Chapter-I

INTRODUCTION
1.1 Introduction to fluid mechanics:

It is well known that a large portion of the earth, including the interior of the earth and the atmosphere above the earth is in a fluid state and hence many scientists from different branches of science and engineering are concentrating on knowing more and about fluids. The studies of fluids in pipes and channels have attracted many civil engineers. The study of fluid machinery such as pumps, compressors, bearings, diffusers and so on has attracted many mechanical engineers. The flow of fluids over obstacles has attracted many aeronautical and space engineers. Since the atmosphere and ocean are fluids, in meteorology and oceanography fluid mechanics play an important role. So, the importance of the fluid mechanics is obvious in engineering, science and technology. The problems in fluid mechanics as cited above are generally nonlinear in nature and are not amicable for analytical study except in some trivial cases. In such situations, we have to resort to the computational fluid dynamics. This requires a proper understanding of numerical analysis and basis concepts of fluid mechanics.

Numerical analysis is a mathematical discipline concerned with methods for generating numerical solutions when analytical procedures are either tedious or nonexistent. Although this subject is as old as classical algebra, it gained momentum from the year 1947 when the Institute of Numerical Analysis was founded at the university of California, Los Angeles, USA. Since then it has become an important tool to solve wide spectrum of non-linear problems that arise not only in fluid mechanics but also in many other practical situations. Accurate simulation of dynamical processes related to metrology and Oceanography, planetary motion, chemical reactions and oscillations, turbulent flow, diffusion and transport processes
and so on, require the knowledge of solving the non-linear equations. If these equations pertain to fluids, then it is called computational fluid dynamics (CFD). As stated above it requires a basic concept of numerical analysis as well as fluid mechanics.

1.2. Basic Principles of Fluid Mechanics:

1.2.1 What is Fluid?

Fluid is a substance which deforms continuously ,without limit, under the action of internal and external forces. In other words, a fluid is a substance which can not resist a shear stress without moving as can a solid.

The main difference between a deformable solid and a fluid is that, a solid material subject to a shearing stress will deform up a certain point and then ceases to deform even it is subjected to shear stress called classic material ,whereas fluids subjected to a shearing stress will deform at a certain rate and continue to deform at the same rate as the stress is applied.

1.2.2 Classification of fluids

The two classifications of fluids are liquids and gases. A liquid has intermolecular forces which hold it together so that it has volume but no definite shape. A liquid poured in to a container will fill the container upto the volume of the liquid and will take the shape of the container on all sides except the surface at the top. If \( v \) is the characteristic velocity of fluid and \( a \) is the velocity of sound then ratio \( \frac{v}{a} = M \) is the dimensionless number called the ‘Match number’. For a liquid generally \( M < 0.5 \).

A gas on the other hand has molecules which collide with each other tending to disperse so that a gas has no definite shape or volume. A gas poured into a
container, regardless of the amount of gass and size of the container, fills the container completely.

The difference between a gas and liquid is very well explained by a simplified molecular theory which states that all fluids are mainly of empty space with a scattering of billions of molecules per cubic inch. The true nature of molecule is very complicated but for our purpose we assume that they are spheres. The molecules have an attraction for each other when they are at a distance greater than the diameter of the molecule and they have strong repulsion for each other when the distance between them is less than the diameter of the molecules. This repulsion acts as a great force to keep the molecules from colliding. Thus the molecules in a gas are so apart in relation to their diameter, whereas in liquids the molecules are much closer together.

A solid has volume and shape, a liquid has a volume but no definite shape, a gas has neither. In solid mechanics, we usually concentrate on a single particle in space and usually ask the question, what is the position of that particle in space as a function of time? From this, we usually find all other physical quantities such as velocity and acceleration. In this case since matter is infinitesimally divisible, it is meaningful to define the velocity of a material at a point as its time rate of displacement and similarly other physical quantities.

