C H A P T E R - I I

A STEADY STATE INFLATIONARY UNIVERSE

$$$

A STEADY STATE INFLATIONARY UNIVERSE

$$$

2.1 Introduction:

The role of irreversible processes in cosmology has been widely discussed by several authors Martinez, Tirapegui, (1985), Misra, (1978), Misra, B. et al (1983). Prigonine et al (1986) have investigated the role of irreversible processes in creation of matter out of gravitational energy. According to the cosmological principle, the observable universe is isotropic and homogeneous; hence, the only irreversible processes that may arise involve either bulk viscosity or creation of matter. Parker (1977) was the first to consider particle creation based on quantum field theory. However, it is important to consider the effect of particle creation on a macroscopic level (that is when incorporated into Einstein's equations or Brans-Dicke (BD) field equations) and to see whether matter can be generated from some space-time background through an irreversible process; if so it will be positive indication of a relation between entropy and creation of matter in the universe. Consequently the entropy generated through the irreversible process should be incorporated in the basic evolution equations. But the classical Einstein's equations are purely adiabatic and reversible; as such they cannot provide an explanation for the origin of cosmological entropy. It was shown by Prigogine (1947) and Prigogine and Glansdorff (1971) that thermodynamics of open systems when applied to cosmology, leads to a reinterpretation in Einstein's equations of the matter-energy stress tensor (1986). With this point of view, Prigogine et.al. (1988, 1989) extended the concept of adiabatic transformation from closed to open systems. They showed that the usual adiabatic energy conservation laws are modified, thereby including irreversible matter creation.

Recently, there has been a lot of interest in alternative theories of gravitation, especially Brans-Dicke theory of gravity. The latest inflationary models such as extended inflation ((1989, 1989) and hyperextended inflatioin (Steinhardt, P.J. et al. (1990)

are based on BD theory which solves the graceful exit problem in a natural way, without taking recourse to any fine tuning as required in relativistic models. The renewed interest in BD theory is also due to the inadequacy of general relativity to contribute to superunification of the basic interactions and to explain satisfactorily the evolution of galactic structure. Moreover, Maeda (1987) has shown that Einstein's gravitational action of ten dimensional superstring theory is equivalent to BD action in four dimensions with scalar field Ø varying as the sixth power of the radius of Calabi-Yau space. As early as 1971, Morganstern (1971) had concluded that although the BD scalar field may be undetectable at the present epoch, if it exists it must play an important role as one approaches the initial singularity. In 1984, Mathiazhagan and Johri (1984) showed that Ø changes very fast in the early universe, due to which quantum gravity effects might come into play in the same scale as that of grand unified theories (i.e. at $T \sim 10^{15}$ GeV) rather than at $T \sim 10^{19}$ GeV. Hence we feel it would be apropriate to investigate the role of irreversible processes corresponding to creation of matter in BD theory, which is the more relevant theory for the early universe.

In this chapter, we present a new inflationary scenario in BD cosmology based on the irreversible thermodynamics of matter creation out of gravitational energy. As discussed below, the inflation is driven purely by creation of matter and entropy is produced due to induction of the internal energy of the created matter in the open system.

Let us consider a homogeneous and isotropic universe arising from a vacuum fluctuation Johri, Kalyani Desikan (1991), Paul Davies (1984), Prigogine, Geheniau, Gunzig, Nardone, Thermodynamics (1988). Accordingly, the line element is given by

$$(2.1) \qquad ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta\, d\phi^2)\right],$$

where R(t) is the scale factor.

For closed systems, adiabatic transformations (dQ=0) are defined by the relation

$$(2.2) \qquad d(\rho V) + pdV = 0 \quad .$$

The extension to open systems is described by the equation

$$(2.3)\qquad d(\rho V) + pdV - (h/n)\, d(nV) = 0 \quad ,$$

where $n = N/V$ is the particle number density and $h = p+\rho$ is the enthalpy per unit volume of the open system. Note that in the presence of particle creation, the number of particles N in a given volume is not fixed to be a constant.

Equation (2.3) can be written as

$$(2.4)\qquad d(\rho V) = -(p+p_c)\, dV = -\bar{p}\, dV, \quad \bar{p}=p+p_c \quad ,$$

where p is the true thermodynamical pressure and

$$(2.5)\qquad p_c = -(h/n)\frac{d(nV)}{dV} = \frac{-(p+\rho)}{n}\frac{d(nV)}{dV} \quad ,$$

p_c is the supplementary pressure corresponding to creation of matter. It is negative or zero depending on the presence of absence of particle production.

It is worth mentioning in this connection that the role of the negative creation pressure is similar to that of bulk viscosity in the expanding universe as considered by Waga et.al. (1986), Pacher et.al. (1987), and Johri and Sudharsan (1988). It has been shown that the effect of bulk viscosity is to reduce the thermodynamic pressure p by a factor (3 H), where is the coefficient of bulk viscosity.

