CHAPTER I
Introduction

Issues pertaining to economic growth have always held special interest for economists – from the classicists to the present day economic theorists – presumably because economic growth is closely related to, and is often seen as a solution to, a more fundamental economic problem, namely that of economic development. A study of the factors determining growth was central to the interest of the classical economists like Smith, Ricardo, Malthus, Mill and Marx. The years that followed (till about the first half of the present century) represent a relatively unproductive period as far as growth theory is concerned. Economists of the next genre were mostly preoccupied with microeconomic issues relating to productive and allocative efficiency in the late nineteenth century neoclassical tradition, and short run macroeconomic problems such as unemployment as it came to the fore during the inter-war period - the only exception to this being the Schumpeterian analysis of growth and business cycles. Modern growth theory in this sense can be said to be a discovery of economics after the Second World War. In fact, the post World War II period saw the study of the nature and causes of economic growth emerging as the preeminent topic of the era and for nearly two decades a thriving interest in this area gave rise to a vast literature on the subject. From the late 60s however, the general enthusiasm about growth, or rather different theoretical models of growth, waned rapidly as economists had nothing new to offer by way of theoretical explanations. Consequently, growth theory as a subject languished for some time, until it was revived again fairly recently (in the decade of the 80s) in the form of the 'new' growth theory or endogenous growth theory.

Modern growth theory owes its origin to the pioneering work of Harrod (1939;1948;1973) and Domar (1946). While Harrod and Domar, following the Keynesian tradition, emphasized the role of demand in analysing the long run growth of an economy, a separate supply side approach – the so-called neoclassical approach – attracted more professional interest during the decades of the 50s and the 60s and went on to represent the mainstream in the theory of economic growth. The
most recent work in this field – the endogenous growth theory – can also be called an offshoot of the traditional neoclassical school of thought.

The mainstream neoclassical growth literature stems from the seminal work of Solow (1956). Solow’s famous paper, which introduced the neoclassical way of thinking to the problems of economic growth, was actually a response to the questions raised by Harrod. Harrod had shown that in a very simple Keynesian model, it was possible for all variables to grow at the same exponential rate (not necessarily the population growth rate); but that this steady state growth path is most unstable, for the slightest deviation from this path takes the economy further and further away from the steady state. A great deal of theoretical work in the postwar period centered around attempts to discover what were the sources of this instability and how to get around it. Solow’s simple but elegant exposition shows that a steady state path with full employment exists and is stable (unlike Harrod) if one allows for continuous substitution between the factors of production. Subsequent to Solow, a variety of models were developed based upon differing assumptions about technology and technical progress, e.g., the conditions satisfied by the production function, whether technical progress was ‘neutral’ or biased, whether technical progress was embodied or disembodied, whether there was substitution between factors before and after capital goods had been constructed, and more recently whether technical progress is exogenous or is influenced by some endogenous economic factors. The essential point is that these studies involved various alternative assumptions about the supply side of the market, but no concern with the problems of demand, and therefore in this very basic sense were neoclassical.

The earlier models of this genre – from Solow to Uzawa to various vintage models – are based on a proportional savings function where a constant proportion of the aggregate income is saved and this proportion is arbitrarily given. In contrast, the neoclassical growth models in recent years have made use of two broad framework, namely the overlapping generations framework and the optimal growth framework, where the savings propensity is determined optimally by utility maximising households. However the types of savings behaviour underlying these
two frameworks are very different in nature. In the case of optimal growth, households are guided by altruistic motives with respect to their progeny: the utility of the future generations of descendants enters into the objective function of the optimizing households. In the overlapping generations framework, successive generations of households may live together but the present generation saves only to finance its own consumption in later periods; the consumption of the future generation is of no importance as far as the present generation is concerned. The purpose of this study is to take a detailed look into the savings behaviour underlying various neoclassical growth models and analyse the implications of these different assumptions about savings behaviour for the long run dynamics – and more specifically for the stability – of the system. The general findings, which we hope will become clear in the course of the theoretical analyses carried out in the next few chapters, are as follows: in the overlapping generations case, the strong stability result of Solow (1956) no longer holds; stability requires more stringent conditions on the production function and/or the utility function. We derive some sufficient conditions for stability and instability in this case. The optimal growth framework is more robust in the sense that unlike the overlapping generations case, it admits very few equilibria (in most cases a single non-trivial equilibrium) and exhibits stable dynamic behaviour. However, even in this framework problems may arise if households are unwilling to lower their consumption below a certain level either because they have some notion of a minimum necessary consumption (not necessarily determined by biological subsistence requirement), or because their rate of time preference is negatively related to the level of consumption that they are enjoying. In either case, the households may optimally choose a consumption path such that over time, the economy approaches the zero production point. We examine the possibility of existence of a floor and a ceiling, which may arrest the movement of the economy away from equilibrium in such cases of instability. We also discuss the role of the government in this context.

In order to put our work in its proper perspective, at this juncture it becomes necessary to have a look at the existing literature in this field. For this purpose we start with the Harrodian instability problem. Though the classical economists were
very much concerned with the nature and causes of economic growth, and amongst them, Marx did indeed draw our attention towards the inherent tendencies of modern capitalism towards instability – an issue which is very close to the central theme of our analysis, we nonetheless have chosen the famous Harrodian questions as our point of departure.¹ There are basically two reasons for this. Firstly, given the historical perspective, the classical writers were looking at the problem of economic growth from a different angle than that of the present day growth theorists. Writing in the wake of the Industrial Revolution in England, the classical economists were primarily concerned with the transition from the pre-capitalist modes of production to modern capitalism and the dramatic changes that it brought about in almost every sphere of economic activity. In contrast, modern growth theory – the systematic formal modelling of growth – as it developed in the post World War II era, has almost exclusively concerned itself with a highly developed capitalist economy, an economy which has left the pre-capitalist production relations far behind. Also, unlike the classical theorists who looked at the problem of economic growth in its relation to the evolution of the political economy in general, the modern growth theory involves a high degree of abstraction from the actual functioning of the economy and is concerned with “rather esoteric issues”.² As Hahn pointed out, “there is no class conflict, no “rising middle class”, no actual government, no labour unions, no war, no financial panic, no history”.³ It is important to note here that the problem of transition from a ‘less-developed’ to a ‘developed’ economy continued to attract a lot of professional interest in the post war period (as it does even today); but somewhere along the line, these studies got segregated from the issues pertaining to economic growth and were relegated to a separate branch of economics that came to be known as ‘development economics’. Since our work, at least as far as the theoretical modelling is concerned, comes closer to the modern ‘growth theory’ rather than the ‘development theory’ (and is therefore necessarily afflicted with all the maladies associated with the former), we have decided

