{dQ}{d\eta} + \left(\frac{2n}{n+1}\right) P + \frac{j(\gamma - 1)Q}{\eta(1 - bD)} = 0. \quad (5.3.10)
\]

By using equations (5.2.6), (5.2.7) and (5.2.10) in (5.2.5), we get

\[
q = -\left[\frac{K_0}{T_0} \frac{\beta_C \rho_0 \delta_C}{\alpha_R} \frac{T^{3\beta_C} \rho^{\beta_C} + 16 \sigma T_0 \beta R \frac{\delta_R}{\rho_0} \frac{T^{3-\beta_R} \rho^{-\delta_R}}{3}}{\alpha_R} \frac{\partial T}{\partial r}\right]. \quad (5.3.11)
\]

Using equations (5.2.8) and (5.3.4) in (5.3.11), we get

\[
Q = -\left[(n + 1) \frac{K_0 \rho_1 \delta_C -1 (CU_0)^\frac{1}{2}}{T_0} \beta_C \rho_0 \delta_C \frac{T^{3\beta_C -1}}{\alpha_R} \frac{1}{3} \frac{T^{3-\beta_R} \rho^{-\delta_R}}{\alpha_R} \frac{T^{3-\beta_R} \delta_R - \delta_R - 3 \gamma 2^{\beta_R}}{\gamma 2} \frac{D_{\delta_C - \beta_C}}{D_{\delta_C - \beta_C}} \right] \times
\]

\[
\frac{d}{d\eta} \left[\frac{P(1 - bD)}{D}\right]. \quad (5.3.12)
\]

Equation (5.3.12) shows that similarity solution of the present problem exists only when

\[
\beta_C = 1 + \frac{1}{2n} \quad \text{and} \quad \beta_R = 2 - \frac{1}{2n}. \quad (5.3.13)
\]

Therefore, equation (5.3.12) becomes

\[
Q = -X \left[\frac{(1 - bD)}{D} \frac{dP}{d\eta} - \frac{P}{D^2} \frac{dP}{d\eta}\right], \quad (5.3.14)
\]

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where
\[ X = (n + 1)^\frac{1}{2} \left[ \frac{P(1 - \bar{b}D)}{D} \right]^{n+1} \left[ \Gamma_C D^\delta + \Gamma_R D^{-\delta_R} \right], \]

and \( \Gamma_C \) and \( \Gamma_R \) are the conductive and radiative non-dimensional heat transfer parameters, respectively. The parameters \( \Gamma_C \) and \( \Gamma_R \) depend on the thermal conductivity \( K \) and the mean free path of radiation \( 1/\alpha_R \), respectively, and also on the exponent \( n \) and they are given by
\[ \Gamma_C = \frac{K_0 \rho_1^{\delta - 1}}{T_0 \Gamma^2 \rho_0^{\delta c} t_0} \left[ \frac{C U_0}{\sqrt{T_0}} \right] \frac{1}{\Gamma}, \]

and
\[ \Gamma_R = \frac{16 \sigma T_0^2 \rho_0^{\delta_R} \rho_1^{\gamma M}}{3 \alpha_R t_0 \Gamma^2} \left[ \frac{C U_0}{\sqrt{T_0}} \right] \frac{1}{\Gamma}. \]

Using the similarity transformations (5.3.4) and equation (5.3.2), we can write equations (5.2.19) as
\[ U(1) = 1 - \beta, \quad (5.3.15a) \]
\[ D(1) = \frac{1}{\beta}, \quad (5.3.15b) \]
\[ H(1) = \frac{M_A^{-1}}{\beta}, \quad (5.3.15c) \]
\[ P(1) = \left[ (1 - \beta) + \frac{1}{\gamma M^2} + \frac{1}{2} M_A^{-2} \left( 1 - \frac{1}{\beta} \right) \right], \quad (5.3.15d) \]
\[ Q(1) = (1 - \beta) \left[ \left( \frac{\bar{b}}{\beta - \bar{b}} \right) \frac{1}{\gamma M^2} - \frac{1}{2} \left( 1 + \beta \right) + \frac{M_A^{-2}}{\beta} \right]. \quad (5.3.15e) \]

Solving equations (5.3.7) to (5.3.10) and (5.3.14) for \( \frac{dU}{d\eta} \), \( \frac{dH}{d\eta} \), \( \frac{dP}{d\eta} \), \( \frac{dQ}{d\eta} \) and \( \frac{dD}{d\eta} \), we have
\[ \frac{dU}{d\eta} = -\left( U - \eta \right) \frac{dD}{D} - \frac{jU}{\eta}, \quad (5.3.16) \]
\[ \frac{dH}{d\eta} = \frac{H}{D} \frac{dD}{d\eta} - \left( \frac{n}{n+1} \right) \frac{H}{U-\eta} + \frac{HU}{\eta(U-\eta)}, \quad (5.3.17) \]

\[ \frac{dP}{d\eta} = \left[ (U-\eta)^2 - \frac{H^2}{D} \right] \frac{dD}{d\eta} + \frac{jUD(U-\eta)}{\eta} - \left( \frac{n}{n+1} \right) UD + \frac{H^2[(2n+1)\eta - 2(n+1)U]}{\eta(n+1)(U-\eta)}, \quad (5.3.18) \]

