CHAPTER-2

CERTAIN TRANSFORMATION FORMULAE FOR HYPERGEOMETRIC SERIES

2.1 Introduction :

In this chapter, we shall make use of of the following Bailey's transformation

if $$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r} \tag{1}$$

and $$\gamma_n = \sum_{r=0}^{n} \delta_{r+n} u_r v_{r+2n}, \tag{2}$$

where α_r, δ_r, u_r and v_r are any functions of r only, such that the series γ_n exists, then

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n, \tag{3}$$

in order to establish certain transformation and summation formulae for hypergeometric functions. In order to obtain transformation and summation formulae, we shall be in need of the following known results:

$${}_7F_6\left[\begin{matrix} a, & 1+\frac{1}{2}a, & b, & c, & d, & 1+2a-b-c-d+n, & -n; & 1 \\ & \frac{1}{2}a, & 1+a-b, & 1+a-c, & 1+a-d, & b+c+d-a, & 1+a+n; & \end{matrix}\right]$$

$$= \frac{(1+a)_n (1+a-b-c)_n (1+a-b-d)_n (1+a-c-d)_n}{(1+a-b)_n (1+a-c)_n (1+a-d)_n (1+a-b-c-d)_n}, \tag{4}$$

provided that $1 + 2a = b + c + d + e - n$.

[Slater [2]; App III (III.13)]

$$ {}_3F_2\left[\begin{matrix} a, & b, & -n; & 1 \\ & 1+a-b, & 1+a+n; & \end{matrix}\right] = \frac{(1+a)_n \left(1+\frac{1}{2}a-b\right)_n}{\left(1+\frac{1}{2}a\right)_n (1+a-b)_n}. \tag{5} $$

[Slater [2]; App III (III.9)]

$$ {}_4F_3\left[\begin{matrix} a, & 1+\frac{1}{2}a, & b, & -n; 1 \\ & \frac{1}{2}a, & 1+a-b, & 1+a+n; \end{matrix}\right] = \frac{(1+a)_n}{(1+a-b)_n}. \tag{6} $$

[Slater [2]; App III(III.11)]

$$ {}_3F_2\left[\begin{matrix} a, & b, & -n; & 1 \\ & 1+a-b, & a+2b-n; & \end{matrix}\right] = \frac{(a-2b)_n \left(1+\frac{1}{2}a-b\right)_n (-b)_n}{(1+a-b)_n \left(\frac{1}{2}a-b\right)_n (-2b)_n}. \tag{7} $$

[Slater [2]; App III(III.16)]

$$ {}_4F_3\left[\begin{matrix} a, & 1+\frac{1}{2}a, & b, & -n; \quad 1 \\ & \frac{1}{2}a, & 1+a-b, & 1+2b-n; \end{matrix}\right] = \frac{(a-2b)_n (-b)_n}{(1+a-b)_n (-2b)_n}. \tag{8} $$

[Slater [2]; App III(III.17)]

$$ {}_4F_3\left[\begin{matrix} a, & 1+\frac{1}{2}a, & b, & -n; & 1 \\ & \frac{1}{2}a, & 1+a-b, & 2+2b-n; & \end{matrix}\right] $$

$$ = \frac{(a-2b-1)_n\left(\frac{1}{2}a+\frac{1}{2}-b\right)_n(-b-1)_n}{(1+a-b)_n\left(\frac{1}{2}a-\frac{1}{2}-b\right)_n(-2b-1)_n}. \tag{9} $$

[Slater [2]; App III(III.18)]

$$ {}_3F_2\left[\begin{matrix} a, & 1+\frac{1}{2}a, & -n; & 1 \\ & \frac{1}{2}a, & 1+a+n; & \end{matrix}\right] = (1+a)_n \tag{10} $$

[Slater [2]; (2.3.4.10)p.57]

$$ {}_2F_1\left[\begin{matrix} a, & b; & -1 \\ & 1+a-b; & \end{matrix}\right] = \frac{\Gamma(1+a-b)\Gamma\left(1+\frac{1}{2}a\right)}{\Gamma(1+a)\Gamma\left(1+\frac{1}{2}a-b\right)}. \tag{11} $$

[Slater [2]; App III(III.5)]

$$ {}_2F_1\left[\begin{matrix} a, & b; & 1 \\ & 1+a-b; & \end{matrix}\right] = \frac{(1+a)_n(1+b)_n}{(1+a+b)_n n!}. \tag{12} $$

[Slater [2]; (2.6.19)p.84]

$$ {}_3F_2[a, \quad b, \quad c; \quad d, \quad a+b+c-d; \quad 1] = \frac{(1+a)_n(1+b)_n(1+c)_n}{n!(d)_n(a+b+c-d)_n}, \tag{13} $$

where $bc+ca+ab = (d-1)(a+b+c-d-1)$.

[Slater [2];(2.6.1.10) p.84]

$$ {}_1F_0[a; \ ;z] = (1-z)^{-a} \tag{14} $$

[Slater [2]; App.III(III.1)]

$$ {}_2F_1[a, \quad b; \quad c; \quad 1] = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \tag{15} $$

[Slater [2]; App.III(III.3)]

$$ {}_3F_2\left[\begin{matrix} x, & 3x+4+n, & -n; & \frac{3}{4} \\ & \frac{3}{2}(x+1), & \frac{3}{2}x+2; & \end{matrix}\right] = \frac{(1)_n(2x+4)_n(x+2)_m(x+3)_{3m}}{(1+x)_n(3x+4)_n(1)_n(2x+4)_{3m}}, \tag{16} $$

provided that m is the greatest integer $\leq \frac{n}{3}$.

[Verma & Jain [1];(1.5)p.1022]

$$ {}_3F_2\left[\begin{matrix} -n, & x, & y; & 1 \\ & -n-x, & -n-y; & \end{matrix}\right] = \frac{(1)_n(1+x+y)_n(1+x)_m(1+y)_m}{(1+x)_n(1+y)_n(1+x+y)_m(1)_m}, \tag{17} $$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (2.6) p. 1024]

$$ {}_3F_2\left[\begin{matrix} -n, & -n-x, & y; & 1 \\ & 1+x, & -n-y; & \end{matrix}\right] = \frac{(1)_n(1+x-y)_m(1+y)_m}{(1+y)_n(1+x)_m(1)_m}, \tag{18} $$

where m is the greatest integer $\leq \frac{n}{2}$.

[verma & Jain [1];(2.7) p. 1024]

$$ {}_3F_2\left[\begin{matrix} -n, & 1+x, & 1+y; & 1 \\ & 1-n-x, & 1-n-y; & \end{matrix}\right] = \frac{(-1)_n(1+x+y)_n(1+x)_m(1+y)_m(1)_n}{(x)_n(y)_n(1+x+y)_m(1)_m}, $$

....(19)

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1];(2.11)p.1025]

$$ {}_3F_2\left[\begin{matrix} -n, & -n-2x, & y; & 1 \\ & -n-x, & 2y+1; & \end{matrix}\right] = \frac{(1)_n(1+x+y)_n(1+x)_m(1+y)_m}{(1+x)_n(1+2y)_n(1+x+y)_m(1)_m}, \quad (20) $$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1];(2.13) p.1026]

$$ {}_3F_2\left[\begin{matrix} -n, & -n-2x, & 1+y; & 1 \\ & 1-n-x, & 2y+1, & \end{matrix}\right] $$

$$ = \frac{(-1)^n(1)_n(1+x+y)_n(1+x)_m(1+y)_m}{(x)_n(1+2y)_n(1+x+y)_m(1)_m}, \quad (21) $$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (2.17) p.1026]

$$ {}_3F_2\left[\begin{matrix} -n, & 1+n, & 2x+2y; & x; 1 \\ & 1+x+y, & 1+2x; & \end{matrix}\right] = \frac{(1)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m}, \quad (22) $$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (2.26) p.1028]

$$ {}_3F_2\left[\begin{matrix} -n, & 1+n+2x+2y, & 1+x; & 1 \\ & 1+x+y, & 1+2x; & \end{matrix}\right] $$

$$=\frac{(-1)^n(1)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m}, \qquad (23)$$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (2.27) p.1028]

$${}_3F_2\left[\begin{matrix}-n, & 2+b+n+2x, & x; & 1\\ & 1+\frac{1}{2}b+x, & 2x+2; & \end{matrix}\right]$$