¹ In one of our later chapters we do discuss the Marxian instability issue and contrast it with the kind of instability problem that we are talking about.
² Sen (1970), pp. 9
generally to leave out the classical writings on economic growth as well as the development economics literature from this survey—except for occasional references.

Secondly, in the latter half of this century, it was Harrod who, to borrow Sen’s words, “set the ball rolling in growth economics”. Much of the work that followed in this field, including the neoclassical growth literature, was inspired—one way or the other—by the propositions put forward by Harrod. In fact, it can be said that the neoclassical growth literature actually developed as a response to the questions raised by Harrod. Therefore in analysing the long run dynamics of the neoclassical growth models, it seems appropriate that we start with the basic Harrodian questions.

Harrod’s theory is a critique of ‘laissez-faire capitalism’. It can be thought of as the dynamic counterpart of the Keynesian doctrine. In the *General Theory*, Keynes focussed on the relation between current investment and current demand (and hence current output). The stock of capital goods and the time path of output unto the present period were seen as historically given; the analysis did not include these predetermined variables explicitly. Harrod went one step further and established a dynamic relationship between the rate of growth of capital stock and the rate of growth of output. The following three postulates constitute the axiomatic basis of Harrod’s theory: “(a) that the level of a community’s income constitutes the most important determinant of its supply of saving; (b) that the rate of increase of its income is an important determinant of its demand for saving; and (c) that demand is equal to supply.” Given this framework, he was concerned with three fundamental questions: (i) is the long run equilibrium consistent with a constant (positive) growth rate? (*Existence of a steady state*); (ii) if so, is this equilibrium stable? (*Stability of the steady state*); (iii) given that population is growing at a constant rate, will the steady growth rate associated with the long run equilibrium be necessarily equal to the population growth rate? (*Possibility of steady state with full employment*).

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4 A similar model was developed independently by Domar (1946). However we focus on Harrod’s formulation only as it brings out more clearly the long run dynamics of the economy and the role of entrepreneurial expectations in determining the long run dynamics.

On the basis of his rather informal analysis, Harrod concludes that there exists a unique warranted line of growth (\(G_w\)), determined jointly by the propensity to save and the quantity of capital required by technological and other considerations per unit of increment of total output, such that if that rate prevails in a period, then it will continue to do so as long as that is physically possible. However, "if the aggregate result by trial and error by numerous producers gives a value for \(G\) (the rate of growth actually realized in the period under consideration) which is different from \(G_w\), there will not be any tendency to adapt production towards \(G_w\), but, on the contrary, a tendency to adapt production still farther away from it, whether on the higher or lower side."\(^6\) Moreover, if \(G_n\) represents 'the rate of advance which the increase of population and technological improvements allow' such that there is always full employment and/or full utilization of the existing capacity, then there is no reason why it should be equal to the warranted rate of growth (unless by mere coincidence). Hence even if the economy is at the warranted path, such a path may (and is most likely to) entail a growing volume of unemployment.

It is important to note here the definition of long run equilibrium vis-à-vis short run equilibrium. We are assuming that the state of the current stock of inventories does not enter into the determination of either ex ante demand or ex ante supply. In other words, the desired stock of inventories that the producers want to maintain is zero; any accumulation or decumulation of inventories is always unwanted. Then short run equilibrium in any time period \(t\) implies that in that period ex ante demand equals ex ante supply; so there is no change in the stock of inventories. On the other hand, an economy will be in long run equilibrium if (a) the economy over time grows at a constant rate and (b) in every period, the entrepreneurs' expectations are fulfilled. Short run equilibrium does not necessarily imply equilibrium in the long run and vice versa.\(^7\) Harrod (as opposed to Keynes) was solely interested in the long run equilibrium and the dynamics associated with

\(^6\) Harrod (1948), pp. 87.

\(^7\) The 'vice versa' part of this statement may not be so obvious. We however have given an example below where an economy, though in equilibrium in the long run, may still be characterised by short run disequilibrium.
it. If the long run equilibrium is found to have a relation with the short run equilibrium, then it is because of the adjustment mechanism implied in the Harrodian framework.

We discuss below the Harrodian instability problem in terms of a demand-constrained model with stock adjustments. A demand-constrained economy can be modeled by introducing specific adjustment mechanisms operating through either changes in the stock of inventories or the degree of capacity utilization. We have assumed that in every period the existing capacity is always fully utilised; any discrepancy between demand and actual production is sorted out by adjusting the stock of inventories. If demand is greater than the output currently produced, then there is a depletion of stocks; on the other hand, if demand falls short of current production, inventories get accumulated. Let us now introduce the following set of notations:

\[ Y_t : \text{realized income (in real terms) in period } t \]
\[ D_t : \text{demand in period } t \]
\[ O_t : \text{output (currently produced) in period } t \]
\[ I_t : \text{investment in period } t \]
\[ K_t : \text{capital stock at the beginning of period } t \]
\[ M_t : \text{stock of inventories at the end of period } t \]

Note that the concept of income we are using here is realized income. If a part of the output is not realized in the market, then that does not constitute a part of the aggregate income by our definition. We further assume that: (a) the price level remains unchanged; (b) there is no depreciation of capital stocks; (c) population grows at a constant rate \( n \); (d) desired stock of inventory accumulation is zero; (e) savings-income ratio is a constant, denoted by \( s \); and (f) the capital-output ratio is given (denoted by \( v \)) – either because technology is represented by a fixed coefficient production function or the rate of interest is given, thereby fixing the capital-output ratio.\(^8\)

\(^8\) For an example of a Harrodian model with adjustments in capacity utilization, see Patnaik (1997), pp. 20-22.

