CHAPTER 1
Chapter — I

Introduction

1.1 — The Boundary Layer Concept:

In the case of fluid motions for which the measured pressure distribution nearly agrees with the perfect fluid theory such as flow past the streamline body the influence of viscosity at high Reynolds numbers is confined to a very thin layer in the immediate neighbourhood of the solid wall. If the condition of no slip were not to be satisfied in the case of a real fluid there would be no appreciable difference between the fields of flow of real fluid as compared with that of a perfect fluid. The fact that at the wall the fluid adheres to it means, however, that friction forces retard the motion of the fluid in a thin layer near the wall. In that thin layer the velocity of the fluid increases from zero at the wall (no slip) to its full value which correspond to external frictionless flow. The layer under consideration is called the boundary layer and the concept is due to Prandtl [31]. The traces left by the particles are proportional to the velocity of flow.
It is seen that there is a very thin layer near the wall in which the velocity is considerably smaller than at a layer distant from it. The thickness of this boundary layer increases along the plate in a downstream direction. It has been observed that the velocity distribution in such a boundary layer at the plate with the dimensions across it considerably exaggerated in front of the leading edge of the plate. The velocity distribution is uniform with increasing distance from the leading edge in the downstream direction in the thickness 'δ' of the retarded layer increases continuously, as increasing quantities of fluid become affected. Evidently the thickness of the boundary layer decreases with decreasing velocity.

We now propose to explain the basic concepts of boundary layer theory with the aid of purely physical ideas and without the use of mathematics. The decelerated fluid particles in the boundary layer do not, in all cases, remain in the thin layer which adheres to the body along the whole wetted length of the wall. In some cases the boundary layer increases its thickness considerably in the downstream direction and the flow in the boundary layer becomes reversed. This causes the decelerated fluid particle to be forced outwards, which means that the
boundary layer is separated from the wall. We then speak of boundary layer separation. This phenomenon is always associated with the formation of vortices and with large energy losses in the wake of body.

In very small viscosities (Large Reynolds Numbers) the frictional shearing stresses \( \tau = \mu \frac{\partial u}{\partial y} \) in the boundary layer are considerable because of large velocity gradient across the flow, whereas outside the boundary layer they are very small. This physical picture suggests that the field of flow in the case of fluids of small viscosity can be divided, for the purpose of mathematical analysis, into two region; the thin boundary layer near the wall, in which friction must be taken into account and the region outside the boundary layer, where the forces due to friction are small and may be neglected, and where, therefore the perfect fluid theory offers a very good approximation. Such a division of the field of flow brings about a considerable simplification of the mathematical theory of the motion of fluids of low viscosity. In fact, the theoretical study of such motion was only made possible by Prandtl [31] when he introduced this concept.
1.2 – Boundary Layer Equations:

In the two dimensional flow assuming the wall to be flat and coinciding with the X-direction. The Y-axis being perpendicular to it, the fundamental equations in the non-dimensional form are

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{dP}{dx} + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{dP}{dx} + \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0.
\end{align*}
\]

(1.2.1)

The boundary conditions are

\[
\begin{align*}
y &= 0; \quad u = v = 0 \\
y &= \infty; \quad u = v
\end{align*}
\]

(1.2.2)

where \( U \) is the free stream velocity assumed to be in the X-direction and \( Re \) is Reynolds number. Further all the non-dimensional quantities are of the order unity.

Considering orders of various terms, it has been estimated that
\[ u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2} \text{ and } \frac{\partial v}{\partial y} \text{ are of } O(1); \]

\[ v, \frac{\partial v}{\partial t}, \frac{\partial v}{\partial x}, \frac{\partial^2 v}{\partial x^2} \text{ are of } O(\delta); \]

\[ \frac{\partial u}{\partial y} \text{ and } \frac{\partial^2 v}{\partial y^2} \text{ are of } O(1/\delta); \]

\[ \text{Re and are of } O(1/\delta^2). \]

Finally since \( \partial P/\partial x \) should be of \( O(\delta) \), the pressure depends only on the \( X \) co-ordinate and on time \( t \). Considering the flow outside the boundary-layer it was obtained that

\[ -\frac{1}{\rho} \frac{\partial P}{\partial x} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x}. \tag{1.2.3} \]

summing up the boundary-layer equation for the two dimensional flow of incompressible fluid over a flat plate are written, returning to dimensional quantities as

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + \frac{\partial U}{\partial x} + U \frac{\partial U}{\partial x} + v \frac{\partial^2 u}{\partial y^2} \tag{1.2.4} \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1.2.5} \]
1.3 – Thermal boundary Layer Equations:

Many cases of the temperature field around a hot body in a fluid stream are of the boundary layer type. There is a very steep temperature gradient normal to the wall and heat flux due to conduction is of the same order of magnitude as that due to convection only across a thin layer near the wall. Hence it may be expected that in conjunction with the velocity boundary-layer there will be formed a thermal boundary layer across which the temperature gradient is very large. If it is therefore possible to introduce into the energy equation simplification of a similar nature to those introduced to the equations of motion.