In fluid mechanics we are not dealing with a single particle but are concerned with a bulk. This is because, the fluids are not infinitely divisible. Here, we usually ask the question at some point in space what is the velocity, acceleration and thermodynamics property as a function of position and time? In fluids, one can comprehend the velocity only of molecules, but the velocity of a geometrical point in empty space between the molecules is physically meaningless. It is very difficult to study the fluid motion directly with molecules. Fortunately, although there is an empty
space between the molecules the number of molecules per unit volume of fluid (liquid or gas) under ordinary conditions is extremely large. A mole of fluid under ordinary conditions has approximately \( 6.024 \times 10^{23} \) molecules (Loschmidt number) and it occupies, under normal conditions, a volume of 22.4 liters, so that 1 cm\(^3\) contains 2.68 \( \times \ 10^{19} \) molecules. For such a small volume this is an exceedingly larger. Thus fluids under ordinary conditions can be considered, for all practical purposes, to be continuum.

1.2.3 CONTINUUM:

What does this mean in a fluid sense? By a continuum we mean a medium in which every sub region however small be is completely filled with matter and hence we can assume that all the mathematical limiting processes such as limit, continuity and so on can be taken in the meaningful sense. This means that a volume of a fluid can be continually subdivided into smaller and smaller volumes while retaining the fluid continuum character and the velocity of a fluid particle can be considered to be a velocity of a material situated at the centre of the mass of fluid particle as a fluid can indefinitely divisible. Obviously this division would break down eventually, but we assume that by that time is dimensions are so small that they are microscopic and of no concern to continuum theory and one has to resort to kinetic theory.

Thus for a continuum hypothesis, the average between fluid particles, more precisely, the mean free path defined as the average distance between two successive collision, is much less than a physical dimension. If \( \lambda \) is the mean free path, and \( \ell \) is physical dimension then \( \frac{\lambda}{\ell} = k \) is a dimensionless number called the Knudsen
number. If $k < 0.01$ the continuum approach is applicable. If $k \geq 1$ the continuum approach fails and we have to use the non-continuum. I.e, kinetic theory approach which is applicable for example to rarefied gases.

1.2.4 COMPRESSIBLE AND INCOMPRESSIBLE FLOWS:

Fluids, as explained in section 1.2 are classified as gases and liquids. Gases are compressible which suffer changes in volume when ever the pressure changes and their density changes readily with temperature and pressure. Liquids are practically incompressible, i.e., their volumes do not change when the pressure changes.

In gases the density is related to temperature and pressure called equation of state, for example for a perfect gas $p = \rho RT$. In liquid, the density is related to temperature by a coefficient of expansion just as solid and the pressure dependence must be written as $dp = \beta \frac{dp}{\rho}$, $\beta$ is the bulk compression modulus. For water $\beta = 3 \times 10^{15}$ psi, so that it requires enormous pressure to effect the slight density change. Before discussing a fluid in motion it is advantageous to know the variation of pressure $p$ and density $\rho$ in a static fluid, i.e., in hydrostatics. These will be discussed, later, in section 2.

1.2.5 VISCOUS AND NON-VISCOUS FLOW

Viscous or non-viscous fluid will depend on its viscosity. To understand viscosity one should know the strain and stress.

1.2.6 Shearing Strain and Rate of Shearing Strain:

Consider a physical configuration shown in fig 1. It consists of fluid between two plate $A$ and $B$. The $A$ is fixed whereas the plate $B$ moves with a uniform velocity, say $u_B$. Let the plates be of infinite length and $y = h$ be the distance
between the plates. Let a constant force $F$ be applied to the plate $B$ in the $x$-direction so that the plate $B$ moves with a uniform velocity $u_B$ with respect to fixed plate $A$ in the $x$-direction.

![Diagram of fluid between two plates](image)

Figure 1: Fluid between two plates

Let us replace the fluid between the two plates by a deformable solid so that the behavior of the plates would be different. The applied force $F$ would produce a displacement, say $d$, a shear stress $\tau$ in the material and a shear strain, say $\gamma$. After a small displacement the upper plate ceases to move.