\(^9\) Though Solow interpreted the constancy of capital-output ratio in terms of technology, Harrod seemed to suggest the latter. He has repeatedly mentioned that the capital-output ratio is constant under the assumption that the rate of interest does not change. See Harrod (1948), pp. 22 and 83.
In any period, output sold is equal to the current production plus change in the stock. Therefore the following relationship holds in the form of an identity:

\[ Y_t \equiv O_t - (M_t - M_{t-1}) \]  

(1.1)

Again, in this demand-constrained economy, demand determines the output sold and hence the income. Thus

\[ Y_t = D_t \]  

(1.2)

The entrepreneurs always produce at full capacity:

\[ O_t = \frac{K_t}{v} \]  

(1.3)

The \textit{ex post} capital-output ratio (which we shall call the capital-income ratio, in order to avoid confusion) is denoted by \( v_t \). Therefore

\[ v_t = \frac{K_t}{Y_t} \]  

(1.4)

The aggregate demand has two components: investment demand and consumption demand. Hence

\[ D_t = I_t + (1-s)Y_t \]  

(1.5)

Now, the rate of growth of output is given by

\[ g_t = \frac{O_{t+1} - O_t}{O_t} \]  

(1.6)

Hence from (1.2) – (1.6), the rate of growth of the economy in any period is determined by the constant savings propensity and the income-capital ratio at that period:

\[ g_t = \frac{s}{v_t} \]  

(1.7)

We can write (1.7) in the following form:

\[ g_t = \frac{s}{v} \cdot \frac{v}{v_t} = \frac{s}{v} \cdot \frac{Y_t}{O_t} \]  

(1.8)

Hence the economy will grow at a constant rate if either \( O_t = Y_t \) (in which case the rate of growth is given by \( s/v \)), or \( O_t \neq Y_t \) but both grow at the same constant rate so that their difference also grows at the same rate (resulting in increasing
accumulation or depletion of stocks according as \( O_t > 0 \) or \( D_t < 0 \). In the long run equilibrium whether the economy grows at the rate \( s/\nu \) (which is Harrod's warranted rate) or some other constant rate depends on the specific expectation formation rule attributed to the entrepreneurs. Moreover, expectations have important role to play for the stability of the long run equilibrium path as well. Expectations enter the picture through the investment function. Note that in the above formulation we have not specified any particular investment behaviour. To see how assumptions about the investment behaviour determine the equilibrium growth rate as well as the dynamics of the economy (when it is out of equilibrium), let us consider the following investment function. Suppose the producers always expect a certain constant rate of growth of output (no matter what the actual rate of growth of demand is) and invest accordingly. Obviously, in this case we have a very straightforward investment function given by \( I_t = \bar{g}K_t \), where \( \bar{g} \) is the expected rate of growth of output. (Since in a demand-constrained economy demands are always met, we do not differentiate between investment \textit{ex ante} and investment \textit{ex post}. The investment function represents the producers' investment plans \textit{ex ante}; but these plans are actually carried out and therefore represents investment in an \textit{ex post} sense also.) In this case, the output in the economy will indeed grow at the constant rate \( \bar{g} \) (this is tautologically true). Demand (and hence income) will also grow at the same rate. Given the above investment function, simplification in terms of (1.1) – (1.7) yields:

\[
Y_t - O_t = M_{t-1} - M_t = \left( \bar{g} - \frac{1}{s} \right) K_t
\]

Since capital stock is growing at a steady rate \( \bar{g} \), if \( \bar{g} > s/\nu \), inventories will get accumulated at an increasing rate; on the other hand if \( \bar{g} < s/\nu \), then the stock of inventories will get depleted at an increasing rate.\(^{10}\) But the entrepreneurs' investment decisions are not affected by the change in the stock of inventories; so

\(^{10}\) We are assuming here that (i) cost of accumulation of stocks is zero; (ii) the expected rate of growth \( \bar{g} \leq n \) where \( n \) is the population growth rate; and (iii) the economy has a sufficiently large initial stock of inventories for the system not to be supply-constrained. In any case, with a finite stock of inventories, running out of stock will act as a ceiling to the rate of expansion - just like the full employment barrier.
the economy nonetheless is in a long run equilibrium path – growing at a constant rate and the entrepreneurial expectations are fulfilled.\footnote{This long run equilibrium path is not unique though. Any positive $\bar{g} \leq n$ can be sustained indefinitely, albeit with an ever increasing or ever decreasing volume of inventories. This example also helps us to illustrate the difference between long run and short run equilibrium. Note that in this case while the economy remains on some long run equilibrium path all the while, it can never be in equilibrium in the short run, unless $\bar{g}$ happens to be equal to $s/v$.} What is more, this equilibrium path is completely stable. In fact the economy can never deviate from the equilibrium path!