The thermal boundary layer equation for an incompressible fluid of constant velocity are estimated as

\[
\begin{align*}
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} + \rho g \beta (T - T_\infty) \\
\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0.
\end{align*}
\]

(1.3.1)
The boundary conditions are

\[ \begin{align*}
    y = 0; & \quad u = v = 0; & \quad T = T_w, \\
    y = \infty; & \quad u = U; & \quad T = T_\infty \\
\end{align*} \right. \]  \tag{1.3.2}

where \( T_w, T \) and \( T_\infty \) are the temperatures respectively at wall, inside and outside the boundary layer, \( \beta \) is the coefficient of thermal expansion, \( k \) thermal conductivity, \( g_x \) is the acceleration due to gravity.

1.4 – Magneto Fluid Dynamic Boundary Layer:

Boundary layer phenomena occur when the influence of a physical quantity is restricted to small regions near confining boundaries. This phenomenon occurs when the non-dimensional diffusion parameters the Reynolds number, Peclet number, or magnetic Reynolds number are large. The boundary layers are then the velocity and thermal or magnetic boundary layer and each thickness is inversely proportional to the square root of the associated diffusion number. Prandtl fathered classical fluid dynamic boundary
theory by observing, from experimental flows that for large Reynolds numbers, the viscosity and thermal conductivity appreciably influenced the flow only near a wall. When distant measurements in the flow direction are compared with a characteristic dimension in that direction, transverse measurements compared with the boundary-layer thickness and velocities compared with the free stream velocity, the Navier–Stokes and energy equations can be considerably simplified by neglecting small quantities. The number of component equations is reduced to those in the flow direction and pressure changes across the boundary layer are negligible. The pressure is then only a function of the flow direction and can be determined from the inviscid flow solution. Also, the number of viscous terms is reduced to the dominant term and the heat conduction in the flow direction is negligible. MFD boundary layer flows are separated into two types by considering the limiting cases of a very large or a negligibly small magnetic Reynolds number. When the magnetic field is oriented in an arbitrary direction relative to a confining surface and the magnetic Reynolds number is very small, the flow direction component of the
magnetic interaction and the corresponding Joule heating is only a function of the transverse magnetic field component and the local velocity in the flow direction. Changes in the transverse magnetic field component and pressure across the boundary layer are negligible. The thickness of the magnetic boundary layer is very large the induced magnetic field is negligible. However, when the magnetic Reynolds number is very large, the magnetic boundary layer thickness is small and nearly the same size as the viscous and thermal boundary layers and then the MFD boundary layer equations must be solved simultaneously. A unique attribute of MFD occurs when the magnetic pressure number in the flow direction is also greater than unity. The boundary layer produces an upstream, as well as a downstream wake, through its coupled action with the magnetic field. If the Reynolds number is also very large, the MFD flow can be described by three regions. The external flow will still be uninfluenced by the viscosity and thermal conductivity which are now important only in very thin layers. A third region that separates these inner and outer regions is produced by the mutual changes in the momentum and magnetic field.
Another unique feature of MFD can be demonstrated by considering a layer adjacent to a solid surface in which both the velocity and induced magnetic field change, called the Hartmann layer. In the limit of negligible viscosity and infinite conductivity the Hartmann layer becomes a vortex sheet and a current sheet at the solid surface. The magnitude of the velocity slip and change in shear across this layer can be determined by considering the change of tangential stress across a thin layer of the electrically conducting fluid very near the solid surface, i.e.

\[ \Delta \left( \tau_x + \frac{B_x B_0}{\mu_0} \right) = 0. \]  

(1.4.1)

Therefore

\[ \mu \frac{du}{dy} + \frac{B_0 B_x}{\mu_0} = \text{constant} \]  

(1.4.2)

since changes in the flow direction are considered to be small compared to transverse changes.
Also from the tangential component of Ohm's law

\[
E_z = -\frac{1}{\mu_0 \sigma} \frac{dB_x}{dy} - uB_0 = \text{constant} \quad (1.4.3)
\]

for a constant tangential electric field.

The solutions to these equations can be obtained by addition and subtraction and are

\[
B_x \pm \mu_0 \left( \mu \sigma \right)^{1/2} u = e^{\pm \kappa y/\ell} + \text{constant} \quad (1.4.4)
\]

where \( \ell \) is a typical length and the sum approaches a constant at large distances from the surface.