\[ \frac{dQ}{d\eta} = \frac{(U-\eta)}{(\gamma - 1)D} \left[ \frac{\gamma P + (1 - bD)H^2 - D(U-\eta)^2(1 - bD)}{D} \right] \frac{dD}{d\eta} - \frac{(U-\eta)(1 - bD)}{(\gamma - 1)} \left[ \frac{jDU(U-\eta)}{\eta} - \left( \frac{n}{n+1} \right) UD + \frac{H^2[(2n+1)\eta - 2(n+1)U]}{\eta(n+1)(U-\eta)} \right] - \left( \frac{2n}{n+1} \right) P \left( \frac{1 - bD}{\gamma - 1} \right) - \frac{jQ}{\eta}, \quad (5.3.19) \]

\[ \frac{dD}{d\eta} = \frac{D^2(1 - bD))}{P - (1 - bD)[(U-\eta)^2D - H^2]} \left[ \frac{Q}{X(1 - bD)} + \frac{jU(U-\eta)}{\eta} \right] \frac{dD}{d\eta} - \left( \frac{n}{n+1} \right) U + \frac{H^2[(2n+1)\eta - 2(n+1)U]}{D\eta(n+1)(U-\eta)}. \quad (5.3.20) \]

Substituting equation (5.3.4) into (5.2.17), we get the expression for reduced isothermal speed of sound as

\[ \frac{a_{\text{isoth}}}{V} = \left[ \frac{P}{D(1 - bD)} \right]^{\frac{1}{2}}. \quad (5.3.21) \]

Also, the total energy of the disturbance is given by

\[ E = 2\pi j \int_{r_p}^{R} \rho \left( e + \frac{u^2}{2} + \frac{\mu h^2}{2\rho} \right) r^3 dr. \quad (5.3.22) \]

With the use of equations (5.3.4) and (5.2.9), equation (5.3.22) becomes

\[ E = 2\pi j \rho_1 (CU_0)^{\frac{2}{n+1}} (n + 1)^{\frac{2n}{n+1}} l_0^{\frac{2n}{n+1}} R^{j+1 + \frac{2n}{n+1}} \times \]

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\[
\int_{\eta_p}^{1} \left[ \left( \frac{1 - \tilde{b}D}{\gamma - 1} \right) P + \frac{DU_2^2}{2} + \frac{H_2^2}{2} \right] \eta^d d\eta.
\]

(5.3.23)

Thus, the total energy of the shock wave is not constant and varies as \(R^{j+1+\frac{2n}{\gamma+1}}\), where \(j = 1\) or \(2\) for a cylindrical or spherical shock.

The piston path coincides with a particle path at \(\eta_p = \frac{r_p}{R}\). Using equations (5.3.1) and (5.3.4), we obtain the following relation:

\[
U(\eta_p) = \eta_p = \frac{1}{C} = \frac{u_p}{V}.
\]

(5.3.24)

In addition to shock conditions (5.3.15), the kinematic condition (5.3.24) at the piston surface must be satisfied.

For presentation of the numerical solutions, it is convenient to write the flow variables in the non-dimensional form as

\[
\frac{u}{u_2} = \frac{U(\eta)}{U(1)}, \quad \frac{\rho}{\rho_2} = \frac{D(\eta)}{D(1)}, \quad \frac{h}{h_2} = \frac{H(\eta)}{H(1)}, \quad \frac{p}{p_2} = \frac{P(\eta)}{P(1)}, \quad \frac{q}{q_2} = \frac{Q(\eta)}{Q(1)}.
\]

(5.3.25)

### 5.4 RESULTS AND DISCUSSION

Distribution of the flow variables behind the shock front are obtained by numerical integration of the equations (3.3.16) to (3.3.20) with the boundary conditions (3.3.15) by the Runge-Kutta method of the fourth order. For the purpose of numerical integration, the values of the constant parameters are taken as (Ghoniem et al. [11], Rosenau and Frankenthal [17], Vishwakarma et al. [26], Rosenau [27]): \(j = 2\); \(\gamma = 5/3\); \(M_A^{-2} = 0.05, 0.1, \delta_C = 1\); \(\delta_R = 2\); \(\Gamma_C = 1\); \(\Gamma_R = 100\); \(n = -1/4\) and \(\tilde{b} = 0, 0.05, 0.1\). The value
$j = 2$ corresponds to a spherical shock, $\bar{b} = 0$ — to the perfect gas case, and $n = -1/4$ — to a decelerated piston. The values $\delta_C = 1$ and $\delta_R = 2$ are inherent for the case of a high-temperature low-density medium (Ghoniem et al. [11]), and the values $\Gamma_C = 1$ and $\Gamma_R = 100$ — for the case of heat transfer by both conduction and radiative diffusion. For a fully ionized gas $\gamma = 5/3$ which is valid for a stellar medium. Rosenau and Frankenthal [17] have shown that the effects of a magnetic field on the flow field behind the shock are significant when $M_A^{-2} \geq 0.01$; because of this, in the present problem the above values of $M_A^{-2}$ are taken for calculations.