Of course we need not assume such utterly naïve and unrealistic investment function to show that long run equilibrium may be consistent with a rate of growth other than $s/v$. (Surely producers are not such a stubborn lot who continuously create more capacity even when the inventory stocks are piling up!) Let us assume instead that in any period, the investors expect that the same absolute level of demand will prevail in the next period as well (static expectations about demand). In the absence of depreciation, the only reason for investment is then to bridge the gap between demand and supply. The investment function in this case is given by $I_t = v(D_t - O_t)$. So the rate of growth of output will be $g_t = \frac{D_t}{O_t} - 1$. Clearly in this case the only steady growth rate that is consistent with entrepreneurial expectation fulfillment is the zero growth rate. The point to note here is that in a Harrod-type demand-constrained model, the Harrodian conclusions about the warranted growth path and its stability do not necessarily hold. In these models, where expectations play a pivotal role in determining the long run behaviour of the economy, assumptions about investment behaviour becomes important not only in determining the long run equilibrium, but also in determining the stability of the system. As Rose has shown, if the producers expect output to grow at the warranted rate, then we in fact get a very stable system – a result which is diametrically opposite to Harrod’s.\footnote{See Rose (1963). Also see Hahn and Matthews (1964).}

Harrod himself considered a specific investment behaviour which is quite different from the investment functions discussed above. He defined long run equilibrium as “that overall rate of advance which, if executed, will leave entrepreneurs in a state of mind in which they are prepared to carry on similar
advance”. Though his words are somewhat ambiguous in this respect, he seems to suggest a particular way of expectation formation where the producers form their expectations about the rate of growth of demand. If investment ex post is justified in any period in the sense that actual production equals demand, then producers in the succeeding periods will increase production in the same proportion as it has just been increased. On the other hand, if in any period investment ex post is more than the justified in the sense that not only the output currently produced has been sold entirely out but there is demand for more (i.e., demand exceeds actual production), then in the next period the producers increase the rate of growth of production. The opposite happens when ex post investment is less than justified. Thus Harrod implicitly assumes an investment function of the following form (as noted by Alexander):

$$\frac{I_t}{K_t} = \frac{I_{t-1}}{K_{t-1}} + F\left(\frac{Y_{t-1}}{O_{t-1}} - 1\right); \quad F(0) = 0 \text{ and } F' > 0$$

(1.10)

Incorporating this investment function in the system of equations represented by (1.1) – (1.7), we find that the only steady growth rate which is consistent with entrepreneurial expectation fulfillment is the one where in every period demand (and hence income) equals the output currently produced (i.e., the economy in each period is at the short run equilibrium). And this long run equilibrium rate of growth is uniquely given by the Harrodian warranted rate $s/v$. 15

13 Harrod (1948), pp. 82.
14 Alexander (1950) has rightly pointed out the ambiguity of the words “to carry on the same way”. The producers, continuing to carry on the same way, may expect that the demand will remain constant at the same absolute level instead of expecting the same rate of growth of demand. As we have seen earlier (in our second example of the investment function), in this case the only possible equilibrium rate of growth is zero.
15 Note that Sen’s representation of Harrod model is very different from ours (see Sen(1970), Introduction). Sen considers an investment function which is based on a constant accelerator. This formulation blurs the distinction that Harrod was making between ‘C’ and ‘Cp’, the first representing the capital required for the production of a unit of potential output whereas the second referring to the actual capital-output ratio in the economy. (In our formulation, these two are analogous to ‘v’ and ‘v’, respectively). Thus in Sen, the adjustment mechanism through which the economy reaches a ‘state of rest’ in the short run, and which has important implications for the long run dynamic behaviour of the economy, remains obscure. (Note however that the short run ‘state of rest’ is different from short run equilibrium, because at this ‘state of rest’ the ex ante demand and ex ante supply do not necessarily match).
What happens when actual rate of growth differs from the warranted rate? Suppose in any period \( g_t \) is greater than \( s/v \). Then from (1.7), \( O_t \) is less than \( Y_t \), and there is a depletion of stock. Producers, taking their cues from this unanticipated decrease in the stock of inventories, revise their expectations about the growth rate upward and invest accordingly. This raises the rate of growth in the next period even more, as is clear from eq. (1.10), thus leading to a further depletion in the stock of inventories. The opposite happens when \( g_t \) is less than \( s/v \). Therefore if the economy is not at the warranted growth path, then over time it moves further and further away from this path.

Thus according to Harrod, not only does 'laissez-faire capitalism' fail to guarantee full employment, but it is also associated with an instability problem, which is the fundamental source of the business cycles that the capitalist world is frequently subjected to. In his own words, "the 'warranted' equilibrium growth rate of laissez-faire capitalism, without management or interference, is unstable. ... There is the further question of whether laissez-faire capitalism tends to bring the economy to a full employment position. It was the central doctrine of Keynes, with which I agree, that this is not so." 16 It is true that the source of Harrodian instability lies in his 'inward looking' investment function where the act of capital accumulation constitutes the only stimuli for further accumulation. (In fact Harrod does not provide any meaningful theory for explaining the phenomenon of economic growth. There is a circular reasoning involved here: growth occurs because producers expect it to occur and producers expect it only because growth has occurred in the past. This leaves the growth rate, if we may use Hicks' terminology, 'hanging by its own bootstraps'!) Whether the Harrodian investment function is a realistic description of the entrepreneurial decision-making process in a capitalist world is a different question altogether and need not detain us here. Suffice it to say that Harrod's theory demonstrates, at the very least, the logical possibility of instability under laissez faire capitalism. It also underscores the point

16 Harrod (1973), pp. 45.
that *laissez faire* capitalism (unstable or not) is likely to be associated with persistent unemployment – either Keynesian or Marxian.

The Harrodian conclusions drew responses from various quarters – both neo-Keynesian and neoclassical. Attempts were made to generate models in which determinants of $s$ and $v$ were defined in ways that tended to bring the economy back towards a steady growth path. One such response came from the so-called ‘Cambridge School’, as represented by Kaldor (1955-56), Robinson (1956), Kahn (1959) and Pasinetti (1961-62). They tried to reconcile full employment with steady growth by making the savings-income ratio a variable.