$$=\frac{(1)_n(2+b+x)_n\left(\frac{3}{2}+x+\frac{b}{2}\right)_m\left(1+\frac{b}{2}\right)_m\left(1+\frac{1}{2}x\right)_{2m}}{(1+x)_n(2+b+2x)_n(1)_m\left(\frac{3}{2}+x\right)_m\left(1+\frac{1}{2}b+\frac{1}{2}x\right)_{2m}}, \qquad (24)$$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (3.3) p.1033]

$${}_3F_2\left[\begin{matrix}-n, & 2b+n+2x, & 1+x; & 1\\ & 1+\frac{1}{2}b+x, & 2+2x; & \end{matrix}\right]$$

$$=\frac{(-)^n(1)_n\left(\frac{3}{2}+\frac{1}{2}b+x\right)_m\left(1+\frac{1}{2}b\right)_m}{(2+b+2x)_n(1)_m\left(\frac{3}{2}+x\right)_m}, \qquad (25)$$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (3.4) p.1033]

$$
{}_4F_3\left[\begin{matrix} n, & \frac{1}{2}x-\frac{1}{2}, & 1+a+n, & -n; & 1 \\ & x-1, & x+1, & \frac{1}{2}+\frac{1}{2}a; & \end{matrix}\right]
$$

$$
=\frac{(1)_n(1+a-x)_n\left(1+\frac{1}{2}a\right)_m\left(\frac{1}{2}+\frac{1}{2}a\right)_m}{(1+a)_n(1+x)_n(1)_m\left(\frac{1}{2}+\frac{1}{2}a-\frac{1}{2}x\right)_m}, \tag{26}
$$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (3.6) p.1033]

$$
{}_3F_2\left[\begin{matrix} \frac{a}{3}, & 1+a-n, & -n; & \frac{3}{4} \\ & \frac{1}{2}+\frac{a}{2}, & 1+\frac{a}{2} & \end{matrix}\right]=\frac{(1)_n\left(1+\frac{1}{3}a\right)_m}{(1+a)_n(1)_m}, \tag{27}
$$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (4.6) p.1036]

$$
{}_3F_2\left[\begin{matrix} \frac{a}{3}, & 1+a+n, & -n; & \frac{3}{4} \\ & \frac{a}{2}, & \frac{1}{2}+\frac{a}{2}; & \end{matrix}\right]=\frac{(1)^{n-m}(1)_n\left(1+\frac{1}{3}a\right)_m}{(1+a)_n(1)_m}, \tag{28}
$$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (4.7) p.1036]

$$
{}_4F_3\left[\begin{matrix} \frac{a}{3}, & 1+\frac{a}{2}, & 1+a+n, & -n; & \frac{3}{4} \\ & \frac{a}{2}, & \frac{1}{2}+\frac{a}{2}, & 2+\frac{a}{2} & \end{matrix}\right]=\frac{(1)_n\left(\frac{a}{2}\right)_n\left(1+\frac{1}{3}a\right)_m\left(2+\frac{a}{6}\right)_m}{(1+a)_n\left(2+\frac{a}{2}\right)_n\left(\frac{a}{6}\right)_m(1)_m}, \tag{29}
$$

where m is the greatest integer $\leq \frac{n}{2}$.

[Verma & Jain [1]; (4.9) p.1037]

2.2 Main Results:

In this chapter, we shall establish our main results.

(i) Choosing

$$u_n = \frac{(1+a-b-c-d)_n}{(1)_n},$$

$$v_n = \frac{(1+2a-b-c-d)_n}{(1+a)_n}$$

and

$$\alpha_n = \frac{(a)_n\left(1+\frac{1}{2}a\right)_n (b)_n (c)_n (d)_n}{n!\left(\frac{1}{2}a\right)_n (1+a-b)_n (1+a-c)_n (1+a-d)_n},$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{\infty} \frac{(a)_r\left(1+\frac{1}{2}a\right)_r (b)_r (c)_r (d)_r}{r!\left(\frac{1}{2}a\right)_r (1+a-b)_r (1+a-c)_r (1+a-d)_r} \times$$

$$\times \frac{(1+a-b-d)_{n-r}}{(1)_{n-r}} \frac{(1+2a-b-d)_{n+r}}{(1+a)_{n+r}}$$

$$= \frac{(1+a-b-c-d)_n (1+2a-b-c-d)_n}{n!(1+a)_n} \times$$

$$\times\sum_{r=0}^{n}\frac{(a)_r\left(1+\frac{1}{2}a\right)_r(b)_r(c)_r(d)_r(1+2a-b-c-d+n)_r}{r!\left(\frac{1}{2}a\right)_r(1+a-b)_r(1+a-c)_r(1+a-d)_r}\times$$

$$\times\frac{(-n)_r}{(b+c+d-n)_r(1+a+n)_r},$$

$$=\frac{(1+a-b-c-d)_n(1+2a-b-c-d)_n}{n!(1+a)_n}\times$$

$$\times{}_7F_6\left[\begin{matrix}a, & 1+\frac{1}{2}a, & b, & c, & d, & 1+2a-b-c+n, & -n; & 1\\ & \frac{1}{2}a, & 1+a-b, & 1+a-c, & 1+a-d, & b+c+d-n, & 1+a+n; & \end{matrix}\right]$$

Now, using (2.1.4) we get :

$$\beta_n=\frac{(1+2a-b-c-d)_n(1+a-b-c)_n(1+a-b-d)_n(1+a-c-d)_n}{n!(1+a-b)_n(1+a-c)_n(1+a-d)_n}.$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n=1$ we get :

$$\gamma_n=\sum_{r=0}^{\infty}1.\frac{(1+a-b-c-d)_r}{r!}\frac{(1+2a-b-c-d)_{r+2n}}{(1+a)_{r+2n}}$$

$$=\frac{(1+2a-b-c-d)_{2n}}{(1+a)_{2n}}\sum_{r=0}^{\infty}\frac{(1+a-b-c-d)_r(1+2a-b-c-d+2n)_r}{r!(1+a+2n)_r}$$

$$=\frac{(1+2a-b-c-d)_{2n}}{(1+a)_{2n}}\,{}_2F_1\left[\begin{matrix}1+a-b-c-d, & 1+2a-b-c-d+2n; & 1\\ & 1+a+2n; & \end{matrix}\right].$$

Now, making use of (2.1.15) we have :

$$\gamma_n = \frac{(1+2a-b-c-d)_{2n}}{(b+c+d)_{2n}} \frac{\Gamma(1+a)\Gamma(2b+2c+2d-2a-1)}{\Gamma(b+c+d)\Gamma(b+c+d-a)},$$

provided Rl(2b+2c+2d-2a-1)>0.

Putting $\beta_n, \gamma_n, \alpha_n$ & δ_n in (2.1.3), we finally set :

$$\sum_{n=0}^{\infty} \frac{(a)_n \left(1+\frac{1}{2}a\right)_n (b)_n (c)_n (d)_n}{n! \left(\frac{1}{2}a\right)_n (1+a-b)_n (1+a-c)_n (1+a-d)_n}$$

$$\times \frac{(1+2a-b-c-d)_{2n}}{(b+c+d)_{2n}} \frac{\Gamma 1+a\ \Gamma 2b+2c+2d-2a-1}{\Gamma b+c+d\ \Gamma b+c+d-a}$$

$$= \sum_{n=0}^{\infty} \frac{(1+2a-b-c-d)_n (1+a-b-c)_n (1+a-b-d)_n (1+a-c-d)_n}{n!\,(1+a-b)_n (1+a-c)_n (1+a-d)_n},$$

$$\frac{\Gamma(1+a)\Gamma(2b+2c+2d-2a-1)}{\Gamma(b+c+d)\Gamma(b+c+d-a)} \times$$

$$\times {}_7F_6\left[\begin{matrix} a, & 1+\frac{1}{2}a, & b, & c, & d, & \frac{1}{2}+a-\frac{b}{2}-\frac{c}{2}-\frac{d}{2}, & 1+a-\frac{b}{2}-\frac{c}{2}-\frac{d}{2}; & 1 \\ & \frac{1}{2}a, & 1+a-b, & 1+a-c, & 1+a-d, & \frac{b+c+d}{2}, & \frac{1}{2}+\frac{b}{2}+\frac{c}{2}+\frac{d}{2}; & \end{matrix}\right]$$

$$= {}_4F_3\left[\begin{matrix} 1+2a-b-c-d, & 1+a-b-c, & 1+a-b-d, & 1+a-c-d; & 1 \\ & 1+a-b, & 1+a-c, & 1+a-d; & \end{matrix}\right],$$

provided Rl (2b+2c+2d-2a) > 1.