The exponential form cannot be accepted for the difference solution since it diverges at large distances. The solutions are then

\[
B_x + \mu_0 \left( \mu \sigma \right)^{1/2} u = e^{-\kappa y/\ell} + \text{constant} \quad (1.4.5)
\]

\[
B_x - \mu_0 \left( \mu \sigma \right)^{1/2} u = \text{constant} \quad (1.4.6)
\]
and \( u = \frac{e^{-H_y y/\ell}}{2\mu_0 (\mu \sigma)^{\nu/2}} + \text{constant} \) \hspace{1cm} (1.4.7)

\[ B_x = \frac{e^{-H_y y/\ell}}{2} + \text{constant}. \]

Since the difference is constant across the Hartmann layer then

\[ \Delta u = \frac{1}{\mu_0 (\mu \sigma)^{\nu/2}} \Delta B_x , \] \hspace{1cm} (1.4.8)

even when the viscosity vanishes and the conductivity becomes infinite. Consequently, the slip at the surface is not arbitrary but depends on the strength of the current sheet since the product \( \mu \sigma \) can have any value as \( H_a \) becomes infinite. The corresponding changes in shear stress are finite and balance the induced force

\[ \Delta \tau_x = -B_0 (\mu \sigma)^{\nu/2} \Delta u . \] \hspace{1cm} (1.4.9)

In the various MFD boundary layer flows will be discussed for both limiting magnetic Reynolds number cases.
1.5 – Newtonian and non-Newtonian Fluids:

A Newtonian fluid is that for which the shearing stress between any two adjacent layers is linearly proportional to the shear rate, that is

\[ \tau = \mu \frac{du}{dy} \]  

(1.5.1)

The above equation is also known as Newton's law of friction, where \( \mu \) is defined to be the measure of the viscosity of fluid and depend to a great extent on its temperature.

Non-Newtonian fluids are usually considered to be those for which the relation between shear stress and shear rate is not linear, that is the viscosity of a non-Newtonian fluid is not constant at a given temperature and pressure but do depends on the rate of shear more generally, on the previous kinematic history of the fluid. Non-linear fluids in shear flow may be classified into two ways

A. Time – independent fluids –

In these fluids the rate of shear at any point is some function of the shear stress at that point and depends on nothing-else, thus
\[
\frac{du}{dy} = \dot{\gamma} = f(\tau). \quad (1.5.2)
\]

This equation shows that the rate of shear \( \dot{\gamma} \) at any point in the fluid is the function of the shear stress \( \tau \) at that point. Such type of fluids are non-Newtonian viscous and termed as generalized Newtonian fluid. These have different categories depending on the nature of the function \( f(\tau) \) in equation (1.5.2)

(a) Bingham Plastic Fluids

(b) Pseudo Plastic fluids

(c) Dilitant fluids

(a) Bingham plastic fluids –

Such type of fluid is characterized by a flow curve which is a straight line having an intercept \( \tau_y \) (which is also known as yield stress) on the shear stress axis. The constitutive equation for such type of fluid is given by

\[
\tau = \tau_y + \eta_p \dot{\gamma} \quad ; \quad \tau > \tau_y
\]
and \( \dot{\gamma} = 0 \); when \( |\tau| \leq \tau_y \)

where \( \eta_p \) is the plastic viscosity, the slope of the flow curve. Common examples of such fluids are slurries, drilling muds, greases, oil paints, toothpaste and sludges. The behaviour of the fluid is that its structural rigidity resists any stress less than yield stress. But when the applied stress exceeded the yield stress then the system behaves as a Newtonian fluid under a shear stress \( \tau - \tau_y \), and when the shear stress falls below \( \tau_y \), the structure is reformed.

(b) **Pseudo plastic fluids** –

This category of fluids show no yield value and the flow curve indicates that the ratio of shear stress to the rate of shear, “which is termed as viscosity” falls increasingly with shear rate and further the flow curve becomes linear at very high rates of shear. The rheological relation is

\[
\tau = k |\dot{\gamma}|^{n-1} \dot{\gamma}
\]  

(1.5.3)
where \( k \) is the measure of viscosity and \( n \) is a measure of the degree of non-Newtonian behaviour, greater its departure from unity the more pronounced are the non-Newtonian properties of the fluid. The viscosity for a power law fluid in terms of \( k \) and \( n \) can be expressed as

\[
\eta = \frac{\tau}{\dot{\gamma}} = k |\dot{\gamma}|^{n-1}
\]  

(1.5.4)

For Pseudo - plastic fluids \( n < 1 \) and hence the viscosity function decreases as the rate of shear increases.

The Herschel – Bulkley equation

\[
\dot{\gamma} = \frac{1}{k} (\tau - \tau_y)^n \quad \text{if} \quad \tau > \tau_y
\]

(1.5.5)

\[
= 0 \quad \text{if} \quad |\tau| \leq \tau_y
\]

(1.5.6)

The equation is reduced to the Bingham fluids when \( n = 1 \), to that for power law fluid when \( \tau_y = 0 \) and that for Newtonian fluid when \( n = 1 \) and \( \tau_y = 0 \).
Other fluids describing Pseudo – plastic behaviours are

Prandtl fluid \[ \tau = A \sin^{-1}\left(\frac{\dot{\gamma}}{C}\right) \] (1.5.7)

Eyring fluid \[ \tau = \frac{\dot{\gamma}}{B} + C \sin\left(\frac{\tau}{A}\right) \] (1.5.8)

Powell – Eyring fluid \[ \tau = A \dot{\gamma} + B \sinh^{-1}(C \dot{\gamma}) \] (1.5.9)

Williamson fluid \[ \tau = \frac{A \dot{\gamma}}{B + |\dot{\gamma}|} + \eta_0 \dot{\gamma} \] (1.5.10)

Ellis fluid \[ \frac{1}{\eta} = \frac{1}{\eta_0} + m^{-1/n} \left(\tau^2\right)^{k/n} \] (1.5.11)

Casson fluid \[ \tau^{1/2} = \tau_y^{1/2} + \eta^{1/2} \dot{\gamma}^{1/2} \] (1.5.12)

(c) Dilitant fluids –

These fluids are similar to pseudo – plastic type showing no yield stress but the viscosity increases with increasing rate of shear.