The Cambridge growth models established a relationship between rate of accumulation and distribution of income. Let $W$ and $P$ be the wage income and the profit income respectively, and $s_w$ and $s_p$ represent the corresponding savings propensities such that $s_p > s_w$. Then total savings in the economy

$$S_t = s_w W_t + s_p P_t$$

and the aggregate savings-income ratio

$$s = \frac{S_t}{Y_t} = s_w \frac{W_t}{Y_t} + s_p \frac{P_t}{Y_t}$$

where $\frac{W}{Y}$ and $\frac{P}{Y}$ are nothing but the share of wages and share of profit in the aggregate income. Let $\frac{P}{Y} = \pi$. Then,

$$s = (s_p - s_w) \pi + s_w$$

(1.11)

Thus one can vary the overall savings ratio in the economy by changing the profit share. Let us now look at equation (1.7) and substitute the value of $s$ from (1.11):

$$g_t = \frac{(s_p - s_w) \pi + s_w}{\nu} \lambda_t$$

(1.12)

Eq. (1.12) gives us the equilibrium growth rate when $\lambda_t = \frac{Y_t}{O_t}$ is a constant. (Along the Harrodian warranted path $\lambda = 1$.) Now if in equilibrium full employment has to be maintained, then $g_t = n$. Therefore, from (1.12)
Therefore given \( \lambda \), there exists a unique value of \( \pi \), say \( \pi^* \), which is consistent long run equilibrium with full employment. This in turn gives an investment-income ratio \((s)\) required for steady growth with full employment.

Note however that in our system with adjustments in the inventory stocks, this distribution of income will not be unique in general. As we have seen before, the long run equilibrium in our formulation may be consistent with any constant value of \( \lambda \) (not necessarily equal to one). Therefore given \( n \), \( s_p \) and \( s_w \), the value of \( \pi \) in equilibrium will depend upon the particular value of \( \lambda \). If \( \lambda \neq 1 \), then the economy will still be in the long run equilibrium with output and income growing at the rate \( n \), but this long run equilibrium will be characterised by either an ever increasing or an ever decreasing stock of inventories. Thus in our formulation, unlike Kaldor, the distribution of income associated with the natural rate of growth \((n)\) need not be unique.

Kaldor was of course assuming that at or in the neighbourhood of full employment real income \((Y_i)\) equals output \(O_i\) through adjustments in the price level; thus \( \lambda \) is always equal to 1. In this case the distribution of income associated with full employment rate of growth will indeed be unique. But the fact that such a distribution exists does not necessarily imply that the economy will actually attain that income distribution. Kaldor suggested an adjustment mechanism of the following kind. Suppose full employment is always maintained. Then output will always grow at the natural rate \( n \). If in any period investment demand rises above the required level, aggregate demand will be greater than the full employment output, leading to a rise in the price level (and also the profit margin). But the rise in price level will result in a fall in real consumption, thereby reducing the demand so that in the end demand is again equal to full employment output. On the other hand, if in any period investment demand falls below the required level, then the price level will fall and the mechanism will work in the opposite direction. There are, however, a few snags in this argument. First of all, the adjustment mechanism is not

\[
\frac{(s_p - s_w)\pi + s_w}{\nu} \lambda = n
\]

(1.13)
symmetric. While it may well work when demand is greater than the full employment output, there is no convincing reason why it should work when demand is less than the full employment output. In the latter case, instead of prices falling, it is more likely that there will be a fall in the capacity utilization or a rise in the stock of inventories.

Secondly, the Kaldorian adjustment mechanism fails to operate at all in a model where supply adjusts to demand through changes in the stock of inventories (for example in the model specified by (1.1) – (1.6)). This is so because Kaldor assumes that whenever demand exceeds full employment output, there is a rise in the price level. However, in a model like ours, a rise in demand above the output currently produced can be accommodated (for some time at least) by depleting the stock of inventories, so that the price level does not rise (at any rate, not immediately).

Thirdly, the Kaldorian investment function does not depend on the profit margin. In fact, in Kaldor, investment is the independent variable; it is the share of profits which depends on the investment-output ratio. The level of investment is such determined so as to maintain full employment in the economy. But it is not clear why the producers would at all be interested in maintaining full employment. Kaldor simply assumes that the producers do so, without providing any justification as to why.

Pasinetti extended Kaldor's model by allowing the workers to own a part of the capital stock. Thus the total profit in the economy is now divided into two parts – one part accruing to the workers, the other part accruing to the capitalists. However, in so far as the workers' propensity to save out of wage income and out of profit income remain the same and remain less than the savings propensity of the capitalists, the share of profit will still be uniquely determined by the full employment rate of growth and the capitalists' propensity to save. The difference between Kaldor and Pasinetti is that in Pasinetti, the workers' propensity to save does not enter at all in determination of the equilibrium profit share. But even Pasinetti did not specify how full employment would always be maintained in the
economy. In fact, all the arguments given above against Kaldor's model hold for Pasinetti as well.

A more powerful attack on Harrod came from the neoclassical school – not surprisingly since Harrod was questioning the very axiomatic basis of the neoclassical framework, namely the efficacy of markets. In fact the neoclassical growth theory, as we know it today, actually developed as a critique of Harrod. The pioneers in this school of thought were Solow (1956) and Swan (1956). The favoured neoclassical solution to the Harrodian problem of inequality between warranted and natural rate was to introduce a variable capital-output ratio as the accommodating variable. Thus the Harrod-Domar model was recast as a special case within a wider neoclassical framework. In his 1956 paper, Solow writes, “the characteristic and powerful conclusion of the Harrod-Domar line of thought is that even for the long run the economic system is at best balanced on a knife-edge of equilibrium growth. Were the magnitudes of the key parameters – the savings ratio, the capital-output ratio, the rate of increase of the labor force – to slip ever so slightly from dead center, the consequence would be either growing unemployment or prolonged inflation. . . . But this fundamental opposition of warranted and natural rates turns out in the end to flow from the crucial assumption that production takes place under conditions of fixed proportions. There is no possibility of substituting labor for capital in production. If this assumption is abandoned, the knife-edge notion of unstable balance seems to go with it. . . . When production takes place under the usual neoclassical conditions of variable proportions and constant returns to scale no simple opposition between natural and warranted rates of growth is possible. There may not be – in fact in the case of the Cobb-Douglas function there can never be – any knife-edge. The system can adjust to any given rate of growth of the labor force, and eventually approach a state of steady proportional expansion.”