(ii) Choosing

$$u_n = \frac{1}{(1)_n} = \frac{1}{n!},$$

$$v_n = \frac{1}{(1+a)_n}$$

and

$$\alpha_n = \frac{(a)_n (b)_n (-1)^n}{n!(1+a-b)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(a)_r (b)_r (-1)^r}{r!(1+a-b)_r} \times \frac{1}{(1)_{n-r}} \times \frac{1}{(1+a)_{n+r}}$$

$$= \frac{1}{n!(1+a)_n} \sum_{r=0}^{n} \frac{(a)_r (b)_r (-n)_r}{r!(1+a-b)_r (1+a+n)_r}$$

$$\beta_n = \frac{1}{n!(1+a)_n} {}_3F_2\left[\begin{matrix} a, & b, & -n; & 1 \\ & 1+a-b, & 1+a-n; & \end{matrix}\right]$$

Now, using (2.1.5), we get

$$\beta_n = \frac{\left(1+\frac{1}{2}a-b\right)_n}{n!\left(1+\frac{1}{2}a\right)_n (1+a-b)_n}.$$

Again putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = (\alpha)_n (\beta)_n$ we get :

$$\gamma_n = \sum_{r=0}^{\infty} (\alpha)_{r+n} (\beta)_{r+n} \frac{1}{r!} \frac{1}{(1+a)_{r+2n}}$$

$$= \frac{(\alpha)_n (\beta)_n}{(1+a)_{2n}} \sum_{r=0}^{\infty} \frac{(\alpha+n)_r (\beta+n)_r}{r!(1+a+2n)_r}$$

$$= \frac{(\alpha)_n (\beta)_n}{(1+a)_{2n}} {}_2F_1[\alpha+n, \quad \beta+n; \quad 1+a+2n; \quad 1].$$

Now, making use of (2.1.15) we have :

$$\gamma_n = \frac{\Gamma(1+a)\,\Gamma(1+a-\alpha-\beta)}{\Gamma(1+a-\alpha)\,\Gamma(1+a-\beta)} \frac{(\alpha)_n (\beta)_n}{(1+a-\alpha)_n (1+a-\beta)_n},$$

provided Rl $(1+a-\alpha-\beta)>0$.

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3), we finally get :

$$\frac{\Gamma(1+a)\Gamma(1+a-\alpha-\beta)}{\Gamma(1+a-\alpha)\,\Gamma(1+a-\beta)} \sum_{n=0}^{\infty} \frac{(a)_n (b)_n (\alpha)_n (\beta)_n (-1)^n}{n!(1+a-b)_n (1+a-\alpha)_n (1+a-\beta)_n}$$

$$= \sum_{n=0}^{\infty} \frac{\left(1+\frac{1}{2}a-b\right)_n (\alpha)_n (\beta)_n}{n!\left(1+\frac{1}{2}a\right)_n (1+a-b)_n}$$

$${}_4F_3\left[\begin{matrix} a, & b, & \alpha, & \beta; & 1 \\ & 1+a-b, & 1+a-\alpha, & 1+a-\beta; & \end{matrix}\right]$$

$$= \frac{\Gamma(1+a-\alpha)\,\Gamma(1+a-\beta)}{\Gamma(1+a)\,\Gamma(1+a-\alpha-\beta)} \times {}_3F_2\left[\begin{matrix} \alpha, & \beta, & 1+\frac{1}{2}a-b; & 1 \\ & 1+\frac{1}{2}a, & 1+a-b; & \end{matrix}\right],$$

provided Rel $(1+a-\alpha-\beta)>0$.

(iii) Choosing

$$u_n = \frac{1}{(1)_n}$$

$$v_n = \frac{1}{(1+a)_n},$$

and

$$\alpha_n = \frac{(a)_n\left(1+\frac{1}{2}a\right)_n(b)_n}{n!\left(\frac{1}{2}a\right)_n(1+a-b)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n}\frac{(a)_r\left(1+\frac{1}{2}a\right)_r(b)_r}{r!\left(\frac{1}{2}a\right)_r(1+a-b)_r}\times\frac{1}{(1)_{n-r}}\times\frac{1}{(1+a)_{n+r}}$$

$$= \frac{1}{n!(1+a)_n}\sum_{r=0}^{n}\frac{(a)_r\left(1+\frac{1}{2}a\right)_r(b)_r(-n)_r(-1)^r}{r!\left(\frac{1}{2}a\right)_r(1+a-b)_r(1+a+n)_r}$$

$$\beta_n = \frac{1}{n!(1+a)_n}\,{}_4F_3\left[\begin{matrix}a, & 1+\frac{1}{2}a, & b, & -n; & -1\\ & \frac{1}{2}a, & 1+a-b, & 1+a+n & \end{matrix}\right].$$

Now, using (2.1.6), we get

$$\beta_n = \frac{1}{n!(1+a-b)_n}.$$

Again putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = (\alpha)_n(\beta)_n$ we get :

$$\gamma_n = \sum_{r=0}^{\infty} (\alpha)_{r+n}(\beta)_{r+n} \frac{1}{r!} \frac{1}{(1+a)_{r+2n}}$$

$$= \frac{(\alpha)_n(\beta)_n}{(1+a)_{2n}} \sum_{r=0}^{\infty} \frac{(\alpha+n)_r(\beta+n)_r}{r!(1+a+2n)_r}$$

$$= \frac{(\alpha)_n(\beta)_n}{(1+a)_{2n}} {}_2F_1[\alpha+n, \quad \beta+n; \quad 1+a+2n; \quad 1]$$

Now, making use of (2.1.15) we have :

$$\gamma_n = \frac{(\alpha)_n(\beta)_n}{(1+a-\alpha)_n(1+a-\beta)_n} \frac{\Gamma(1+a)\,\Gamma(1+a-\alpha-\beta)}{\Gamma(1+a-\alpha)\,\Gamma(1+a-\beta)},$$

provided $\mathrm{Rel}(1-a-\alpha-\beta) > 0$.

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3), we finally get :

$$\frac{\Gamma(1+a)\Gamma(1+a-\alpha-\beta)}{\Gamma(1+a-\alpha)\,\Gamma(1+a-\beta)} \sum_{n=0}^{\infty} \frac{(a)_n\left(1+\frac{1}{2}a\right)_n (b)_n(\alpha)_n(\beta)_n}{n!\left(\frac{1}{2}a\right)_n (1+a-b)_n(1+a-\alpha)_n(1+a-\beta)_n}$$

$$= \sum_{n=0}^{\infty} \frac{(\alpha)_n(\beta)_n}{n!(1+a-b)_n}$$

$$\frac{\Gamma(1+a)\,\Gamma(1+a-\alpha-\beta)}{\Gamma(1+a-\alpha)\,\Gamma(1+a-\beta)} \times {}_5F_4\left[\begin{matrix} a, & 1+\frac{1}{2}a, & b, & \alpha, & \beta; & 1 \\ & \frac{1}{2}a, & 1+a-b, & 1+a-\alpha, & 1+a-\beta; & \end{matrix}\right]$$

$$= {}_2F_1[\alpha, \quad \beta; \quad 1+a-b; \quad 1]$$

Now, making use of (2.1.15) we finally get :

$${}_5F_4\left[\begin{matrix} a, & 1+\frac{1}{2}a, & b, & \alpha, & \beta; & 1 \\ & \frac{1}{2}a, & 1+a-b, & 1+a-\alpha, & 1+a-\beta; & \end{matrix}\right]$$

$$= \frac{\Gamma(1+a-\alpha)\Gamma(1+a-\beta)}{\Gamma(1+a)\,\Gamma(1+a-\alpha-\beta)} \times \frac{\Gamma(1+a-b)\,\Gamma(1+a-b-\alpha-\beta)}{\Gamma(1+a-b-\alpha)\,\Gamma(1+a-b-\beta)},$$

provided $\mathrm{Rel}(1+a-\alpha-\beta) > 0$.