Then replace the deformable solid by a fluid and again apply the force $F$ in the $x$-direction, so that the plate $B$ will move continuously with a velocity $u_B$. At time $t=0$ the fluid is denoted by a vertical dashed line in figure 1.1. The angle between the vertical line and inclined line is defined as shearing strain denoted by $\gamma$. Then the derivative of $\gamma$ with respect to time, namely $\frac{d\gamma}{dt}$ is known as the rate of shearing strain. For small displacements $\tan d\gamma \approx d\gamma$

By definition

$$\tan d\gamma \approx \frac{\text{opposite}}{\text{adjacent}} = \frac{u_B dt}{h}$$
Then, the rate of shearing strain can be written as

$$\frac{d\gamma}{dt} = \frac{u_B}{h}$$

Which has the units of \( \frac{1}{\text{second}} = \frac{1}{s} \) and is denoted by \( \gamma \).

We note that the velocity of the fluid adhering to the fixed plate A is zero and that at the moving plate will be the velocity of the plate \( u_B \). If \( u \) is the velocity of the fluid then \( u = 0 \) at the fixed plate A, at \( y = 0 \) and \( u = u_B \) at the moving plate B, at \( y = h \). This is called slip-boundary condition in the case of viscous fluid, defined as the velocity of the fluid at the boundaries is that of the boundaries. If \( u \) is the velocity then \( \frac{du}{dy} \) is the velocity gradient.

1.2.7 Shear Stress:

In figure 1, the shear stress, denoted by \( \tau \) on the top of the element of fluid results in a force that moves the element down stream. The shear stress at the bottom of the element resists that movement. In figure 1.1, the fluid elements moves with a uniform velocity without rotation then the shear stress, say \( \tau \), on the element must be the same as the shear stress, \( \tau \). Then

$$\frac{d\tau}{dy} = 0 \quad \text{and} \quad \tau_{\lambda} = \tau_{\text{wall}}$$

By definition, the shearing stress at the walls may be represented by

$$\tau_{\lambda} = \tau_{\text{wall}} = \frac{\text{force}}{\text{plate area}} = \frac{F}{A}$$
The shearing stress, $\tau$ on a fluid is related to the strain, $\gamma$. The relationship between the shearing stress, $\tau$ and the rate of shearing strain, $\frac{d\gamma}{dt}$, is determined by the fluid property known as viscosity, denoted by $\mu$.

### 1.2.8 Viscosity:

Usually, viscosity $\mu$ is visualized through the graph of stress $\tau$ against the rate of shearing strain $\gamma$ as shown in following figure 2.

![Figure 2: Stress verses rate of shearing strain for different Fluids.](image)

### 1.2.9 Newtonian and non-Newtonian fluids:

The viscosity $\mu$ is measured by the slope of stress –shearing rate curve. For natural fluids like water, air, oil, and so on, viscosity does not vary with rate of strain. That is fluids with constant viscosity known as Newtonian fluids. For Newtonian fluids shear is linearly proportional to the rate of strain. That is

$$\tau = \mu\gamma$$  \hspace{1cm} 1.1
Where $\tau$ is the shear stress, $\mu$ is the viscosity of fluid and $\gamma$ is the rate of shearing strain. For non-Newtonian fluids there exists a nonlinear relationship between $\tau$ and $\gamma$. For such non-Newtonian fluids viscosity $\mu$ will change and will be a function of the rate of shearing strain that is $\mu = \mu(\gamma)$.

This means the slope of the shear stress/shear rate curve is not a constant. Non-Newtonian fluids are usually classified as shear thinning, shear thickening and Bingham plastic.

In shear thinning non-Newtonian fluids the apparent viscosity $\mu_{ap}$, decreases with an increase in shear rate. Apparent viscosity is defined as the instantaneous slope of the shear stress/shear rate curve. Paints are good example of shear thinning fluids. At low shear rate blood is also shear thinning fluid.