Power law relation, for the index n greater than unity, is applicable for these fluids.
Shear Stress vs Shear Rate Relation for Bingham Fluids

(a) Pseudo – Plastic Fluids (b) and Dilatant Fluids

(c) The Dashed Line Show Newtonian Behaviour
1.6 – Visco-Elastic Fluids:

These fluids possess a certain degree of elasticity in addition to that of viscosity. When a visco-elastic fluid is in motion, a certain amount of energy is stored up in the material as strain energy while some energy is lost due to viscous dissipation. In this class of fluids unlike the inelastic viscous fluids, one can not neglect the strain, however small it may be, as it is responsible for the recovery to the original state and for the possible reverse flow that follows the removal of the stress. During the flow, the natural state of the fluid changes constantly and tries to attain the instantaneous state of the deformed state, but it does never succeed completely. This lag is the measure of the elasticity or the so called “memory” of the fluid. We write the constitutive equations of various visco-elastic fluids.

(a) Rivlin – Ericksen fluids –

Rivlin – Ericksen introduced the constitutive equation as

\[ S = -p I + \phi_1 A_1 + \phi_2 A_2 + \phi_3 A_1^2 + \phi_4 A_2^2 + \phi_5 (A_1 A_2 + A_2 A_1) \]

\[ + \phi_6 \left( A_1^2 A_2 + A_2 A_1^2 \right) + \phi_7 \left( A_1 A_2^2 + A_2^2 A_1 \right) \]

\[ + \phi_8 \left( A_1 A_2^2 + A_2^2 A_1 \right) \]  

(1.6.1)
where, p is an arbitrary hydrostatic pressure and φ's are polynomial functions of the traces of the various tensors occurring in the representation. Matrices $A_1$ and $A_2$ are defined by

$$A_{ij}^{(1)} = \left( v_{i,j} + v_{j,i} \right),$$

$$A_{ij}^{(2)} = \frac{\partial A_{ij}^{(1)}}{\partial t} + v_p A_{ij}^{(1)} + A_{i,p}^{(1)} v_{p,j} + A_{p,j}^{(1)} v_{p,i},$$

$v_p$ being velocity vector.

On neglecting the squares and products of $A_2$, we have

$$S = -p I + \phi_1 A_1 + \phi_2 A_2 + \phi_3 A_1^2; \quad (1.6.2)$$

where $\phi_1$, $\phi_2$ and $\phi_3$ are constants. Usually, $\phi_1$ denotes the coefficient of Newtonian viscosity, $\phi_2$ the coefficient of visco-elasticity and $\phi_3$ the coefficient of cross-viscosity.

Coleman [8] and co-workers have adopted a different approach to obtain the constitutive equation of the form

$$S = -p I + \phi_1 E^{(1)} + \phi_2 E^{(2)} + \phi_3 E^{(1)2} \quad (1.6.3)$$
where \( E^{(1)}_{ij} = v_{i,j} + v_{j,i}, \)

\[ E^{(2)}_{ij} = a_{i,j} + a_{j,i} + 2v_{i,m}v_{m,j}. \]

In the above equations, \( S \) is the stress tensor, \( v_i \) and \( a_i \) are the components of velocity and acceleration in the direction of \( i^{th} \) co-ordinate \( x_i \), \( p \) is an indeterminate hydrostatic pressure and the coefficients \( \phi_1, \phi_2 \) and \( \phi_3 \) are the measure of viscosity, visco-elasticity and cross-viscosity, respectively.

(b) Walters 'Liquid B' with short memory –

Walters proposed the constitutive equation for visco-elastic fluid of short memory in the form,

\[ S^{ik} = -p g^{ik} + \tau^{ik} \quad (1.6.4) \]

where \( S^{ik} \) is the stress tensor, \( p \) is the isotropic pressure, \( g^{ik} \) is the metric tensor.

\[ \tau^{ik} = 2\eta e^{ik} - 2k_0 \frac{\delta}{\delta t} e^{ik}, \quad (1.6.5) \]

where \( \tau^{ik} \) is the deviatoric stress tensor.
\[
\frac{\partial e_{ik}}{\partial t} = \frac{\partial e_{ik}}{\partial t} + v^i e_{ik,j} - v^k e_{ij} - v^i e_{ik} + e_{ik} v^j
\]

= Convected derivative of \(e^{ik}\)

\[e_{ik} = \frac{1}{2} (v_{i,k} + v_{k,i}) = \text{Strain rate tensor.}\]

where \(\eta\) – the limiting viscosity at small rate of shear
\(k_0\) – the elastic coefficient.