(iv) Choosing

$$u_n = \frac{(-2b)_n}{(1)_n}$$

$$v_n = 1,$$

and

$$\alpha_n = \frac{(a)_n (b)_n}{n!(1+a-b)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(a)_r (b)_r}{r!(1+a-b)_r} \cdot \frac{(-2b)_{n-r}}{(1)_{n-r}} .1$$

$$= \frac{(-2b)_n}{n!} \sum_{r=0}^{n} \frac{(a)_r (b)_r (-n)_r}{r!(1+a-b)_r (1+2b-n)_r}$$

$$= \frac{(-2b)_n}{n!} {}_3F_2[a,\ b,\ \ -n;\ \ 1+a-b,\ \ 1+2b-n;\ \ 1]$$

Now, using (2.1.7), we get

$$\beta_n = \frac{(a-2b)_n \left(1+\frac{1}{2}a-b\right)_n (-b)_n}{n!(1+a-b)_n \left(\frac{1}{2}a-b\right)_n}.$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we get :

$$\gamma_n = z^n \sum_{r=0}^{\infty} \frac{(-2b)_r}{r!} z^r$$

$$= z^n {}_1F_0[-2b;\ \ ...;\ \ z]$$

Now, making use of (2.1.14) we have :

$$\gamma_n = z^n (1-z)^{2b}.$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3), we finally get :

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{n!(1+a-b)_n} z^n (1-z)^{2b} = \sum_{n=0}^{\infty} \frac{(a-2b)_n \left(1+\frac{1}{2}a-b\right)_n (-b)_n}{n!(1+a-b)_n \left(\frac{1}{2}a-b\right)_n} z^n.$$

$$(1-z)^{2b} {}_2F_1[a,\ \ b;\ \ 1+a-b;\ \ z]$$

$$={}_3F_2\left[a-2b,\quad 1+\frac{1}{2}a-b,-b;\quad 1+a-b,\frac{1}{2}a-b;\quad z\right].$$

(v) Choosing

$$u_n=\frac{(-2b)_n}{(1)_n}$$

$$v_n=1$$

and

$$\alpha_n=\frac{(a)_n\left(1+\frac{1}{2}a\right)_n(b)_n}{n!\left(\frac{1}{2}a\right)_n(1+a-b)_n}.$$

in (2.1.1) we get :

$$\beta_n=\sum_{r=0}^{n}\frac{(a)_r\left(1+\frac{1}{2}a\right)_r(b)_r}{r!\left(\frac{1}{2}a\right)_r(1+a-b)_r}.\frac{(-2b)_{n-r}}{(1)_{n-r}}.1$$

$$=\frac{(-2b)_n}{n!}\sum_{r=0}^{n}\frac{(a)_r\left(1+\frac{1}{2}a\right)_r(b)_r(-n)_r}{r!\left(\frac{1}{2}a\right)_r(1+a-b)_r(1+2b-n)_r}.$$

$$=\frac{(-2b)_n}{n!}{}_4F_3\left[\begin{matrix}a, & 1+\frac{1}{2}a, & b, & -n; & 1\\ & \frac{1}{2}a, & 1+a-b, & 1+2b-n; & \end{matrix}\right]$$

Now, using (2.1.8), we get

$$\beta_n = \frac{(a-2b)_n(-b)_n}{n!(1+a-b)_n}.$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we get :

$$\gamma_n = z^n \sum_{r=0}^{\infty} \frac{(-2b)_r}{r!} z^r$$

$$= z^n {}_1F_0[-2b; \quad ...; \quad z]$$

Now, making use of (2.1.14) we have :

$$\gamma_n = z^n (1-z)^{2b}.$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3), we finally get :

$$\sum_{n=0}^{\infty} \frac{(a)_n \left(1+\frac{1}{2}a\right)_n (b)_n}{n!\left(\frac{1}{2}a\right)_n (1+a-b)_n} z^n (1-z)^{2b} = \sum_{n=0}^{\infty} \frac{(a-2b)_n(-b)_n}{n!(1+a-b)_n} z^n$$

$$(1-z)^{2b} {}_3F_2\left[\begin{matrix} a, & 1+\frac{1}{2}a, & b; & z \\ & \frac{1}{2}a, & 1+a-b; & \end{matrix}\right] = {}_2F_1[a-2b, \quad -b; \quad 1+a-b; \quad z].$$

(vi) Choosing

$$u_n = \frac{(-1-2b)_n}{(1)_n},$$

$$v_n = 1$$

and

$$\alpha_n = \frac{(a)_n\left(1+\frac{1}{2}a\right)_n(b)_n}{n!\left(\frac{1}{2}a\right)_n(1+a-b)_n}.$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n}\frac{(a)_r\left(1+\frac{1}{2}a\right)_r(b)_r}{r!\left(\frac{1}{2}a\right)_r(1+a-b)_r}.\frac{(-1-2b)_{n-r}}{(1)_{n-r}}.1$$

$$=\frac{(-1-2b)_n}{n!}\sum_{r=0}^{n}\frac{(a)_r\left(1+\frac{1}{2}a\right)_r(b)_r(-n)_r}{r!\left(\frac{1}{2}a\right)_r(1+a-b)_r(2+2b-n)_r}.$$

$$=\frac{(-1-2b)_n}{n!}\,{}_4F_3\left[\begin{matrix}a, & 1+\frac{1}{2}a, & b, & -n; & 1\\ & \frac{1}{2}a, & 1+a-b, & 1+2b-n; & \end{matrix}\right]$$

Now, using (2.1.9), we get

$$\beta_n = \frac{(a-2b-1)_n\left(\frac{1}{2}+\frac{1}{2}a-b\right)(-b-1)_n}{n!(1+a-b)_n\left(\frac{1}{2}a-\frac{1}{2}-b\right)_n}.$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we get :

$$\gamma_n = z^{ni}\sum_{r=0}^{\infty}\frac{(-1-2b)_r}{r!}z^r$$

$\gamma_n = z^n(1-z)^{1+2b}$ using (2.1.14) we have :

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3), we finally get :

$$\sum_{n=0}^{\infty}\frac{(a)_n\left(1+\frac{1}{2}a\right)_n(b)_n}{n!\left(\frac{1}{2}a\right)_n(1+a-b)_n}z^n(1-z)^{1+2b}$$

$$=\sum_{n=0}^{\infty}\frac{(a-2b-1)_n\left(\frac{1}{2}a+\frac{1}{2}-b\right)_n(-b-1)_n}{n!(1+a-b)_n\left(\frac{1}{2}a-\frac{1}{2}-b\right)_n}z^n$$

$$(1-z)^{1+2b}\,{}_3F_2\left[\begin{matrix}a, & 1+\frac{1}{2}a, & b; & z\\ & \frac{1}{2}a, & 1+a-b; & \end{matrix}\right]$$

$$={}_3F_2\left[\begin{matrix}a-2b-1, & \frac{1}{2}a+\frac{1}{2}-b, & -b-1; & z\\ & 1+a-b, & \frac{1}{2}a-\frac{1}{2}-b; & \end{matrix}\right]$$

(vii) Choosing

$$u_n = \frac{1}{(1)_n},$$

$$v_n = \frac{1}{(1+a)_n}$$

and

$$\alpha_n = \frac{(a)_n\left(1+\frac{1}{2}a\right)_n(-1)_n}{n!\left(\frac{1}{2}a\right)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(a)_r\left(1+\frac{1}{2}a\right)_r(-1)^r}{r!\left(\frac{1}{2}a\right)_r} \times \frac{1}{(1)_{n-r}} \times \frac{1}{(1+a)_{n+r}}$$

$$\beta_n = \frac{1}{n!(1+a)_n}\,{}_3F_2\left[\begin{matrix} a, & 1+\frac{1}{2}a, & -n; & 1 \\ & \frac{1}{2}a, & 1+a+n; & \end{matrix}\right]$$

Now, using (2.1.10), we get

$$\beta_n = \frac{1}{n!}.$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = (\alpha)_n(\beta)_n$ we have :

$$\gamma_n = \sum_{r=0}^{\infty} (\alpha)_{r+n}(\beta)_{r+n} \frac{1}{r!} \frac{1}{(1+a)_{r+2n}}$$

$$= \frac{(\alpha)_n(\beta)_n}{(1+a)_{2n}}\,{}_2F_1[\alpha+n, \quad \beta+n; \quad 1+a+2n; \quad 1]$$

Now, making use of (2.1.15) we have :

$$\gamma_n = \frac{\Gamma(1+a)\,\Gamma(1+a-\alpha-\beta)}{\Gamma(1+a-\alpha)\,\Gamma(1+a-\beta)} \frac{(\alpha)_n(\beta)_n}{(1+a-\alpha)_n(1+a-\beta)_n},$$

provided $\mathrm{Rel}\,(1-a-\alpha-\beta)>0$.

and either α or β is a negative integer.

