The potential of graphs lie in their ability to abstract a situation, and to present an overview of a given problem by means of points and links representing relationships. The applications of graphs range from the field of social sciences where they are used to model organizational structure or social hierarchy, to that of electrical engineering to analyze electrical networks. Their applications in the field of computer science are numerous: the use of dependency graphs to represent data dependencies in relational databases, flow graphs in code optimization of compilers, petrinets in modelling and simulation are just to mention a few. Needless to say, the curiosity of an interested researcher will lead him to probe further into labeled graphs. And what better tool to do so than to use matrices which enable a thorough and efficient algebraic manipulation of graphs, at the same time allowing us to be within the familiar precincts of matrix theory.

It will be accepted, in general, that every matrix has a labeled graph corresponding to it. For example, in the case of real matrices the labels will be real numbers and similarly for other kinds of matrices. It should be clear that the study
of labeled graphs amounts to the same thing as the study of matrices. The hierarchy we have chosen is from the most structured one to the most primitive one. Once the equivalence between labeled graphs and matrices is accepted, it is difficult to resist the temptation to look for those concepts of graphs which correspond to the ideas in matrix theory. For example, can we talk about an eigenvalue of an automaton. It is questions of this kind that we have considered in this thesis.

The hierarchy we consider in this thesis is,

- real matrices
- nonnegative matrices
- natural matrices
- boolean matrices, and
- regular matrices

and hence the chapters are titled accordingly. We briefly talk about each of these.

Real matrices [8] are discussed in chapter 2. The chapter mainly consists of definitions and basic results on real matrices, building the base for all the other chapters. Compound matrices [1] have been explained in quite some detail due to their extensive use in later chapters. Two variations row compounds and column compounds are defined. The matchant of a symmetric matrix is defined on similar lines as the pfaffian. Another important definition given here is that of the intrinsic polynomial of a matrix A.

Chapter 3 of the thesis deals with matrices with nonnegative elements [17]. It is known that irreducible matrices [12,17] have strongly connected graphs corresponding to them. An attempt has been made in this chapter to give graph theoretic proofs and interpretation of the results for irreducible matrices. An intuitive proof of the Frobenius–König theorem [17] is given. New terms like cyclant and circuit polynomial are also defined here.

In the chapter on natural matrices, we have attempted to study graph properties in terms of its associated matrices. In the process of fixing up new terminology
to suit our definitions, we have used terms like *lasso* and *noose*. The relation between permanents and matchants in the adjacency matrix and matchings in the bigraph has been studied. Theorems relating compounds of the adjacency matrix to matchings are then stated. An interpretation of the Frobenius–König theorem in relation to complete matchings in a bigraph is also given.

Matrices with boolean elements have been discussed in chapter 5 of the thesis. Here the adjacency matrix of a graph is written as a 0–1 boolean matrix. This means every nonzero element of the natural matrix is replaced by '1', which is equivalent to shrinking all parallel edges to a single edge. It is seen that the row and column compounds of boolean adjacency and incidence matrices are related to the cover and independent sets of the graph. More results on irreducible matrices have been discussed in relation to the corresponding strongly connected graph.

Chapter 6 of the thesis deals with matrices with regular expressions [15] as their elements. A *finite automaton* (FA) is defined as a labeled digraph, the labels of edges being symbols from an alphabet [22]. The adjacency matrix of a finite automaton is a *regular matrix*, conversely, there is a FA corresponding to every regular matrix. The *closure* $A^*$ of a regular matrix $A$ is defined as,

$$ A^* = I + A + A^2 + \cdots $$

Various methods of computing $A^*$ are discussed. The transition between regular matrices and natural matrices, when the FA has only one symbol, is considered. The closure of a matrix with real elements is also discussed.

From what has been said above, it is clear that the complex algebraic structure called matrices can be visualized physically as labeled graphs. A vast amount of literature available on matrices talks about a plethora of properties on matrices. Can we visualize some of these properties in terms of graphs, that is the thrust of the investigations in this thesis. For a brief listing of our accomplishments, see the conclusions at the end.
Before we end this chapter, a word about our notations and conventions. One would agree that a thoughtfully chosen set of notations and a well defined terminology is a great aid to the understanding of any subject. Motivated by this fact, we have chosen our notations to be simple so as to facilitate an easy reading and understanding of the text. Even though the notations of graph theory and matrix theory are obviously well known, we found it appropriate to recall it when they are used for the first time. In all the chapters, definitions are given at the beginning and are supplemented later as and when required. An attempt has been made to include the definitions of most of the terms used to avoid ambiguity and to make the discussion self-contained. This has lead sometimes to a repetition of what is already present in the literature. Although most of the definitions are conventional, at times we have deviated from the convention to maintain symmetry of definition. Examples have been included for all the new terms and for most of the results and theorems.