Solow's exposition can be summarised as follows:

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17 Solow (1956); reprinted in Sen (1970), pp. 161-162. Solow was obviously wrong in identifying the source of instability in Harrod. But we shall come back to this point later.
(a) technology is represented by a production function which satisfies the standard neoclassical properties, namely, continuity, concavity and constant returns to scale. Thus there is continuous substitutability between capital and labour;
(b) a fixed proportion (s) of the aggregate income is saved and invested;
(c) there is always full employment of both the factors;
(d) perfect competition ensures that the wage rate and the rental rate are equal to the marginal product of labour and capital respectively;
(e) population grows at a constant exogenous rate n.

The economy functions in the following way: at each point of time, the capital stock and the labour force are historically given. Therefore the respective marginal products at full employment uniquely determine the wage rate and the rate of interest. On the other hand, the given stock of capital and labour give us a certain output (from the production function) and out of this output a constant proportion is saved and invested – which constitute the capital stock for tomorrow. Thus in each period, the economy is at a momentary equilibrium. The economy will be in long run equilibrium if the dynamic path, which consists all these momentary equilibria occurring at different points of time, approaches some steady state over time. From propositions (a) – (e) above, one can easily derive the fundamental dynamic equation of Solow:

$$\dot{k} = sf(k) - nk$$

where \( k \) is the capital-labour ratio and \( f(k) \) is the per capita output. In long run equilibrium (steady state) \( k \) does not change.\(^{18}\) This gives us the condition for equilibrium as:

$$sf(k) = nk$$

Equation (1.15) determines the long run equilibrium value of the capital-labour ratio. In equilibrium, capital stock as well as output grows at the exogenously given population growth rate, \( n \). Given the assumption about technology, it can be easily shown that if an equilibrium exists, it will be unique and stable. Solow himself recognised that there might be problems of existence of equilibrium; but that can be

\(^{18}\) Note that since there is no independent investment function here, expectations do not enter the picture. Thus any steady state will be a long run equilibrium.
avoided if one assumes the Inada conditions (which are highly restrictive though). The point to note here is that, even without Inada conditions, if an equilibrium exists, the concavity property of the production function ensures that the equilibrium is stable.

The argument is simple but it depends on strong assumptions. It is assumed at the outset that there is full employment and the level of investment is determined by the full-employment savings. The Keynesian problem of effective demand is thus ruled out. If full employment is always maintained and the labour force grows at some exogenous rate \( n \), then it is hard to imagine how the growth rate of production could differ from \( n \) in the long run (in the absence of technical progress). Furthermore, instability of the growth path finds no place in a model where there is no independent investment function. As we have seen before, the Harrodian instability problem depends on the behavioural assumptions about the determinants of the investment function. Sen pointed out that “once an independent investment function is introduced, the instability problem of Harrod quickly reappears in the Solow-Swan model, in spite of replacing the assumption of a constant capital-output ratio by a neoclassical production function.” 19 (Swan did suggest an adjustment mechanism by which the level of investment always equals the full-employment savings. According to him, “effective demand is so regulated (via the rate of interest or otherwise) that all savings are profitably invested, productive capacity is fully utilized, and the level of employment can never be increased merely by raising the level of spending. The forces of perfect competition drive the rate of profit or interest \( r \) and the (real) wage rate \( w \) into equality with the marginal productivities of capital and labour.” 20 But it is not at all clear how the rate of interest alone can perform the two different jobs, namely, that of equating savings with investment and at the same time the job of equating the return to capital with its marginal product at full employment.) The neoclassical growth theory thus eschews the Harrodian instability problem by simply assuming it away.

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19 Sen (1970), pp. 23
20 Swan (1956), pp. 335.
Though the Swan-Solow model failed to provide a satisfactory answer to the Harrodian questions, it nonetheless became a major source of inspiration for a variety of other models, which tried to extend this framework in different ways while maintaining its basic neoclassical character. The popularity of this framework probably lies in the fact that by introducing a variable capital-output ratio, it enables one to tell a logically consistent story involving permanent full employment. The simple Solow-Swan one-sector neoclassical growth model has been extended to incorporate more than one sector; technical progress – exogenous as well as endogenous; and different types of savings behaviour. We shall first discuss the two-sector neoclassical growth model, which is a natural extension of the simple Solow-type one sector model.

The two-sector growth model originated in Meade (1961), but Uzawa (1961; 1963) was the main contributor on the subject. Important insights were also provided by Solow (1961), Inada (1963; 1964), Drandakis (1963), Takayama (1963), Amano (1964), Hahn (1965) and Burmeister (1968). The two-sector models are similar to the neoclassical one sector model in their basic structure. Only in stead of the assumption that the same good is used for consumption purposes as well as capital, we now consider two separate commodities – one consumption good and one investment good. However, the introduction of this rather simple and innocuous looking assumption immediately poses problems for the existence, uniqueness and stability of both short run (momentary) equilibrium and long run steady growth path. Firstly, for a predetermined capital-labour ratio (historically given), there may not exist any momentary equilibrium. Secondly, even if the momentary equilibrium exists and is determinate, it may not be unique; so the long run dynamic path becomes indeterminate. Thirdly, there may not be any equilibrium steady growth path (or balanced growth path), or there may be more than one such path – some stable, some unstable. Like the one sector case, Inada conditions guarantee the existence of at least one balanced growth path, but unlike the one sector case, even the Inada conditions cannot ensure the uniqueness of the balanced growth path or the uniqueness of the momentary equilibrium. Additional and more restrictive assumptions are needed; for example, we may assume that the consumption good
industry is more capital intensive than the capital good industry for all wage-rental ratios (Uzawa); or that the elasticities of substitution are greater than unity in both sectors (Drandakis). Nature of the equilibrium depends also on the assumptions about savings behaviour – whether the savings function is classical or Keynesian. We summarise below some important results that have been derived in this context. 21

1. All of wages are consumed, constant fraction of profits saved.
   
   (a) Sufficient conditions for uniqueness of momentary equilibrium:
      
      (i) capital intensity in consumption good sector ≥ capital intensity in investment good sector;
      
      or
      
      (ii) sum of elasticities of substitution greater than 1.
   