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3), we finally get :

$$\sum_{n=0}^{\infty} \frac{(a)_n\left(1+\frac{1}{2}a\right)_n(\alpha)_n(\beta)_n}{n!\left(\frac{1}{2}a\right)_n(1+a-\alpha)_n(1+a-\beta)_n} \frac{\Gamma 1+a \Gamma 1+a-\alpha-\beta}{\Gamma 1+a-\alpha\, \Gamma 1+a-\beta}$$

$$= \sum_{n=0}^{\infty} \frac{(\alpha)_n(\beta)_n}{n!}$$

$${}_4F_3\left[\begin{matrix} a, & 1+\frac{1}{2}a, & \alpha, \beta; & 1 \\ & \frac{1}{2}a, & 1+a-\alpha, & 1+a-\beta; \end{matrix}\right]$$

$$\frac{\Gamma 1+a-\alpha\, \Gamma 1+a-\beta}{\Gamma 1+a\, \Gamma 1+a-\alpha-\beta} \times {}_2F_0[\alpha, \quad \beta; \quad ...; \quad]$$

provided $\mathrm{Rel}\,(1+a-\alpha-\beta)>0$

(viii) Choosing

$u_n = 1 = v_n$

and

$$\alpha_n = \frac{(a)_n (b)_n}{n!(1+a+b)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(a)_r (b)_r}{r!(1+a+b)_r} .1.1 = {}_2F_1[a, \quad b; \quad 1+a+b; \quad 1]_n$$

Now, using (2.1.11) we get

$$\beta_n = \frac{(1+a)_n (1+b)_n}{n!(1+a+b)_n}.$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we get :

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n}$$

$$\gamma_n = \frac{z^n}{(1-z)}$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3), we finally get :

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{n!(1+a+b)_n} \frac{z^n}{(1-z)} = \sum_{n=0}^{\infty} \frac{(1+a)_n (1+b)_n}{n!(1+a-b)_n} z^n$$

$${}_2F_1[a, \quad b; \quad 1+a+b; \quad z] = (1-z)_2F_1[1+a, \quad 1+b; \quad 1+a+b; \quad z].$$

(ix) Choosing

$$u_n = 1 = v_n$$

and

$$\alpha_n = \frac{(a)_n (b)_n (c)_n}{n!(a+b+c-d)_n (d)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(a)_r (b)_r (c)_r}{r!(a+b+c-d)_r (d)_r} .1.1$$

$$= {}_3F_2 \left[\begin{matrix} a, & b, & c; & 1 \\ & d, & (a+b+c-d); & \end{matrix} \right]_n$$

Now using (2.1.13) we get :

$$\beta_n = \frac{(1+a)_n (1+b)_n (1+c)_n}{n!(d)_n (a+b+c-d)_n}$$

provided $bc + ca + ab = (d-1)(a+b+c-d-1)$.

Again putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} z^{n+r} .1.1$$

$$= z^n \sum_{r=0}^{\infty} z^r = \frac{z^n}{(1-z)}$$

$$\gamma_n = \frac{z^n}{(1-z)}$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3), we finally get :

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n (c)_n}{n!(d)_n (a+b+c-d)_n} \frac{z^n}{(1-z)} = \sum_{n=0}^{\infty} \frac{(1+a)_n (1+b)_n (1+c)_n}{n!(d)_n (a+b+c-d)_n} z^n$$

$$ {}_3F_2\left[\begin{matrix} a, & b, & c; & z \\ & d, & a+b+c-d; & \end{matrix}\right] $$

$$ =(1-z)_3F_2\left[\begin{matrix} 1+a, & 1+b, & 1+c; & z \\ & d, & a+b+c-d; & \end{matrix}\right], $$

provided $bc+ca+ab=(d-1)(a+b+c-d-1)$.

(x) Choosing

$$ u_n=\frac{1}{(1)_n}, $$

$$ v_n=(3x+4)_n $$

and

$$ \alpha_n=\frac{(x)_n(3/4)^n(-1)^n}{n!\left(\frac{3}{2}+\frac{3}{2}x\right)_n\left(\frac{3}{2}x+2\right)_n} $$

in (2.1.1) we get :

$$ \beta_n=\sum_{r=0}^{n}\frac{(x)_r(3/4)^r(-1)^r}{r!\left(\frac{3}{2}+\frac{3}{2}x\right)_r\left(\frac{3}{2}x+2\right)_r}\cdot\frac{1}{(1)_{n-r}}.(3x+4)_{n+r} $$

$$ =\frac{(3x+4)_n}{n!}\sum_{r=0}^{n}\frac{(x)_r(3/4)^r(-n)_r(3x+4+n)_n}{r!\left(\frac{3}{2}+\frac{3}{2}x\right)_r\left(\frac{3}{2}x+2\right)_r} $$

$$ =\frac{(3x+4)_n}{n!}\,{}_3F_2\left[\begin{matrix} x, & 3x+4+n, & -n; & 3/4 \\ & 3/2x+3/2 & 3/2x+2; & \end{matrix}\right] $$

Now using (2.1.16) we get :

$$\beta_n = \frac{(2x+4)_n (x+2)_m (x+3)_{3m}}{(1+x)_n (1)_m (2x+4)_{3m}},$$

where m is the greatest inter $\leq n/3$.

Again putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} \frac{z^{r+n}(3x+4)_{r+2n}}{r!}$$

$$= z^n (3x+4)_{2n} \sum_{r=0}^{\infty} \frac{z^r (3x+4+2n)_r}{r!}$$

$$= z^n (3x+4)_{2n}\, {}_1F_0[3x+4+2n; \;\; ; \;\; z]$$

Now using (2.1.14) we get :

$$\gamma_n = (3x+4)_{2n} z^n / (1-z)^{3x+4+2n}.$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(x)_n (3/4)^n (-1)^n}{\left(\frac{3}{2}x+\frac{3}{2}\right)_n \left(\frac{3}{2}x+2\right)_n n!} \frac{(3x+4)_{2n} z^n}{(1-z)^{3x+4+2n}}$$

$$= \sum_{n=0}^{\infty} \frac{(2x+4)_n (x+2)_m (x+3)_{3m} z^n}{(1)_m (1+x)_n (2x+4)_{3m}}$$

$$\sum_{n=0}^{\infty} \frac{(x)_n (3/4)^n (-1)^n}{n!\left(\frac{3}{2}x+\frac{3}{2}\right)_n} \left(\frac{3}{2}x+\frac{5}{2}\right)_n \left(\frac{2}{1-z}\right)^{2n} z^n$$

$$= (1-z)^{3x+4} \sum_{n=0}^{\infty} \frac{(2x+4)_n (x+2)_m (x+3)_{3m}}{(1+x)_n (1)_m (2x+4)_{3m}} z^n ,$$

where m is the greatest integer $\leq n/3$.

(xi) Choosing

$$u_n = \frac{(1+x)_n (1+y)_n}{(1)_n} ,$$

$$v_n = 1$$

and

$$\alpha_n = \frac{(x)_n (y)_n (-1)^n}{n!}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(x)_r (y)_r (-1)^r}{r!} . \frac{(1+x)_{n-r} (1+y)_{n-r}}{(1)_{n-r}} .1$$

$$= \frac{(1+x)_n (1+y)_n}{n!} {}_3F_2 \left[\begin{matrix} x, & y, & -n; & 1 \\ & -n-x, & -n-y, & \end{matrix} \right]$$

Now, using (2.1.17) we get :

$$\beta_n = \frac{(1+x)_m (1+y)_m (1+x+y)_n}{(1)_m (1+x+y)_m} ,$$

where m is the greatest inter $\leq n/2$.

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} \frac{1}{r!} (1+x)_r (1+y)_r .1$$

$$= z^n \sum_{r=0}^{\infty} \frac{z^r (1+x)_r (1+y)_r}{r!}$$

$$\gamma_n = z^n {}_2F_0[1+x, \quad 1+y; \quad ;z].$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(x)_n (y)_n (-1)^n z^n}{n!} {}_2F_0[1+x, \quad 1+y; \quad ;z]$$

$$= \sum_{n=0}^{\infty} \frac{(1+x+y)_n (1+x)_m (1+y)_m z^n}{(1)_m (1+x+y)_m},$$

where m is the greatest integer $\leq n/2$.

