CHAPTER 0

INTRODUCTION

Optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function\[2\]. Optimization is applied in almost all fields such as Engineering\[!!\], Medicine, Computer Science, Economics, Agriculture and also to real life problems\[58\]. Hence, numerical optimization techniques\[39,79\] have made deep inroads into almost all branches after the publication of the fundamental papers by Kuhn and Tucker\[53\], Dantzig\[13\] and Arrow, Hurwicz and Uzawa\[3\]. Optimization problems can be broadly classified into Linear and Non-linear programming\[2, 85\]. During the last two decades the area of Linear and Non-linear programming has grown considerably\[22\]. There is no single method available for solving all optimization problems efficiently\[69, 70\].

It is interesting to note that the major developments in the area of numerical methods of unconstrained optimization were achieved in the United Kingdom only in the 1960's. The development of the Simplex method by Dantzig in 1947 for Linear programming problems paved the way for the development of the methods of constrained optimization\[14\].
Originally Linear programming (LP) was developed to investigate the problem of resource allocation, which is at the heart of most, if not all, business decisions. LP is a mathematical technique concerned with optimizing some desired objective under conditions of limited resources. Since its inception, however, applications of LP have become extremely diverse, ranging from financial organizations investigating portfolio and cash flow management, through production departments and marketing organizations to problems of waste disposal and pollution control[5]. In the real world, LP is carried out via computer based analysis[26, 52].

A widespread popular interest in recent years has led to improved methods for solving Linear programming models. Linear programming problems (LPP) possess certain properties which make these problems easier to solve. These properties of a Linear programming problem are: (i) the set of feasible solutions is a convex set, (ii) every local optimal solution is a global optimal solution, and, (iii) an optimal solution, if it exists, lies on the boundary of the convex set of feasible solutions. For a non-linear program, some or all of these properties may not be true. For example, the feasible region may not be convex. Also, even when the feasible region is convex, an optimal solution may be an interior point, and not a boundary or an extreme point of this set. Further, there can be many local optima, each different from a global optimum. In short, non-linearity in optimization problems, in general, makes such problems difficult to solve.
In 1826, Fourier found a method for solving systems of linear inequalities by successive elimination of variables. The method may be regarded as an extension of Gauss elimination for systems of linear equations. Since elimination works well for solving linear equations, it is natural to investigate a similar method for linear inequalities as was done by Fourier[25] and Motzkin[64]. It was forgotten until rediscovered in this century by Motzkin. One feature that stimulated the interest beyond the realm of pure Mathematics was the fact that the method can be used to solve linear Optimization problems. Several authors have discussed various algorithms for solving Linear programming problems[2, 14, 48, 50].

In [25], Fourier describes a rudimentary version of the simplex method. When an LP algorithm is first conceived, it is useful to be able to implement it quickly in a suitable high-level language, preferably one available within an interactive computing environment. The computational experience thus obtained often results in new insights and developments and helps to lay out the basic features of an algorithm[63]. Such a language should permit programs to be written with relative ease in the vernacular of applied mathematics and it serves as a medium for communicating algorithmic ideas precisely. The major drawback of the method for practical purposes is the rapid increase in the number of constraints that happens during elimination. Williams[92] solved the LPP using the Fourier variable elimination method. In his algorithm, the variables are chosen arbitrarily for elimination.
Non-linear programming methods for solving optimization problems can be classified broadly into two categories: i) direct search methods, and, 2) gradient methods (derivative based methods). Gradient methods require the evaluation of the gradient of the function, while the direct search methods do not require gradient information^, 9, 37, 60).

Almost all the multidimensional algorithms make use of unidimensional or line search scheme to evaluate the optimum step length in the required direction[24, 89]. Golden-Section, Quadratic Interpolation and Cubic Interpolation methods are the most widely used and popularly known line search schemes[22, 37, 42, 72]. Recently, Root mean square method[73], Arithmetic Mean(AM) method, Geometric Mean(GM) method and Harmonic Mean(HM) method[74] have been developed. Further, these line search schemes AM, GM and HM have been converted into implementable algorithms [86] which satisfy the acceptability criteria for the step size and global convergence conditions[34].

The performance of the line search schemes AM, GM and HM, coupled with three quadratically convergent algorithms namely, Broyden's Rank one algorithm (Algorithm I), Projection algorithm (Algorithm II) and Fletcher-Reeves algorithm (Algorithm III), through non-quadratic examples, have also been studied[12, 27, 37]. In[95], Sherif and Boice have modified the
Hookes and Jeeves method for solving unconstrained minimization problems which does not involve the derivatives of the given function. Recently, Goh has developed algorithms for solving unconstrained optimization problems via control theory[31]. In these algorithms, Goh has assumed that the given function should be twice differentiable.

**CRITERIA FOR EVALUATING ALGORITHMS FOR EFFICIENCY**

Algorithms can be examined from a theoretical viewpoint, as well as by experimentation. The former method can be applied only to a rather restricted class of problems. Hence, this study will be concerned with the evaluation of the effectiveness of algorithms by experimentation, i.e., by solving test problems. Algorithms can be tested on problems with small as well as large number of variables, on problems with varying degrees of non-linearity and on problems evolving from practical applications, such as least squares and solution of sets of non-linear equations. By examining the effectiveness of an algorithm in treating a variety of problems, one can hope to predict the general effectiveness of an algorithm in solving other problems[1].
The criteria to be considered in the evaluation of the algorithms are:

- Robustness-success in obtaining an optimal solution for a wide range of problems.
- Number of functional evaluations.
- Computer time to termination.