   (b) Balanced growth path always uniquely determined.
   
   (c) System stable if (sufficient conditions)
      
      (i) capital intensity in consumption good sector ≥ capital intensity in investment good sector;
      
      or
      
      (ii) sum of elasticities of substitution greater than 1.

2. Some wages saved, but a smaller proportion than out of profits.
   
   (a) Sufficient conditions for uniqueness of momentary equilibrium:
      
      same as (1a).
   
   (b) Sufficient conditions for uniqueness and stability of balanced growth:
      
      (i) capital intensity in consumption good sector ≥ capital intensity in investment good sector;
      
      or
      
      (ii) elasticity of substitution in each sector is not smaller than 1.

3. Same proportion of profit and wages saved.
   
   (a) Momentary equilibrium always uniquely determined.
   
   (b) Sufficient conditions for uniqueness and stability of balanced growth:

21 The list has been taken from Gadolfo (1971), pp. 454 – 455.
(i) capital intensity in consumption good sector ≥ capital intensity in investment
good sector;
or
(ii) elasticity of substitution in each sector is not smaller than 1.

Things become even more complicated if multiple sectors are introduced. As Hahn (1966) has shown, if one introduces heterogeneous capital goods in a simple neoclassical framework, then not only does the momentary equilibrium become indeterminate, but it also becomes underdetermined in the sense that the number of variable in the system exceeds the number of equations. Far more stringent assumptions are needed in this case to ensure stable dynamic behaviour.

Technical progress has been introduced in the Solovian structure in a variety of ways. The simplest of these approaches is the one which views technical knowledge as "manna from heaven" that brings about an increase in output over time at a given rate. Hence the production function can be written as:

\[ Y(t) = F(L(t), K(t), A(t)) \text{ such that } \frac{\dot{A}}{A} = m \text{ (given)} \]  

But another approach – the vintage approach – considers technical progress to be embodied in the newly produced capital goods so that machines of different vintages have different productivity. These models can be further classified as putty-clay or clay-clay depending on whether the framework allows for ex ante input substitution or not. Models of this genre have been developed, among others, by Johansen (1959), Slater (1960), Solow (1962; 1963), Phelps (1963), Solow-Tobin-Weizsäcker-Yaari (1966), and Sheshinski (1967). In these kinds of models, the possibility of balanced growth can be shown to exist under certain assumptions. Unlike the one sector neoclassical growth model, expectations play an important role here in determining the steady state.

In all the models discussed so far, technical progress – embodied or disembodied– is an exogenous phenomenon, determined by factors lying outside the scope of economic analysis. As Hahn and Matthews pointed out, even the embodied technical progress of the various vintage models “does not as such involve...
necessarily any departure from the assumption that technical progress takes place at an externally given rate. The difference from the alternative approach is merely that now the manna of technical progress falls only on the latest machines." 22 Attempts have been made more recently to incorporate endogenous technical progress in the neoclassical framework. Though one could trace this line of thought to Arrow (1962), it was only in 1980s that this avenue was explored more extensively and a whole new branch of growth theory came into vogue, which sought to endogenise changes in technology through models of market externalities. Some important contributors in this field are Romer (1986), Lucas (1988), Barro (1990) and Grossman and Helpman (1991). But research in this area is still very much in progress and many others have contributed in the substantial volume of literature that has already developed.

Romer revived the concept of endogenous growth based on learning-by-doing, initially employed by Arrow. Romer postulates constant returns to scale at the firm level but increasing returns to scale at the economy level. The source of aggregate increasing returns to scale is the positive externalities to capital investment, not recognised by individual firms, which increases the productivity of the aggregate capital stock. Lucas presented another variant in which technology and endogenous growth are modelled through the existence of two distinct constant returns to scale inputs - physical capital ($k$) and human capital ($h$). Lucas postulated a positive social externality through human capital accumulation; productivity of any individual in the economy is an increasing function of the aggregate stock of human capital. In Barro, government expenditure ($G$) enters as an input in production and is financed entirely through an income tax levied at a constant proportional rate. The source of positive externality lies in the fact that any rise in the physical capital stock leads, through the balanced budget, to a proportional rise in the other input, $G$. A fourth type of endogeneity in technical progress was formulated by Grossman and Helpman, who analysed the open economy implications of endogenous growth based on research and development (R&D). Investment in R&D serves two functions. First it accelerates the introduction of new

capital goods of higher productivity. Second, it provides spillovers onto the aggregate stock of knowledge, reducing the cost both of producing manufactured goods and of further investment. Thus Grossman and Helpman model the spillover effects of R&D investment as the positive externality driving endogenous growth.

It is important to note here that the positive externalities incorporated in various endogenous growth models lead to a divergence between private and social returns, and therefore, between private and social optimal levels of investment. As a result, these models necessitate government intervention in order to drive the economy to the Pareto-efficient social optima. But the endogenous growth models (in most cases) do not allow for the existence of Keynesian unemployment, as there is no effective demand problem. Hence the purpose of the government policy is very different here than what was envisaged by Keynes or Harrod.

The neoclassical growth models discussed so far (with the exception of the endogenous growth theory) share one common feature: they are all based on a proportional savings function where a constant proportion of either the whole national income or of each factor income is saved and this constant proportion is arbitrarily given. While this feature of the savings function may be justifiable on the grounds of convenience and realism, one may nevertheless raise the following question: why should not the savings behaviour of the economy be determined within the system through some optimization exercise? After all, if profit maximisation considerations are assumed — and generally accepted — to guide the entrepreneurial decisions in choice of techniques, utility maximization motives may also regulate the savings behaviour of the households. This argument becomes more relevant in the context of neoclassical growth theory, since "the neoclassicists are committed to an economic theory derived from some kind of rational behaviour" and rationality under conventional economics is defined in terms of optimizing behaviour. It is therefore not at all surprising that two different frameworks have developed within the bounds of neoclassical theory where savings propensity is determined optimally by utility maximising households. These two are the optimal growth framework and the overlapping-generations framework. The neoclassical

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growth theory in recent years, including the endogenous growth models, has made use of either of these two frameworks to determine the aggregate savings in the economy.