(xii) Choosing

$$u_n = \frac{1}{(1)_n} \frac{(1+y)_n}{(1+x)_n},$$

$$v_n = 1$$

and

$$\alpha_n = \frac{(y)_n (-1)^n}{n!(1+x)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(y)_r (-1)^r}{(1+x)_r r!} \cdot \frac{(1+y)_{n-r}}{(1)_{n-r} (1+x)_{n-r}} .1$$

$$= \frac{(1+y)_n}{(1+x)_n n!} \sum_{r=0}^{\infty} \frac{(y)_r (-x-n)_r (-n)_r}{r!(1+x)_r (-n-y)_r}$$

$$=\frac{(1+y)_n}{(1+x)_n n!}{}_3F_2\left[\begin{matrix}-n, & -n-x, & y; & 1\\ & 1+x, & -n-y, & \end{matrix}\right]$$

Now, using (2.1.18) we get :

$$\beta_n = \frac{(1+y)_m (1+x+y)_m}{(1)_m (1+x+y)_m (1+x)_n},$$

where m is the greatest integer $\leq n/2$.

Again putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = (\alpha)_n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} (\alpha)_{n+r} \frac{1}{r!} \frac{(1+y)_{r+2n}}{(1+x)_{r+2n}}$$

$$= \frac{(1+y)_{2n} (\alpha)_n}{(1+x)_{2n}} \times {}_2F_1[\alpha+n, \quad 1+y+2n; \quad 1+x+2n; \quad 1]$$

Now using (2.1.15) we get :

$$\gamma_n = \frac{(1+y)_{2n} \Gamma(1+x)\, \Gamma(x-\alpha)(\alpha)_n (-1)^n}{(1+x-\alpha)_n (1-x+\alpha)_n \Gamma(1+x-\alpha) \Gamma(x-y)},$$

provided $Rl(x-\alpha-n) > 0$.

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(y)_n (1+y)_{2n} (\alpha)_n}{n!(1+x)_n (1+x-\alpha)_n (1-x+\alpha)_n}$$

$$= \frac{\Gamma(1+x-\alpha)\Gamma(x-y)}{\Gamma(1+x)\Gamma(x-\alpha)} \sum_{n=0}^{\infty} \frac{(1+x-y)_m (1+y)_m (\alpha)_n}{(1+x)_n (1+x)_m (1)_m},$$

where m is the greatest integer $\leq n/2$.

(xiii) Choosing

$$u_n = \frac{(x)_n (y)_n}{(1)_n},$$

$$v_n = 1$$

and

$$\alpha_n = \frac{(1+x)_n (1+y)_n (-1)^n}{n!}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(1+x)_r (1+y)_r (-1)^r}{r!} \cdot \frac{(x)_{n-r} (y)_{n-r}}{(1)_{n-r}} .1$$

$$= \frac{(x)_n (y)_n}{n!} {}_3F_2\left[\begin{matrix} -n & 1+x, & 1+y; \\ & 1-n-x, & 1-n-y \end{matrix}\right]$$

Now, using (2.1.19) we get :

$$\beta_n = \frac{(-1)^n (1+x+y)_n (1+x)_m (1+y)_m}{(1)_m (1+x+y)_m},$$

where m is the greatest inter $\leq n/2$.

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = 1$ we have

$$\gamma_n = \sum_{r=0}^{\infty} 1.\frac{(x)_r (y)_r}{r!} .1.$$

$\gamma_n = {}_2F_0[x, \quad y; \quad ;1].$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(1+x)_n (1+y)_n (-1)^n}{n!} {}_2F_0[x, \quad y; \quad ;1]$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (1+x+y)_n (1+x)_m (1+y)_m}{(1)_m (1+x+y)_m},$$

where m is the greatest integer $\leq n/2$.

(xiv) Choosing

$$u_n = \frac{1}{(1)_n} \frac{(1+x)_n}{(1+2x)_n},$$

$$v_n = 1$$

and

$$\alpha_n = \frac{(y)_n (-1)^n}{n!(2y+1)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(y)_r (-1)^r}{(2y+1)_r r!} \times \frac{(1+x)_{n-r}}{(1)_{n-r} (1+2x)_{n-r}} \times 1$$

$$= \frac{(1+x)_n}{(1+2x)_n n!} {}_3F_2\left[\begin{matrix} -n, & -n-2x, & y; & 1 \\ & -n-x & 2y+1 & \end{matrix}\right]$$

Now, using (2.1.20) we get :

$$\beta_n = \frac{(1+x)_m (1+y)_m (1+x+y)_n}{(1)_m (1+x+y)_m (1+2x)_n (1+2y)_n},$$

where m is the greatest inter $\leq n/2$.

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = (\alpha)_n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} (\alpha)_{n+r} \frac{1}{r!} \frac{(1+x)_r}{(1+2x)_{2r}} .1$$

$$= (\alpha)_n \times {}_2F_1[\alpha+n, \quad 1+x; \quad 1+2x; \quad 1]$$

Now, using (2.1.15) we get :

$$\gamma_n = \frac{(\alpha)_n (\alpha-2x)_n \Gamma(x-a)\Gamma(1+2x)}{(1-x+\alpha)_n \Gamma(x)\Gamma(1+2x-\alpha)},$$

provided Rel $(x-\alpha-n) > 0$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(y)_n (\alpha)_n (\alpha-2x)_n}{n!(2y+1)_n (1-x+\alpha)_n} \frac{\Gamma 1+2x \Gamma x-\alpha}{\Gamma x \Gamma 1+2x-\alpha}$$

$$= \sum_{n=0}^{\infty} \frac{(\alpha)_n (1+x+y)_n (1+x)_m (1+y)_m}{(1+2x)_n (1+2y)_n (1+x+y)_m (1)_m},$$

where m is the greatest integer $\leq n/2$.

$${}_3F_2\left[\begin{matrix} y, & \alpha-2x, & \alpha; & -1 \\ & 2y+1, & 1-x-\alpha; & \end{matrix}\right]$$

$$= \frac{\Gamma x \Gamma 1+2x-\alpha}{\Gamma 1+2x \Gamma x-\alpha} \times \sum_{n=0}^{\infty} \frac{(\alpha)_n (1+x+y)_n (1+x)_m (1+y)_m}{(1+2x)_n (1+2y)_n (1+x+y)_m (1)_m};$$

where m is the greatest integer $\leq n/2$.

(xv) Choosing

$$u_n = \frac{1}{(1)_n}\frac{(x)_n}{(1+2x)_n},$$

$$v_n = 1$$

and

$$\alpha_n = \frac{(1+y)_n(-1)^n}{n!(2y+1)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{\infty}\frac{(1+y)_r(-1)^r}{r!(2y+1)_r}\frac{1}{(1)_{n-r}}\frac{(x)_{n-r}}{(1+2x)_{n-r}}$$

$$= \frac{(x)_n}{n!(1+2x)_n} \times {}_3F_2\left[\begin{matrix}-n, & -n-2x, & 1+y; & z\\ & 1-n-x, & 2y+1; & \end{matrix}\right]$$

Now, using (2.1.21) we get :

$$\beta_n = \frac{(-1)^n(1+x+y)_n(1+x)_n(1+y)_n}{(1+2x)_n(1+2y)_n(1+x+y)_m(1)_m},$$

where m is the greatest integer $\leq n/2$.

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$, we have:

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} \frac{(x)_r}{r!(1+2x)_r}$$

$$\gamma_n = z^n {}_1F_1[x; \quad 1+2x; \quad z].$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(1+y)_n (-1)^n}{n!(2y+1)_n} z^n {}_1F_1[x; \quad 1+2x; \quad z]$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (1+x+y)_n (1+x)_m (1+y)_m}{(1+2x)_n (1+2y)_n (1+x+y)_m (1)_m} z^n,$$

where m is the greatest integer $\leq n/2$.

(xvi) Choosing

$$u_n = \frac{1}{(1)_n},$$

$$v_n = (1+2x+2y)_n$$

and

$$\alpha_n = \frac{(x)_n (-1)^n}{n!(1+x+y)_n (1+2x)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(x)_r (-1)^r}{r!(1+x+y)_r (1+2x)_r} \times \frac{1}{(1)_{n-r}} (1+2x+2y)_{n+r}$$

$$=\frac{(1+2x+2y)_n}{n!}\,{}_3F_2\left[\begin{matrix}-n, & 1+n+2x+2y, & x; & 1\\ & 1+x+y, & 1+2x; & \end{matrix}\right]$$

Now, using (2.1.22) we get :

$$\beta_n=\frac{(1+2x+2y)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m},$$

where m is the greatest inter $\le n/2$.