In the case of isotropic and homogeneous universe, V is chosen to be

$$V = R^3 (t) \quad ,$$

then

(2.6) $$P_c = - (1+\gamma) \frac{\rho}{N} - \frac{dN}{dt} \frac{1}{3H}$$

where $H = \dot{R}/R$ is the Hubble function and

(2.7) $$p = \gamma\rho \quad , \qquad 0 \leq \gamma \leq 1$$

is the equation of state.

Now equation (2.4) can be rewritten as

(2.8) $$\dot{\rho} + 3H (\rho + p + p_c) = 0 \quad ,$$

$$\text{or } \dot{\rho} + 3 (1+\gamma) \rho H = - 3p_c H \quad .$$

For adiabatic transformation in a closed system, as in the case of traditional cosmology, the entropy change dS, vanishes.

While, the entropy change dS, for adiabatic transformation in open systems is given by

$$(2.9) \quad TdS = (h/n)\, d(nV) - \mu d\, (nV) = T(s/n)\, d(nV) ,$$

where $\mu n = h - Ts$ is the chemical potential and $s = (S/V)$ is the specific entropy.

Equation (2.9) gives

$$(2.10) \quad dS/S = dN/N$$

Now the second law of thermodynamics requires that $ds \geq 0$. Therefore, the only particle number variations admitted are such that

$$dN = d(nV) \geq 0 .$$

From this it follows that the transformation from gravitational to matter energy is irreversible.

2.2 Field Equations of BD Theory with Creation of Matter:

In the BD theory, the field equations are given by

$$G_{ab} = -\frac{8\pi}{\phi}T_{ab} - \frac{w}{\phi^2}\left[\phi_{;a}\phi_{;b} - \frac{1}{2}g_{ab}\phi_{;c}\phi^{;c}\right] - \frac{1}{\phi}\left[\phi_{;a;b} - g_{ab}\Box^2\phi\right], \tag{2.11}$$

$$\Box^2\phi = \frac{8\pi}{3+2w}T^a{}_a \tag{2.12}$$

where ϕ is the long range scalar field (which approximately varies as the inverse of the gravitational constant G), T_{ab} is the energy momentum tensor of the universe and w the BD coupling parameter.

The BD field equations (2.11) and (2.12) with the metric given by (2.1) and equation of state (2.7) now become

$$3\left(\frac{\dot{R}}{R}\right)^2 + 3\frac{k}{R^2} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{w}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \frac{8\pi\rho}{\phi}, \tag{2.13}$$

$$(2.14) \qquad 2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{\ddot{\phi}}{\phi} + \frac{w}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 2\,\frac{\dot{R}}{R}\,\frac{\dot{\phi}}{\phi} + \frac{k}{R^2} = -\frac{8\pi\gamma\rho}{\phi} - \frac{8\pi\, p_c}{\phi} ,$$

$$(2.15) \qquad \ddot{\phi} + 3\,\frac{\dot{R}}{R}\,\dot{\phi} = \frac{8\pi}{3+2w}\,[1-3\gamma)\rho - 3p_c]$$

$$(2.16) \qquad \dot{\rho} + 3\,(1+\gamma)\,\rho H = -\,3p_c H \ .$$

In a homogeneous space-time, the scalar field ϕ is a function of time only as such it can be expressed as a function of the scale factor R(t). For the sake of simplicity we assume a power-law relation

$$(2.17) \qquad \phi\,(t) = KR^{\alpha}$$

where K is the proportionality constant and α is the power index.

This assumption is justified in view of the nature of the solution found by Dicke (1962) and the inflationary solution found by Mathiazhagan and Johri (1984).

Neglecting the curvature parameter k for the early universe, with (2.17), the field equations (2.13-2.15) simplify to

(2.18) $$(3+ 3\alpha - \frac{w}{2}\alpha^2)\ (\frac{\dot{R}}{R})^2 = \frac{8\pi\rho}{\phi}\ ,$$

(2.19) $$(2+ \alpha)\frac{\ddot{R}}{R} + (1+ \frac{w}{2}\alpha^2 + \alpha + \alpha^2)\ (\frac{\dot{R}}{R})^2 = - \frac{8\pi\gamma\rho}{\phi} - \frac{8\pi p_c}{\phi}\ ,$$

(2.20) $$[\alpha(\alpha+2)\ (\frac{\dot{R}}{R})^2 + \alpha\frac{\ddot{R}}{R}]\ (3+2w) = \frac{8\pi}{\phi}\ [(1-3\gamma)\rho - 3p_c]\ .$$

2.3 General Solution:

Eliminating $\rho(t)$ and p_c from (2.18), (2.19) and (2.20), we have

(2.21) $$\frac{\ddot{R}}{R} + \beta(\frac{\dot{R}}{R})^2 = 0$$

(2.22) where $$\beta = \frac{w\alpha^2 + 4w\alpha - 6}{2(w\alpha - 3)}\ ,$$

which leads to the exact solution

(2.23) $$R(t) = (1+ t/t_c)^{1/(1+\beta)}$$

$$R(0) = 1,\ t_c = \text{a constant.}$$

Without loss of generality, we can choose the arbitrary constant of integration so that $t_c = 1$.