Though the central principles of the optimal growth framework may be traced way back to the 1920s (to the pioneering work of Ramsey (1928)), it was only in the 1960s that the economists turned their attention to the various interesting possibilities that this framework had to offer. Since then a thriving interest in this area has given rise to a vast literature on the subject, which constitutes a substantial part of the neoclassical growth theory. The post World War II revival of the optimal growth framework can be attributed to the works of Tinbergen (1956; 1960), Chakravarty (1962), Cass (1965), Koopmans (1965), and von Weizsäcker (1965). The question of optimal growth was initially posed as a problem of the central planner; however the framework is easily adaptable to a decentralized economy where the households are price takers. In the latter case of course one has to assume that the households, though they take the wage rate and the rental rate as given, are nonetheless blessed with perfect foresight so that they can correctly guess the future time path of the wage rate and the rate of interest. Without the perfect foresight assumption, the optimal growth path of the households in a decentralised economy may be indeterminate.\textsuperscript{24}

The optimal growth framework in the context of a decentralised economy typically consists of a representative dynastic household whose generations are bonded together by strong parental altruism. The representative household determines its optimal consumption and accumulation paths by maximising the present discounted value of its utility over infinite horizon. The rate at which future utilities are discounted is generally treated as a constant. The household follows an optimal path along which it saves if it is relatively poor, and dissaves if it is relatively rich, until it reaches the maximum sustainable per capita consumption (given the rate of time preference). The infinite horizon utility maximisation exercise implies that the households are not only concerned about their own welfare, but they are also concerned about the well-being of their children (though in case of

\textsuperscript{24}We shall come back to this point in chapter 4.
positive time preference, the concern about their descendants' welfare becomes progressively less with successive generations of descendants).

The overlapping-generations framework, in contrast, deals with households which are less altruistic in nature. This framework was originally developed by Samuelson (1958) in the context of an exchange economy without production. Diamond (1965) later used this framework in a neoclassical growth model. More recently, Marglin (1984) and Galor and Ryder (1989) have provided useful insights into the dynamic behaviour associated with this framework.

The overlapping-generations framework consists of individuals who live for two periods – working in the first period of their life and being retired in the next. Thus, at every point of time, there are two and only two generations living together – one young and one old. The representative individual of the young generation decides upon his current and future consumption (expected) by maximising his own utility over the two periods that he is going to live. In fact, in the second period of his life, he consumes not only his entire income but also his assets. Thus in this framework, the households are not at all concerned about the welfare of the future generations; savings plans are carried out only to finance consumption after the age of retirement.

As is clear from the basic descriptions of the two frameworks, the types of savings behaviour underlying these two frameworks are very different and will have different implications for the dynamic behaviour of the economy. The question that arises is, if savings behaviour in the economy is determined by households' optimization exercise, does the neoclassical framework still maintain its strongly stable character? In other words, could there be some destabilizing elements operating through the savings propensity, which is no longer a constant? The present study runs precisely along this line. Our study is motivated by the query: what happens to the long run dynamics of the neoclassical growth models, in particular to the stability of the system, if savings behaviour is determined either by life-cycle savings as in the case of overlapping-generations, or by optimal savings in an infinite horizon framework? We show that under both types of savings
behaviour, problems regarding stability may arise and one has to make very restrictive assumptions in order to avoid this problem.

The sequence of chapters in the thesis is as follows: in the first theoretical chapter (chapter two), we introduce the overlapping-generations framework and life-cycle savings behaviour in a standard neoclassical one-sector growth model (Solow-type) and examine the question of stability. We show that the strong stability result of Solow no longer holds in this case; stability requires more stringent conditions on the production function and/or the utility function. We derive certain sufficient conditions for instability. Also, in order to elucidate the nature of the problem, we analyse the issue of stability with specific production functions and utility functions.

In chapter three, we extend this line of inquiry to the standard two-sector neoclassical growth model (Uzawa) and derive similar results.

The next three chapters deal with an infinite horizon optimal growth framework in the context of a decentralised economy. In chapter four, which can be considered as a prelude to chapter five, we consider utility function similar to that of Stone and Geary, i.e., we allow for a minimum necessary consumption level. However, in our formulation, this 'subsistence' consumption level does not necessarily denote biological subsistence. We define it to be some kind of a desired minimum level of consumption that the households wish to maintain. With this kind of modified 'subsistence' consumption, we work out the dynamics of the model. In this chapter we also examine the importance of the perfect foresight assumption in the standard optimal growth model.

The fifth chapter continues with the optimal growth framework. However, unlike the standard case, we now postulate variable time preference, where poor people are assumed to be more impatient. We assume that the households' instantaneous rate of time preference is negatively related to the level of current consumption, and the nature of this relationship is linear. With this assumption, we analyse the long run dynamics and the conditions for stability and instability. We explore the possibility of a low level equilibrium trap such that economies starting
with per capita capital stocks less than a certain critical minimum value face economic retrogression over time.

Chapter six extends this variable time preference model to incorporate non-linear instantaneous time preference function. While analysis of the long run dynamics in general terms becomes difficult in this case, we nonetheless construct an example which illustrates that the basic results derived in the preceding chapter regarding stability and instability of the optimal path holds in this case as well.

Chapter seven analyses the nature and sources of instability in our models and compares them with other models of instability that exist in the literature in relation to *laissez-faire* capitalism. We also discuss the possible upper and lower bounds in our models that may restrict the movement away from equilibrium when the long run equilibrium is not stable.

In the eighth and final chapter of our study, we discuss the role of the state in stabilizing the capitalist system, which may otherwise be unstable. We also analyse the justification for state intervention (if any) in this context. This chapter concludes our discussion.