The values of the density ratio $\beta$ across the shock front and of the piston position $\eta_p$ are tabulated in table 1 for different values of $\bar{b}$ and $M_A^{-2}$ with $j = 2$, $\gamma = 5/3$, $n = -1/4$, $\delta_C = 1$, $\delta_R = 2$, $\Gamma_C = 1$ and $\Gamma_R = 100$.

<table>
<thead>
<tr>
<th>$\bar{b}$</th>
<th>$M_A^{-2}$</th>
<th>$\beta$</th>
<th>$\eta_p$</th>
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Table 1. Density ratio $\beta \left( = \frac{\rho_1}{\rho_2} \right)$ across the shock front and piston position $\eta_p$ for different values of $\bar{b}$ and $M_A^{-2}$.
Fig. 1. Variation of reduced velocity $\frac{u}{u_2}$ in the region behind the shock front
Fig. 2. Variation of reduced density $\frac{\rho}{\rho_2}$ in the region behind the shock.
Fig. 3. Variation of the reduced magnetic field $\frac{\hat{h}}{h_2}$ in the region behind the shock
Fig. 4. Variation of the reduced pressure $\frac{p}{p_2}$ in the region behind the shock
Fig. 5. Variation of the reduced total heat flux $\frac{q_1}{q_2}$ in the region behind the shock.
Figures 1 - 5 show the variation of the flow variables $\frac{u}{u_2}, \frac{\rho}{\rho_2}, \frac{h}{h_2}, \frac{p}{p_2}$ and $\frac{q}{q_2}$ with $\eta$ at various values of the parameters $\tilde{b}$ and $M_A^{-2}$. Figure 1 shows that as we move inward from the shock front towards the inner contact surface (piston), the reduced velocity $\frac{u}{u_2}$ increases. Figures 2 and 3 show that the reduced density $\frac{\rho}{\rho_2}$ decreases from the shock front to the inner expanding surface, whereas the reduced magnetic field $\frac{h}{h_2}$ increases. Figure 4 shows that the reduced pressure $\frac{p}{p_2}$ increases when $\tilde{b} \neq 0$ and decreases when $\tilde{b} = 0$ (perfect gas case). Also, the reduced pressure $\frac{p}{p_2}$ decreases sharply near the piston in the perfect gas case. Figure 5 shows that in a non-ideal gas the reduced heat flux $\frac{q}{q_2}$ increases from the shock front, attains a maximum, and then decreases abruptly near the piston, whereas for the perfect gas it increases throughout.

As can be seen from equation (5.3.20) for $D$, there is a singularity at the piston, where $U = \eta$, because this equation becomes singular there. The singularity is non-removable, and the derivative of the density tends to negative infinity, as shown in figure 2. This singularity can be physically interpreted as follows (Steiner and Hirchler [7]): the path of the decelerated piston diverges from the path of a particle immediately ahead rarefying the gas.

It is found that the effects of an increase in the value of the parameter of non-idealness $\tilde{b}$ of the gas are:

(i) to increase the value of $\beta$ (i.e. to decrease the shock strength, see Table 1);
(ii) to increase the distance of the piston \((1 - \eta_p)\) from the shock front (see Table 1), i.e. the flow field behind the shock becomes somewhat rarefied. This leads to the decrease in the shock strength;

(iii) to decrease the magnetic field \(\frac{h}{h_2}\) and the total heat flux \(\frac{q}{q_2}\), and to increase the density \(\frac{\rho}{\rho_2}\) and the pressure \(\frac{p}{p_2}\) at any point in the flow field behind the shock;

(iv) to decrease the velocity \(\frac{u}{u_2}\) when \(M_A^{-2} = 0.05\) and to increase it when \(M_A^{-2} = 0.1\), i.e. when the initial magnetic field is strong (see Fig. 1).

In fact, the non-idealness of the gas lowers the compressibility of the gas, which results in (i), (ii), (iii) and (iv) given above.

The effects of an increase in the value of \(M_A^{-2}\) (i.e. the effects of an increase in the strength of ambient magnetic field) are (from Figs. 1 to 5)

(i) to increase the value of \(\beta\) (i.e. to decrease the shock strength, see Table 1);

(ii) to increase the distance of the piston from the shock front (see Table 1).

Physically, it means that the gas behind the shock is less compressed, i.e. the shock strength is reduced, which is the same as given in (i) above;

(iii) to increase the velocity \(\frac{u}{u_2}\), the density \(\frac{\rho}{\rho_2}\), and the total heat flux \(\frac{q}{q_2}\), and to decrease the magnetic field \(\frac{h}{h_2}\) at any point in the flow field behind the shock;
(iv) to decrease the pressure $\frac{p}{p_2}$ when $\bar{b} \neq 0$ (non-ideal case) and to increase it when the medium is perfect gas.

The present self-similar model may be used to describe some of the overall features of a ‘driven’ shock wave produced by a time-dependent flare energy release $E (= E_0 t^{\omega_1}$, where $\omega_1 = j(n+1)+3n+1$).
REFERENCES