(c) Maxwell Fluid

The Maxwell fluid model of constitutive equation representing the stress and strain relation is

\[
\left(1 + \lambda \frac{\delta}{\delta t}\right) \tau^{ik} = 2 \eta e^{ik} \quad (1.6.6)
\]

where \(\tau^{ik}\), is the stress tensor
\(\lambda\), the relaxation time
\(\frac{\delta}{\delta t}\), the convected derivative
\(\eta\), the constant having dimension of viscosity
\(e^{ik}\), the rate of strain tensor.
1.7 – Histological Background of MHD:

Magneto fluid dynamics deals with the study of the motion of electrically conducting fluids in the presence of electric and magnetic fields. It unifies in a common frame work the electromagnetic and fluid dynamic theories to yield a description of the concurrent effects of the magnetic field. Magneto fluid dynamics (MFD) deals with an electrically conducting fluid, whereas its sub topics, magneto hydrodynamics (MHD) and magneto gas dynamics (MGD) are specifically concerned with electrically conducting liquids and ionized compressible gases.

The name of the new branch of science born from such a merger is usually an accurate indication of its origin, astrophysics, physical chemistry, biochemistry and biomechanics being typical example. Magneto-fluid dynamics is useful in astrophysics because much of the universe is filled with widely spaced, charged particles and permeated by magnetic fields. Geophysicists encounter MFD phenomena in the interactions of conducting fluids and magnetic fields that are present in and around heavenly bodies. Engineers employ MFD principles in the design of heat exchangers, pumps and
flow meters; in solving space vehicle propulsion, control and reentry problems; in designing communication and radar system; in creating novel power generating systems and in developing confinement schemes for controlled fusion. It should also be noted that MHD phenomena can be produced without applying a magnetic field to a moving conducting medium. From very beginning it has been demonstrated experimentally that motion of an electric charge or charged medium is accompanied by a magnetic field. This fact is also incorporated into Maxwell’s equations, a cornerstone of classical electrodynamics. Due to the motion of a conducting fluid across the magnetic field the currents are generated and consequently, an induced magnetic field appears that modifies the original magnetic field. The action of the magnetic field on these currents gives rise to an electromagnetic force which perturbs the original motion. Thus MFD is the study of complex interaction between the fluid velocity and electromagnetic fields.

There are many natural phenomena and engineering problems susceptible to magneto fluid dynamic analysis. It is used in astrophysics because much of the universe is filled with widely
spaced, charged particles and permeated by magnetic fields and so the continuum assumption becomes applicable. Again geophysicists encounter MFD phenomena in the interaction of conducting fluids and magnetic fields that are present in and around heavenly bodies. Engineers employ MFD principles in the design of heat exchangers, pumps and flow meters; in solving space vehicle propulsion, control and reentry problems; in designing communications and radar systems; in creating novel power generating systems and in developing confinement schemes for controlled fusion.

The principal MFD effects were first demonstrated in experiments of Faraday [12] and Ritchie [35]. Faraday [12] experimented with flow of mercury in glass tubes placed between poles of a magnet and discovered that a voltage was induced across the tube by the motion of the mercury across the magnetic field, perpendicular to the direction of flow and to the magnetic field. Faraday observed that the current generated by this included voltage interacted with the magnetic field to slow down the motion of the fluid and he was aware of the fact that the current produced its own magnetic field that obeyed Ampere's right hand rule two and thus, in
turn, distorted the field of the magnet. He conjectured that similar phenomena could occur in nature, and in 1832 attempted to measure the voltage induced by the flow of the Thames through the earth's magnetic field, but was unsuccessful because the river bed short-circuited the resultant voltage. However, his theory was confirmed in 1881 by Wallaston [68] who did measure an induced voltage in the much deeper English Channel. Ritchie [35], a contemporary of Faraday, discovered in 1832 that when an electric field was applied to a conducting fluid in a direction perpendicular to both fields.

Faraday [12] also suggested that electrical power could degenerate in a load circuit by the interaction of a flowing conducting fluid and a magnetic field. The practical applications of Faraday's ideas come with Smith and Slepian [55] invention of an instrument for measurement of ship's speed and with Williams MFD flow meter, which was based on the principle that the induced voltage is proportional to the flow rate. Young et al. [74] were the first to study tidal motion with an induced voltage device, a technique since then widely used in oceanography. Further fundamental work in this area
was done by Hart [16] and Hartmann and Lazarus [17], who like Faraday studied channel flow of mercury.

The first astronomical application of the MFD theory occurred in 1930, when Williams [70] suggested that the Sun was a gigantic magnetic system. It remained, however, for Alfven [3] to make a most significant contribution by discovering MFD waves in the Sun. These waves are produced by disturbance which propagates simultaneously in the conducting fluid and the magnetic field. The analogy that explains the generation of an Alfven wave is that of a sharp string pluck while submerged in a fluid. The string provides the elastic force and the inertial force and they combine to propagate a perturbing wave through the fluid and the string. More recently, Ahlstrom's experiments [4] demonstrated that disturbances generated by an aerodynamic body could propagate in the magnetic field and the fluid to produce both upstream and downstream wakes.