Again putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n=z^n$ we have

$$\gamma_n=\sum_{r=0}^{\infty}z^{r+n}\frac{1}{r!}(1+2x+2y)_{r+2n}$$

$$\gamma_n=z^n(1+2x+2y)_{2n}\sum_{r=0}^{\infty}\frac{z^r(1+2x+2y+2n)_r}{r!}$$

$$=z^n(1+2x+2y)_{2n}\big/(1-z)^{1+2x+2y+2n}.$$

Putting $\beta_n,\gamma_n,\alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty}\frac{(x)_n(-1)^n(1+2x+2y)_{2n}}{n!(1+x+y)_n(1+2x)_n}\times\frac{z^n}{(1-z)^{2n}}$$

$$=(1-z)^{1+2x+2y}\sum_{n=0}^{\infty}\frac{(1+2x+2y)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m}z^n$$

where m is the greatest integer $\le n/2$.

(xvii) Choosing

$$u_n=\frac{1}{(1)_n}$$

$$v_n = (1+2x+2y)_n$$

and

$$\alpha_n = \frac{(1+x)_n(-1)^n}{n!(1+x+y)_n(1+2x)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(1+x)_r(-1)^r}{r!(1+x+y)_r(1+2x)_r} \cdot \frac{1}{(1)_{n-r}}(1+2x+2y)_{n+r}$$

$$= \frac{(1+2x+2y)_n}{n!} {}_3F_2\left[\begin{matrix}-n, & 1+n+2x+2y, & 1+x; & 1 \\ & 1+x+y, & 1+2x; & \end{matrix}\right]$$

Now using (2.1.23) we get :

$$\beta_n = \frac{(-1)(1+2x+2y)_n(1+x)_n(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m},$$

where m is the greatest integer $\leq n/2$.

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} \frac{1}{r!}(1+2x+2y)_{r+2n}$$

$$\gamma_n = (1+2x+2y)_{2n} z^n \times {}_1F_0[1+2x+2y+2n; \ ; \ z]$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(1+x)_n(-1)^n(1+2x+2y)_{2n}}{n!(1+x+y)_n(1+2x)_n} \times \frac{z^n}{(1-z)^{2n}}$$

$$= (1-z)^{1+2x+2y} \sum_{n=0}^{\infty} \frac{(-1)^n (1+2x+2y)_n (1+x)_m (1+y)_m z^n}{(1+2x)_n (1+x+y)_m (1)_m},$$

where m is the greatest integer $\le n/2$.

(xviii) Choosing

$$u_n = \frac{1}{(1)_n}$$

$$v_n = (2+b+2x)_n$$

and

$$\alpha_n = \frac{(x)_n (-1)^n}{n!\left(1+\frac{1}{2}b+x\right)_n (2x+2)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(x)_r (-1)^r}{r!\left(1+\frac{1}{2}b+x\right)_r (2x+2)_r} \cdot \frac{1}{(1)_{n-r}} (2+b+2x)_{n+r}$$

$$= \frac{(2+b+2x)_n}{n!} \times {}_3F_2\left[\begin{matrix} -n, & 2+b+n+2x, & x; & 1 \\ & 1+\frac{1}{2}b+x, & 2x+2; & \end{matrix}\right]$$

Now, using (2.1.24) we get :

$$\beta_n = \frac{(2+b+x)_n \left(\frac{3}{2}+x+\frac{b}{2}\right)_m \left(1+\frac{1}{2}b\right)_m \left(1+\frac{1}{2}x\right)_{2m}}{(1+x)_n (1)_m \left(\frac{3}{2}+x\right)_m \left(1+\frac{b}{2}+\frac{x}{2}\right)_{2m}},$$

where m is the greatest integer $\le n/2$.

Again putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} \frac{1}{r!}(1+b+2x)_{r+2n}$$

$$\gamma_n = z^n \times {}_1F_0[2+b+2x+2n;\quad ;z\quad] \times (2+b+2x)_{2n}$$

Now, using (2.1.14), we get

$$\gamma_n = \frac{(2+b+2x)_{2n} z^n}{(1-z)^{2+b+2x+2n}}.$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x)_n \left(\frac{3}{2}+b+x\right)_n}{n!(2x+2)_n} \times \left(\frac{2}{(1-z)}\right)^{2n} \times z^n$$

$$= (1-z)^{2+b+2x} \sum_{n=0}^{\infty} \frac{(2+b+x)_n \left(\frac{3}{2}+x+\frac{b}{2}\right)_m \left(1+\frac{b}{2}\right)_m \left(1+\frac{x}{2}\right)_{2m} z^n}{(1+x)_n (1)_m \left(\frac{3}{2}+x\right)_m \left(1+\frac{3}{2}+\frac{x}{2}\right)_{2m}},$$

where m is the greatest integer $\le n/2$.

(xix) Choosing

$$u_n = \frac{1}{(1)_n},$$

$$v_n = (2+b+2x)_n$$

and

$$\alpha_n = \frac{(1+x)_n(-1)^n}{n!\left(1+\frac{1}{2}b+x\right)_n(2+2x)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(1+x)_r(-1)^r}{r!\left(1+\frac{1}{2}b+x\right)_r(2+2x)_r} \cdot \frac{1}{(1)_{n-r}}(2+b+2x)_{n+r}$$

$$= \frac{(2+b+2x)_n}{n!} \times {}_3F_2\left[\begin{matrix} -n, & 2+n+b+2x, & 1+x; \\ & 1+\frac{1}{2}b+x, & 2x+2; \end{matrix}\right]$$

Now, using (2.1.25) we get :

$$\beta_n = \frac{(-1)^n\left(\frac{3}{2}+\frac{b}{2}+x\right)_m\left(1+\frac{1}{2}b\right)_m}{(1)_m\left(\frac{3}{2}+x\right)_m},$$

where m is the greatest integer $\leq n/2$.

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} \frac{1}{r!}(2+b+2x)_{r+2n}$$

$$\gamma_n = z^n(2+b+2x)_{2n} \times \sum_{r=0}^{\infty} \frac{z^r(2+b+2x+2n)_r}{r!},$$

Now, using (2.1.14) we get

$$\gamma_n = z^n(2+b+2x)_{2n} / (1-z)^{2+b+2x+2n}$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(-1)^n (1+x)_n \left(\frac{3}{2}+\frac{b}{2}+x\right)_n}{n!(2x+2)_n} \times \left(\frac{2}{(1-z)}\right)^{2n} \times z^n$$

$$= (1-z)^{2+b+2x} \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{3}{2}+x+\frac{b}{2}\right)_n \left(1+\frac{b}{2}\right)_n z^n}{(1)_m \left(\frac{3}{2}+x\right)_n},$$

where m is the greatest integer $\leq n/2$.

(xx) Choosing

$$u_n = \frac{1}{(1)_n},$$

$$v_n = (1+a)_n$$

and

$$\alpha_n = \frac{(x)_n \left(\frac{1}{2}x - \frac{1}{2}\right)_n (-1)^n}{n!(x-1)_n (x+1)_n \left(\frac{1}{2}+\frac{1}{2}a\right)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(x)_r \left(\frac{1}{2}x - \frac{1}{2}\right)_r (-1)^r}{r!(x-1)_r (x+1)_r \left(\frac{1}{2}+\frac{1}{2}a\right)_r} \cdot \frac{1}{(1)_{n-r}} (1+a)_{n+r}$$

$$= \frac{(1+a)_n}{n!} \times {}_3F_2\left[\begin{matrix} x, & \frac{1}{2}x-\frac{1}{2}, & 1+a+n, & -n; & 1 \\ & x-1, & x+1, & \frac{1}{2}+\frac{1}{2}a & \end{matrix}\right]$$

Now, using (2.1.26) we get :

$$\beta_n = \frac{(1+a-x)_n\left(1+\frac{1}{2}a\right)_m\left(\frac{1}{2}+\frac{1}{2}a\right)_m}{(1+x)_n(1)_m\left(\frac{1}{2}+\frac{1}{2}a-\frac{1}{2}x\right)_m},$$

where m is the greatest integer $\leq n/2$.