Hence, in this thesis, with the aim of developing efficient algorithms, a new method called "Modified Fourier Variable Elimination Method" is developed for solving Linear programming problems. Besides, several efficient algorithms for solving non-linear programming problems are developed, which are proved to be more efficient than the existing algorithms by implementing the newly developed algorithms for several typically chosen test functions.

A BRIEF SUMMARY OF THE RESEARCH CONTRIBUTIONS DESCRIBED IN THIS THESIS ARE LISTED BELOW

- An optimized version of Modified Fourier variable elimination method and its implementation.
- Efficient unidimensional direct search algorithms developed and tested for efficiency using test functions by implementing the algorithms.
• Modifications of certain unidimensional direct search algorithms developed in this thesis into implementable algorithms for unconstrained multidimensional optimization problems.

• Performance analysis of some of the direct search algorithms in Quadratically convergent algorithms

• Development and implementation of accelerated pattern search algorithms

• Development of an efficient algorithm for solving unconstrained optimization problems via control theory and its implementation.

BRIEF CHAPTER-WISE OUTLINE OF THE THESIS

CHAPTER I

In recent years, a good deal of research has gone into improving the available methods for solving Linear programming problems[82]. In 1977, Shor[83] described a new algorithm for solving Linear programming problems. In the year 1979, Khachian[50] modified the algorithm of Shor and his method is known as the Ellipsoid method, which is a polynomial time algorithm. In 1984, Karmarker[48] developed another
algorithm which is also polynomially bounded. The Simplex method of Dantzig[14] and his revised Simplex method are being used in commercial applications.

In 1986, Williams[92] showed that the method discovered by Fourier[25] in 1826 for manipulating Linear inequalities can be used to solve Linear programming problems. However, in the work of Williams[92], the application of Fourier variable elimination method for solving Linear programming problems generates additional and redundant constraints at each step. Further, even after using Kohler's rule[51] to eliminate redundant constraints, it is found that some of the constraints generated still remain redundant. In this chapter, the method of Fourier, as developed by Williams[92], is modified for solving a Linear programming problem by choosing a variable for elimination[45] which will generate minimum number of constraints at each step. It is observed that the new method[45] generates less number of constraints in comparison with that of Williams. As a result of this modified method, the CPU-time is considerably reduced. This has been demonstrated by suitably chosen typical examples of Linear programming problems[88] for implementation.
CHAPTER II

In this chapter an efficient unidimensional search algorithm based on Identric Mean (IM)\cite{46} is developed. The efficiency of this new algorithm in terms of number of function evaluations and CPU-time has been demonstrated in comparison with the RMS method.

CHAPTER III

In this chapter three efficient unidimensional direct search algorithms based on Weighted Arithmetic Mean (WAM), Weighted Geometric Mean (WGM) and Weighted Harmonic Mean (WHM) are developed\cite{47}. The efficiency of these new algorithms in terms of number of function evaluations and CPU-time has been demonstrated in comparison with other well known algorithms. The WAM method is compared with the AM method, the WGM method with the GM method and the WHM method with the HM method\cite{74}.

CHAPTER IV

An implementable algorithm\cite{76} is defined as an algorithm which takes only a finite number of arithmetic operations and function evaluations in contrast with the conceptual algorithm which usually takes an arbitrary number of arithmetical operations and function evaluations. In this chapter, the line search algorithms developed in chapter III, viz. the WAM method, the WGM method and the WHM
method are modified into implementable algorithms for solving the unconstrained optimization problems. The efficiency of these newly developed algorithms is demonstrated by comparing the existing implementable algorithms in the literature[86].

CHAPTER V

In [40], the Fletcher-Reeves algorithm, the rank-one algorithm and the projection algorithm have been studied for multidimensional unconstrained optimization problems using Golden section, Quadratic interpolation and Cubic interpolation for finding the optimum step length. Recently, Rao and Subbaraj[75] have modified the above three algorithms using AM, GM and HM for finding the step length. In this chapter, more efficient algorithms than those of Rao and Subbaraj using WAM, WGM, and WHM for finding optimum step length are developed.

CHAPTER VI

The Hooke and Jeeves Pattern search method consists of searching the local nature of the objective function in space and then moving in a favourable direction for reducing the function value. Sherif and Boice[95] have modified the Hookes and Jeeves method by eliminating unnecessary exploratory moves around the pattern search. In this chapter, the pattern search has been further
accelerated by using Golden section ratio[62], which will eliminate more number of unnecessary exploratory moves around the pattern search and it is proved that this method is more efficient than that of Sherif and Boice, by demonstration with the help of typically chosen multidimensional optimization problems.

CHAPTER VII

Recently, Goh[31] has developed an algorithm for solving unconstrained optimization problems via control theory. In this algorithm, Goh[31] has used Newton's method[67] for prescribing the direction vector by computing the Hessian matrix using the second order derivative. But the existence of the second order derivative for a function is a stronger mathematical condition and therefore this algorithm cannot be used for a wider class of optimization problems. In this chapter, algorithms for unconstrained optimization problems in which the functions are assumed to be differentiable ones are developed. For developing this efficient algorithm, Broyden's update[71] formula for approximating the Hessian matrix using gradient has been used. It is demonstrated that this new algorithm is more efficient than the existing algorithm in the literature[31].