Shear stress verses rate of shearing strain for some non Newtonian Fluids.

Shear thickening fluids are non-Newtonian fluids whose apparent viscosity, $\mu_a$, increases with an increase in shear rate. Good examples of shear thickening non-Newtonian fluids are a mixture of com-starch and water, Quicks and so on, Bingham plastic is neither a fluid nor a solid and it can with stand shear load but can flow as a fluid when that shear stress is exceeded. Good examples of Bingham plastics are tooth paste, muddy fluid and blood. Blood behaves as a solid at shear rates very close to zero.

1.2.10 Kinematic viscosity:

$$\nu = \frac{\mu}{\rho},$$

is called kinematic viscosity where $\rho$ is the density and $\mu$ is the viscosity of fluid. The SI units of $\mu$ are $\text{Ns/}m^2$ and that of $\nu$ are $m^2/s$.

The viscosity is measured by $\mu = \frac{\tau}{\gamma}$ and $\gamma = \frac{u}{h}$. 
All fluids have viscosity which causes friction depending on the type of fluid and physical configuration, or flow pattern. If the friction is negligible, then the fluid is called ideal or non-viscous or inviscid.

A non-viscous (or Ideal) fluid is a continuous fluid substance which is incapable of exerting a shearing stress, however small, during the motion of fluid. A viscous fluid is a continuous fluid substance which can exert shearing stress during the motion of fluid. The viscosity will arise due to the internal friction in the fluid and it is measure of the fluid’s resistance to shear when the fluid is in motion. That is, if \( \mu \) is the viscosity, then

\[
\mu = \frac{\tau}{\gamma} = \frac{\tau}{\gamma}
\]

The difference between viscous and non-viscous fluids:

(i) In the case of viscous liquid there is no-slip at the rigid boundary i.e., the liquid will adhere to the rigid boundary and the velocity of liquid at the rigid boundary will be that of the boundary. If the boundary is at rest, then both normal and tangential velocities are zero at the boundary. In the case of non-viscous fluid there is a slip at the boundary i.e., liquid will not adhere to the boundary and in this case only the normal component of velocity is continuous at the boundary (solid or a free surface).

(ii) In the case viscous fluid there is resistance or internal friction whereas in the case of non-viscous fluid there is no internal friction. Physically this means that in the case of viscous fluid, there is energy dissipation between the fluid and the boundary where as in the non-viscous case
there is no-energy dissipation. That is why non-viscous fluid is called non-dissipative fluid and viscous fluid is called dissipative fluid.

(iii) Another important distinction between viscous and non-viscous flow will be based on the dimensionless number is called **Reynolds Number**. Before defining the Reynolds number ,Re, we will review the dimensionl analysis afterderiving the basic equation. Re will be small for viscous fluids and it will be large for non-viscous fluids.

### 1.2.11 Steady and Unsteady Flow:

The fluid flow, in which all the physical properties such as velocity, density, pressure, temperature and so on do not change in time at every point in space, is called the steady flow. When one of the physical quantities change with the flow is said to be unsteady. It is important to note even in steady flow a fluid may have acceleration, because of inertia called inertial acceleration.

### 1.2.12 Laminar and Turbulent flows:

Laminar flow is one in which fluid flows in layers where the flow variables such as velocity, pressure, density etc., are deterministic functions of position and time. The laminar flow is called the stream line flow. Turbulent flow is one in which the velocity components have random fluctuations imposed on their mean values.

The laminar or turbulent flow for a given fluid is determined by its velocity and channel configuration or size. As the velocity increases the flow will change from laminar to turbulent passing through a transition regime. The problem of finding transition from laminar to turbulent flow is called the stability problem.
1.2.13 Boundary Layer:

There are two types of fluid configuration:

(i) External flow $\rightarrow$ flow of fluid over a body.

(ii) Internal flow $\rightarrow$ fluid flows within a body.