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n}\frac{1}{r!}(1+a)_{r+2n}$$

$$\gamma_n = (1+a)_{2n} z^n \times \sum_{r=0}^{\infty}\frac{z^r(1+a+2n)_r}{r!}$$

Now, using (2.1.14) we get :

$$\gamma_n = \frac{(1+a)_{2n} z^n}{(1-z)^{1+a+2n}}.$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty}\frac{(-1)^n(x)_n\left(\frac{1}{2}x-\frac{1}{2}\right)_n\left(1+\frac{a}{2}\right)_n}{n!(x-1)_n(x+1)_n}\times\left(\frac{2}{(1-z)}\right)^{2n}\times z^n$$

$$=(1-z)^{1+a}\sum_{n=0}^{\infty}\frac{(1+a+x)_n\left(1+\frac{1}{2}a\right)_m\left(\frac{1}{2}+\frac{1}{2}a\right)_m z^n}{(1+x)_m(1)_m\left(\frac{1}{2}+\frac{a}{2}-\frac{x}{2}\right)_m},$$

where m is the greatest integer $\leq n/2$.

(xxi) Choosing

$$u_n=\frac{1}{(1)_n},$$

$$v_n=(1+a)_n$$

and

$$\alpha_n=\frac{(a/3)_n(3/4)^r(-1)^n}{n!\left(\frac{1}{2}+\frac{a}{2}\right)_n\left(1+\frac{a}{2}\right)_n}$$

in (2.1.1) we get :

$$\beta_n=\sum_{r=0}^{n}\frac{(a/3)_r(3/4)^r(-1)^r}{r!\left(\frac{1}{2}+\frac{1}{2}a\right)_r\left(1+\frac{a}{2}\right)_r}\cdot\frac{1}{(1)_{n-r}}(1+a)_{n+r}$$

$$=\frac{(1+a)_n}{n!}\times{}_3F_2\left[\begin{matrix}\frac{a}{3}, & 1+a+n, & -n; & 3/4\\ & \frac{1}{2},+\frac{1}{2}a, & 1+\frac{a}{2}; & \end{matrix}\right]$$

Now, using (2.1.27) we get :

$$\beta_n=\frac{\left(1+\frac{a}{3}\right)_m}{(1)_m}$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = \left(\frac{\alpha}{\beta}\right)_n$ we have :

$$\gamma_n = \sum_{r=0}^{\infty} \left(\frac{\alpha}{\beta}\right)_{r+n} \frac{1}{r!}(1+a)_{r+2n}$$

$$\gamma_n = \frac{(1+a)_{2n}(\alpha)_n}{(\beta)_n} \times {}_2F_1[\alpha+n, \quad 1+a+2n; \quad \beta+n; \quad 1]$$

Now, using (2.1.15) we get

$$\gamma_n = \frac{(1+a)_{2n}(\alpha)_n(\beta-a-1)_n}{(\beta-\alpha-a-1)_n} \times \frac{\Gamma(\beta)\Gamma(\beta-\alpha-a-1)}{\Gamma(\beta-\alpha)\Gamma(\beta-a-1)},$$

provided Rl $(\beta-\alpha-a-1-n) > 0$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(a/3)_n (3/4)^r (-1)^n}{n!\left(\frac{1}{2}+\frac{1}{2}a\right)_n \left(1+\frac{1}{2}a\right)_n} \times \frac{(1+a)_{2n}(\alpha)_n(\beta-a-1)_n}{(\beta-\alpha-a-1)_n} \frac{\Gamma\beta\Gamma\beta-\alpha-a-1}{\Gamma\beta-\alpha\Gamma\beta-a-1}$$

$$= \sum_{n=0}^{\infty} \frac{(\alpha)_n \left(1+\frac{a}{3}\right)_m}{(\beta)_n (1)_m},$$

where m is the greatest integer $\leq n/2$.

(xxii) Choosing

$$u_n = \frac{1}{(1)_n},$$

$$v_n = (1+a)_n$$

and

$$\alpha_n = \frac{(a/3)_n (-1)^n (3/4)^n \left(1+\frac{a}{2}\right)_n}{n!\left(\frac{a}{2}\right)_n \left(\frac{1}{2}+\frac{a}{2}\right)_n \left(2+\frac{a}{2}\right)_n}$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(a/3)_r (3/4)^r (-1)^r}{r!\left(\frac{a}{2}\right)_r \left(\frac{1}{2}+\frac{1}{2}a\right)_r} \cdot \frac{1}{(1)_{n-r}} (1+a)_{n+r} \times \frac{\left(1+\frac{a}{2}\right)_n}{\left(2+\frac{a}{2}\right)_n}$$

$$= \frac{(1+a)_n}{n!} \times {}_4F_3 \left[\begin{matrix} \frac{a}{3}, & 1+\frac{a}{2}, & 1+a+n, & -n; & 3/4 \\ & 2+\frac{a}{2}, & \frac{a}{2}, & \frac{1}{2}+\frac{1}{2}a; & \end{matrix} \right]$$

Now, using (2.1.28) we get :

$$\beta_n = \frac{(-1)^{n-m}\left(1+\frac{a}{3}\right)_m}{(1)_m}$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have:

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} \frac{1}{r!} (1+a)_{r+2n}$$

$$\gamma_n = (1+a)_{2n} z^n \times {}_1F_0 [1+a+2n; \ ; \ z]$$

Now, using (2.1.14) we get :

$$\gamma_n = (1+a)_{2n} z^n / (1-z)^{1+a+2n}.$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{a}{3}\right)_n \left(1+\frac{1}{2}a\right)_n \left(\frac{3}{4}\right)^n}{n!(a/2)_n} \times \left(\frac{2}{(1-z)}\right)^{2n} \times z^n$$

$$= (1-z)^{1+a} \sum_{n=0}^{\infty} \frac{(-1)^{n-m} \left(1+\frac{a}{3}\right)_m z^n}{(1)_m},$$

where m is the greatest integer $\leq n/2$.

(xxiii) Choosing

$$u_n = \frac{1}{(1)_n},$$

$$v_n = (1+a)_n$$

and

$$\alpha_n = \frac{(a/3)_n \left(1+\frac{a}{2}\right)_n (3/4)^n (-1)^n}{n!\left(\frac{1}{2}+\frac{a}{2}\right)_n \left(2+\frac{a}{2}\right)_n}.$$

in (2.1.1) we get :

$$\beta_n = \sum_{r=0}^{n} \frac{(a/3)_r (1+a/2)_r (3/4)^n (-1)^r}{r!\left(\frac{1}{2}+\frac{1}{2}a\right)_r \left(2+\frac{a}{2}\right)_r} \times \frac{1}{(1)_{n-r}} (1+a)_{n+r}$$

$$= \frac{(1+a)_n}{n!} \times {}_4F_3\left[\begin{matrix} \frac{a}{3}, 1+\frac{a}{2}, & 1+a+n, & -n; & 3/4 \\ \frac{a}{2}, & \frac{1}{2}+\frac{1}{2}a, & 2+\frac{a}{2}; & \end{matrix}\right]$$

Now, using (2.1.29) we get :

$$\beta_n = \frac{\left(\frac{a}{2}\right)_n \left(1+\frac{a}{3}\right)_m \left(2+\frac{a}{6}\right)_m}{\left(2+\frac{a}{2}\right)_n (1)_m \left(\frac{a}{6}\right)_m}.$$

Again, putting the above values of u_n, v_n in (2.1.2) and taking $\delta_n = z^n$ we have:

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} \frac{1}{r!} (1+a)_{r+2n}$$

$$\gamma_n = (1+a)_{2n} z^n \times {}_1F_0[1+a+2n; \ ; \ z]$$

Now using (2.1.14) we get :

$$\gamma_n = (1+a)_{2n} z^n / (1-z)^{1+a+2n}.$$

Putting $\beta_n, \gamma_n, \alpha_n$ and δ_n in (2.1.3); we finally get :

$$\sum_{n=0}^{\infty} \frac{\left(\frac{a}{3}\right)_n \left(1+\frac{1}{2}a\right)_n \left(\frac{3}{4}\right)^n (-1)^n}{n!(a/2)_n \left(2+\frac{a}{2}\right)_n} \times (1+a)_{2n} z^n$$

$$= (1-z)^{1+a+2n} \sum_{n=0}^{\infty} \frac{\left(\frac{a}{2}\right)_n \left(1+\frac{a}{3}\right)_m \left(2+\frac{a}{6}\right)_m}{\left(2+\frac{a}{2}\right)_n (1)_m \left(\frac{a}{6}\right)_m} z^n,$$

where m is the greatest integer $\leq n/2$.