Hence, we ahve

(2.24) $$R(t) = (1+t)^{1/(1+\beta)} .$$

The above solution represents an expanding universe provided $(1+\beta) > 0$. Further, from (2.21) we see that the expansion would be accelerating ($\ddot{R} > 0$) if $\beta < 0$.

By virtue of (2.22), this constrains the exponent in (2.7) to lie in the range

(2.25) $$\frac{3}{2w} < \alpha < \frac{2}{w} ,$$

which turns out to be a necessary condition for inflation to occur.

From (2.17) we have

(2.26) $$\phi(t) = K(1+t)^{\alpha/(1+\beta)} .$$

Using (2.24) in (2.18) we get

(2.27) $$\rho(t) = \rho_0 (1+t)^{(\alpha/1+\beta))-2} , \quad \rho(0) = \rho_0 ,$$

where $$\rho_0 = \frac{3+3\alpha - w\alpha^2/2}{(1+\beta)^2} \cdot \frac{K}{8\pi}$$

Using (2.24) and (2.27) in (2.16) we have

(2.28) $$P_c = -(\rho_0/3)[-(2+2\beta - \alpha)+3(1+\gamma)] (1+t)^{(\alpha/1+\beta)-2} .$$

2.4 Particular Solutiion:

As a particular case we present here a simple inflationary model driven by creation mechanism

Choose

(2.29) $$\alpha = 4/(2w+1) .$$

Note that it lies in the range given by (2.25). From (2.22) we have

(2.30) $$\beta = (\alpha/2) - 1 \quad \text{or} \quad \alpha = 2\,(1+\beta)\ .$$

This leads to an inflationary model with scale factor, BD scalar field and energy density being given by

(2.31) $$R(t) = (1+t)^{w+1/2}\ ,$$
$$\phi(t) = K\,(1+t)^2\ ,$$
$$\rho(t) = \rho_o\ ,$$

respectively. This model has a non-singular origin with expansion starting at t = 0 when

$$R\,(0) = 1\ ,$$
$$\dot{R}\,(0) = 1/2\,(2w+1) = 0(w)\ ,$$
$$\ddot{R}\,(0) = w^2 - 1/4 = 0\,(w^2)\ ;\ w >> 1\ .$$

Again from (2.28) we have the creation pressure

(2.32) $$p_c = -\,(1+\gamma)\,\rho_o\ .$$

But by (2.6)

$$p_c = -(1+\gamma)\frac{\rho}{N}\frac{dN}{dt}\frac{1}{3H} = -(1+\gamma)\frac{\rho_0}{N}\frac{dN}{dt}\frac{1}{3H} \quad (2.33)$$

Hence by comparing (2.32) and (2.33), we have

$$\frac{1}{N}\frac{dN}{dt} = 3H = 3\,(w+1/2)\,(1+t)^{-1} \quad (2.34)$$

It follows from (2.10) that

$$\frac{\dot{S}}{S} = \frac{\dot{N}}{N} = 3\,\frac{\dot{R}}{R} \quad (2.35)$$

From (2.34) and (2.35)

$$\left.\begin{aligned} N(t) &= N_o\,(1+t)^{3(w+1/2)} \\ S(t) &= S_o\,(1+t)^{3(w+1/2)} \\ n &\equiv N/R^2 = N_o,\ \text{a const} \end{aligned}\right\} \quad (2.36)$$

Since $\rho = \rho_o$ and $N = N_o$ initially when $t = 0$, the universe starts from a vacuum fluctuation with finite density and N_o particles in it. Obviously, the number of particles and the entropy increase indefinitely in this model, while the energy density, thermodynamic pressure, creation pressure and particle number density remain stationary. It furnishes an example of an inflationary steady-state model.

2.5 Concluding Remarks:

By including irreversible creation mechanism in BD theory, we can obtain a variety of inflationary models without any fine tuning or artificial assumptions, corresponding to various values of α within the range prescribed by (2.25).

The particular choice of α in (2.29) leads to the steady state inflationary model dealt with in section 4. It is evident from (2.31), (2.35) and (2.36) that the rate of creation of particles in this inflationary model just compensates for the dilution of density due to expansion. As such the three physical parameters energy density, pressure and particle number density remain constant throughout. However, this technique can be used to discuss evolutionary models with inflation also. It is worth pointing out that Mathiazhagan and Johri (1984), Steinhardt and La (1989) have obtained the same expressions for the scale factor R(t) and the scalar field $\phi(t)$ in the inflationary scenarios discussed by them in the context of the evolutionary models.

*****