When a fluid flows past a solid body, attached to the body will be a thin layer of fluid called the boundary layer in which the viscosity is significant and outside this boundary layer the viscosity is not so significant. Thus, inside the boundary layer the fluid is considered to be viscous and outside the fluid is considered to be non-viscous. The streamlines very near to the boundary will be bent because the fluid resistance that is viscosity whereas for away from the boundary the streamlines are straight as there is no resistance. Near the body the fluid develops a shear layer (since the fluid velocity must be zero relative to the body on the surface of the body) where viscosity is important. This friction layer is called the boundary layer. The boundary layer may be laminar or turbulent. Behind a body wake overlaps and is generally a region of high turbulence and low pressure (hence drag due to wake). The wake is due to the separation of the boundary layer from the surface of the body. The boundary layer brought about because of the viscosity in the fluid is the cause of the wake. If the fluid is non-viscous, then there could be separation and hence no wake.

If there were no wakes the flow pattern would be symmetrical from front to back of the body and the pressure would be the same at the front as on the back and there would be no drag on an object inserted into a flowing fluid. (De Alembert's paradox). This absence of drag is contrary to experience as we realize that all fluids must have some internal friction. In the early days of development of fluid mechanics
as a science, it was thought that viscosity was rather negligible and mathematically
the fluid would be ideal i.e., non-viscous everywhere and hence there would be no
drag predicted by the theory. Since this conclusion was contrary to experience and
hence it was known as De Alembert’s paradox. It was not until the boundary layer
concept was introduced by Prandtl (in 1904), it became apparent that there is no
paradox at all, but any amount of viscosity, no matter how small, could bring about a
wake and consequent drag. If a body is streamlined i.e., trailing edge is varied into a
gradual smooth contour, no separation will occur and the boundary layer will the body
all way around. Streamlining substantially reduces the drag and most aerodynamic
objects wings etc., are streamlined. In such cases the fluid is ideal around the body
except in the boundary layer and in the wake.

1.2.14 Pressure:

Pressure is defined as normal stress or surface force per unit area. If $p$ is the pressure,
$\Delta F$ is the surface force acting on the area $\Delta A$ then

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA} \quad \therefore F = \int p dA$$

1.2.15 Density:

In fluid mechanics the density is defined as mass per unit volume. If $\Delta V$ is
the volume and $\Delta M$ is the mass of fluid within this volume, then the average density,
say $\rho_a$, of fluid within the volume is given by

$$\rho_a = \frac{\Delta M}{\Delta V}$$

However, to define the density $\rho$ at a point inside this volume, we need that the
volume should shrink to that point. Therefore:
\[ \rho = \lim_{\Delta V \to 0} \frac{\Delta M}{\Delta V} \]

The density is a function of position and time.

|s the force per unit area.

### 1.3: Basic Equations:

#### Continuity Equation

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (1)

#### Momentum Equations

\[ \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \]  \hspace{1cm} (2)

\[ \rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \]  \hspace{1cm} (3)

Where \( \rho \) is the constant fluid density and the individual stress components become

\[ \tau_{xx} = -p + 2\eta_0 \frac{\partial u}{\partial x} - 2k_0 \left[ u \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 u}{\partial x \partial y} - 2 \left( \frac{\partial u}{\partial x} \right)^2 - \frac{\partial u}{\partial x} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \]

\[ \tau_{yy} = -p + 2\eta_0 \frac{\partial v}{\partial y} - 2k_0 \left[ u \frac{\partial^2 v}{\partial x \partial y} + v \frac{\partial^2 v}{\partial y^2} - 2 \left( \frac{\partial v}{\partial y} \right)^2 - \frac{\partial v}{\partial y} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \]

\[ \tau_{xy} = \tau_{yx} = \eta_0 \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - k_0 \left[ u \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) + v \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} \right) - 2 \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} - 2 \frac{\partial v}{\partial x} \frac{\partial u}{\partial x} \right] \]  \hspace{1cm} (4)

#### Energy Equation

\[ \rho c_p \left( u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) = k \nabla^2 T + \frac{J^2}{\sigma} \]  \hspace{1cm} (5)

Where \( c_p \) is the specific heat and \( k \) is the thermal conductivity of the fluid.