Even though the important MFD effects were demonstrated long ago, most of the practical work in this field has been done since 1850 because of the era's interest in high-temperature gases, nuclear
engineering and space technology. Among the conspicuous application of MFD theory are meters for liquid-metal heat transfer systems employed in nuclear reactors. Since 1960, MFD power generation has been the object of a world wide research and development effort.

The first ever Magneto-hydro-dynamics (MHD) power plant in India has been established as a joint research project of the Department of Science and Technology, “Bhabha Atomic Research Centre” and “Bharat Heavy Electricals Limited” at Tiruchirapalli in Tamil Nadu in March 1983. The significance of this pilot plant is that high ash low grade coal, which is found in abundance in India, is being used in this process. The efficiency of this system goes upto 52 percent as against 32 percent of the conventional thermal power plants, besides, it needs 40 percent less water than a conventional thermal plant. It will reduce the atmospheric pollution and if the existing thermal plants are replaced by MHD power plants then the life of coal deposits of our country will be lengthened by over 250 years.
In summary, MFD phenomena result from the mutual effect of a magnetic field and a conducting fluid flowing across it. Thus, an electromagnetic force is produced in a fluid flowing across a transverse magnetic field and the resulting current and magnetic field combine to produce a force that resist the fluid’s motion. The current also generates its own magnetic field which distorts the original magnetic field. An opposing, or pumping force on the fluid can be produced by applying an electric field perpendicularly to the magnetic field. The science of magnetic fluid dynamics is the detailed study of these phenomena, which occur in nature and are produced in engineering devices. Combined free and forced convective hydromagnetic flows through a porous channel with Hall and Wall conductance effect have been studied by Pandey [30], Ravikant [34], Singh [48], and Soundalgekar [56]. Watanke [69] describes the behaviour of two dimensional steady laminar boundary layer flow of an incompressible electrically conducting fluid past a semi-infinite flat plate in the presence of transverse magnetic field by using the difference differential equation. Recently Dormy [10] and Hart [16] discussed the super rotating shear layer in MHD spherical Couette flow.
1.8 – Summary of the Electromagnetic Equations:

The Maxwell equations represent mathematical expressions of certain experiments and can not be proved. However, their applicability can be verified and, as result of extensive experimental research, these equations are known to apply for almost all macroscopic electro magnetic phenomena. The Maxwell equations are now accepted as guiding principles. However, they were obtained for a medium at rest which could be set in motion. The equations can be reconstructed applying relativistic considerations with the result that they are unaltered except that the consecutive equations for contain an additional term. These terms are negligible when the mediums velocity is small compared to the light speed. Thus the constitutive relations for a medium at rest are approximately correct for a medium in motion at modest speeds. The electromagnetic equations are listed again for convenience and the first five are the Maxwell equations. The constitutive equations are –

\[ \nabla \cdot D = \rho_e \quad \text{(Charge continuity)} \]

\[ \nabla \cdot J = -\frac{\partial \rho_e}{\partial t} \quad \text{(Current continuity)} \]
\[ \nabla \cdot \mathbf{B} = 0 \quad \text{(Magnetic field continuity)} \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{(Amperes law)} \]
\[ \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \quad \text{(Faraday’s law)} \]
\[ \mathbf{D} = \varepsilon \mathbf{E} \quad \text{(Constitutive)} \]

and \( \mathbf{B} = \mu_0 \mathbf{H} \)

\[ \mathbf{F}_p = q \left( \mathbf{E} + \mathbf{V}_p \times \mathbf{B} \right) \quad \text{(Lorentz force on a charge)} \]
\[ \omega = \frac{1}{2} \int \left( \mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B} \right) \, dv \quad \text{(Electromagnetic energy)} \]
\[ \mathbf{J} = \sigma \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) + \rho_e \mathbf{v} \quad \text{(Current density)} \]
\[ \zeta = \mathbf{J} - \mathbf{W}_e \mathbf{I} \quad \text{(Maxwell stress tenser)} \]
\[ \mathbf{G} = \mathbf{D} \times \mathbf{B} \quad \text{(Electromagnetic momentum)} \]
\[ \mathbf{P} = \mathbf{E} \times \mathbf{H} \quad \text{(Paynting vector)} \]

1.9 – Magnetohydrodynamic Equations:

In magnetofluiddynamics, we consider a conducting fluid that is approximately grossly neutral. The charge density in the Maxwell equations must then be interpreted as an excess charge density which
is generally not large, so we neglect the displacement current. Thus, the displacement current, excess charge density, excess body force and current due to convection of the excess charge are small. The complete sets of MHD equations are —

\[ \nabla \cdot D = 0, \quad D = \varepsilon \mathbf{E}, \quad \nabla \cdot J = 0 \]

\[ B = \mu_e \mathbf{H}, \quad \nabla \cdot B = 0, \quad J = \sigma [ \mathbf{E} + \mathbf{v} \times B ] \]

\[ \nabla \times \mathbf{H} = \mathbf{J}, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho v_i) = 0 \] Continuity equation

\[ \rho \frac{Dv_i}{Dt} = \rho F_i - \frac{\partial p}{\partial x_i} + (\mathbf{j} \times \mathbf{B})_i + \frac{\partial \tau_{ij}}{\partial x_j} \] Momentum equation

\[ \rho T \frac{DS}{Dt} = \phi + \frac{J^2}{\sigma} + \nabla \cdot \mathbf{q} + \rho \mathbf{R} \] Energy equation

\[ \frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{H}) - \nabla \times [\mu_e (\nabla \times \mathbf{H})] \] Magnetic field equation

the symbols have their usual meaning.
1.10 – Equation of Motion:

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) + u \frac{d p}{dx} + v \frac{d p}{dy} + w \frac{d p}{dz} = X - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{1.10.1}
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) + \frac{d v}{dt} = Y - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \tag{1.10.2}
\]

\[
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) + \frac{d w}{dt} = Z - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \tag{1.10.3}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1.10.4}
\]

In vector notation –

\[
\rho \frac{Dw}{Dt} = F - \nabla p + \mu \nabla^2 w
\]
The Navier–Stokes equations in indicial notation –

\[
\rho \left( \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = X_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left\{ \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial v_k}{\partial x_k} \right) \right\}
\]

(i, j, k = 1, 2, 3)

The Navier–Stokes equation for incompressible fluid is

\[
\rho \frac{Dw}{Dt} = F - \nabla p + \mu \nabla^2 w
\]

where \( \nabla^2 \) denote Laplace operator.

**Navier–Stokes equation to cylindrical co-ordinate –**

If \( r, \phi, z \) denote the radial, azimuthal and axial co-ordinates and \( v_r, v_\phi, v_z \) denote the velocity components in the

\[
\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\phi}{r} \frac{\partial v_r}{\partial \phi} - \frac{v_z^2}{r} + v_z \frac{\partial v_r}{\partial z} \right)
\]

\[
= F_r - \frac{\partial p}{\partial r} + \mu \left( \frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \phi^2} - \frac{2}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial^2 v_r}{\partial z^2} \right)
\]

(A)
\[
\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_{\phi} \frac{\partial v_r}{\partial \phi} + v_z \frac{\partial v_r}{\partial z} \right)
\]

\[
= F_r - \frac{1}{r} \frac{\partial p}{\partial \phi} + \mu \left( \frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \phi^2} + 2 \frac{\partial v_r}{\partial r} + \frac{\partial^2 v_r}{\partial z^2} \right) \tag{B}
\]

\[
\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_{\phi} \frac{\partial v_z}{\partial \phi} + v_z \frac{\partial v_z}{\partial z} \right)
\]

\[
= F_r - \frac{\partial p}{\partial r} + \mu \left( \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \phi^2} + \frac{\partial^2 v_z}{\partial z^2} \right) \tag{C}
\]

\[
\frac{\partial v_z}{\partial r} + \frac{v_z}{r} + \frac{1}{r} \frac{\partial v_z}{\partial \phi} + \frac{\partial v_z}{\partial z} = 0. \tag{D}
\]

The stress component assume the form

\[
\sigma_r = -p + 2\mu \frac{\partial v_r}{\partial r},
\]

\[
T_{ra} = \mu \left[ \frac{r}{\partial r} \left( \frac{v_\phi}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \phi} \right],
\]

\[
\sigma_\phi = -p + 2\mu \left( \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} \right),
\]

\[
T_{\phi z} = \mu \left( \frac{\partial v_\phi}{\partial z} + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} \right),
\]

35
\[ \sigma_z = -p + 2\mu \frac{\partial v_z}{\partial z}, \]

\[ T_{rz} = \mu \left( \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial z} \right). \]

**Cylindrical co-ordinate-**

If \( r, \phi, z \) denote the radial azimuthal and axial co-ordinate. Then the relation between stress strain tensor

\[ \sigma_{rr} = -p + 2\eta \frac{\partial v_r}{\partial r}, \]

\[ \sigma_{r\phi} = \eta \left( \frac{1}{r} \frac{\partial v_r}{\partial \phi} + \frac{\partial v_\phi}{\partial r} - \frac{v_\phi}{r} \right), \]

\[ \sigma_{\phi\phi} = -p + 2\eta \left( \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} \right), \]

\[ \sigma_{r\phi} = \eta \left( \frac{\partial v_\phi}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \phi} \right), \]

\[ \sigma_{zz} = -p + 2\eta \frac{\partial v_z}{\partial z}, \]

\[ \sigma_{x\phi} = \eta \left( \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right). \]
The Navier – Stokes equation in cylindrical co-ordinate-

\[
\frac{\partial v_r}{\partial t} + (v \cdot \nabla)v_r - \frac{v_r^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + v \left( \frac{\partial (v \cdot v_r)}{\partial r} - \frac{1}{r^2} \frac{\partial v_\phi}{\partial \phi} \right)
\]

\[
\frac{\partial v_\phi}{\partial t} + (v \cdot \nabla)v_\phi + \frac{v_r v_\phi}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial \phi} + v \left( \frac{\partial (v \cdot v_\phi)}{\partial \phi} + \frac{2}{r^2} \frac{\partial v_z}{\partial \phi} \right)
\]

\[
\frac{\partial v_z}{\partial t} + (v \cdot \nabla)v_z = -\frac{1}{\rho} \frac{\partial p}{\partial z} + v \Delta v_z
\]

where \( (v \cdot \nabla)f = v_r \frac{\partial f}{\partial r} + v_\phi \frac{\partial f}{\partial \phi} + v_z \frac{\partial f}{\partial z} \)

\[
\nabla f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}
\]

and continuity equation

\[
\frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z} = 0
\]

Spherical co-ordinate-

If \( r, \phi, \theta \) be the radial azimuthal, axial co-ordinate. Then

the stress – strain relation

\[
\sigma_{rr} = -p + 2\eta \frac{\partial v_r}{\partial r}
\]
\[
\sigma_{rr} = -p + 2\eta \left( \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right)
\]

\[
\sigma_{\theta\theta} = -p + 2\eta \left( \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta}{r} \right)
\]

\[
\sigma_{r\phi} = \eta \left( \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} - \frac{v_\theta}{r} \right)
\]

\[
\sigma_{\phi\phi} = \eta \left( \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\phi}{r} \right)
\]

The Navier – Stokes equation -

\[
\frac{\partial v_r}{\partial t} + (v \nabla) v_r - \frac{v_r^2 + v_\phi^2}{r}
\]

\[
= -\frac{1}{\rho} \frac{\partial p}{\partial r} + v \left[ \Delta v_r - \frac{2v_r}{r^2} - \frac{2}{r^2 \sin^2 \theta} \frac{\partial (v_\theta)}{\partial \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi} \right]
\]

\[
\frac{\partial v_\theta}{\partial t} + (v \nabla) v_\theta - \frac{v_r v_\theta}{r} - \frac{v_\theta^2}{r} \sin \theta
\]

\[
= -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + v \left[ \Delta v_\theta + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2 \sin^2 \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi} \right]
\]
\[
\frac{\partial v_\phi}{\partial t} + (v \nabla) v_\phi + \frac{v_r v_\theta}{r} + \frac{v_\theta v_\phi \sin \theta}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + v \left[ \Delta v_\theta + \frac{2}{r^2 \cos \theta} \frac{\partial v_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\theta}{\partial \phi} - \frac{v_\phi}{r^2 \cos^2 \theta} \right]
\]

where 
\[
(v \nabla)f = v_r \frac{\partial f}{\partial r} + v_\theta \frac{\partial f}{\partial \theta} + \frac{v_\phi}{r \cos \theta} \frac{\partial f}{\partial z}
\]

\[
\nabla f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial^2 f}{\partial r^2} \right) + \frac{1}{r \cos \theta} \frac{\partial}{\partial \phi} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}
\]

and the continuity equation are

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 v_z \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta v_\theta \right) + \frac{1}{r \sin \theta} \frac{2 v_\phi}{\partial \phi} = 0
\]

**Energy equation**-

\[
\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \sigma B_0^2 u^2
\]

where

- \( T \) – temperature field,
- \( k \) – conductivity of fluid
- \( \sigma \) – current density,
- \( B_0 \) – magnetic intensity
- \( C_p \) – specific heat at constant pressure
The earliest fundamental works in laminar MHD flow were done in 1937 by Hartmann [18]. In later years several authors contributed much towards the development of laminar boundary layer theory. Important contributions are due to Abbott and Bethel [1], Bansal [5], Chaturani and Bharatiya [6], Gupta [14], Hasan [15], Hassanian [19], Hayasi [20], Katagin and Pop [22], Liao [23], Lighthill [25], Meksyn [26], Mishra et al. [28], Nazar et al. [29], Rosenhead [36], Roy [37], Sacheti and Singh [38], Sakalak and Wang [40], Sarangi and Jose [41, 42], Schilchting [44], Singh [49], Soundalgekar [57, 58], Soundalgekar and Bhat [59], Soundalgekar et al. [60, 61, 62, 63], Stewartson [64], Terril [65], and Yang [72, 73].

Recent contributions to laminar MHD free convection flows have been made by Acharya et al. [2], Chandrakala and Antony [7], Das et al. [9], El-Hakiem [11], Géindreau and Auriault [13], Kafoussius and Williams [21], Lien et al. [24], Merkin [27], Rachana and Agarwal [32], Rahman, and Sakar [33], Sahoo et al. [39], Saville and Churchill [43], Sharma et al. [45], Singh and Singh [46, 47], Singh and Singh [50], Singh et al. [51], Singh and Khem Chand [52], Singh et al. [53, 54], Varshney and Kaushlendra [66], Varshney and Pawsn Kumar [67], Xu and Liao [71] and Zakaria [75].
References


Fluids Caused by an Impulsively Stretching Plate".


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