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]
Applying the usual boundary layer approximations we obtain the following boundary layer equations for two dimensional unsteady state viscoelastic fluid flows.

**Equation of Continuity**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(6)

**Momentum Equation**

Making use of the usual boundary layer approximations, the equation of motion for unsteady flow are derived in the following simplified form

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^3 u}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} - k_\nu \left\{ \frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right\}
\]  

(7)

For steady flow the boundary layer equation of motion is

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^3 u}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} - k_\nu \left\{ \frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right\}
\]  

(8)

**Equation of Energy**

\[
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial y^2} \right) + \mu \phi
\]  

(9)

Boundary layer energy equation for two-dimensional steady incompressible fluid flow is

\[
\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial y^2} \right) + \mu \phi
\]  

(10)

**1.4. Boundary Conditions:**

The boundary conditions on velocity and temperature depend on the nature of the fluid flow and geometry of the boundary wall. Here, in this thesis, we consider a steady state two dimensional boundary layer flow of incompressible, viscous and
viscoelastic fluid past a flat elastic sheet. Boundary is assumed to be moving axially with a velocity by applying two equal and opposite forces along the x-axis, the sheet is being stretched with a speed proportional to the distance from the fixed origin \( x = 0 \). The resulting motion of the fluid is thus caused solely by the moving surface. The mathematical forms of velocity boundary conditions are as follows.

(a) **Linear Boundary stretching for flow:**

The boundary conditions on velocity depend on the nature of the fluid flow and geometry of the boundary wall. Here, in this thesis, we consider a steady, incompressible, non-Newtonian fluid of Walters’ liquid B past a flat elastic sheet, By applying two equal and opposite forces along x-axis, the sheet is being stretched with a speed proportional to the distance from the origin \( x = 0 \). The mathematical forms of velocity boundary conditions are as follows.

\[
\begin{align*}
  u &= bx, & v &= 0 & \text{at } y &= 0 \\
  u &\to 0, & \frac{\partial u}{\partial y} &\to 0 & \text{as } y &\to \infty
\end{align*}
\]  

(b) **Linear Boundary stretching for heat transfer:**

Thermal boundary conditions depend on the type of the heating process under consideration. Here, we consider two different types of heating processes, namely,

(i) Prescribed surface temperature (PST Case)

(ii) Prescribed wall heat flux (PHF Case).

Mathematical forms of such temperature boundary conditions are

**PST CASE**

\[
\begin{align*}
  T &= T_w = T_\infty + A \left( \frac{x}{l} \right)^2 & \text{at } y &= 0, \\
  T &\to T_\infty & \text{as } y &\to \infty
\end{align*}
\]  

1.5. Dimensionless Numbers or Non-Dimensional Parameters:

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic forces as this is a ratio of one force to the other force, it will be a dimensionless number. These dimensionless numbers are also called non-dimension parameters.

The following are some important dimensionless numbers.

(I) **Reynold’s Number** (\( R_e \)): It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid.

\[
R_e = \frac{F_i}{F_v} = \frac{\text{inertial force}}{\text{viscous force}}
\]

\[
= \frac{\rho l^2 v^2}{\mu l} = \frac{\rho l v}{\mu} = \frac{vl}{\left(\frac{\mu}{\rho}\right)} = \frac{vl}{\nu}
\]

Where \( R_e = \frac{vl}{\nu} \), \( \nu = \frac{\mu}{\rho} \) = Kinematic viscosity

\( F_i \) = Inertia force = \( \rho l^2 v^2 \), \( F_v \) = viscous force = \( \mu vl \)

In case of pipe flow, the linear dimension “1” is taken diameter “d”, hence Reynold’s number for pipeflow is
\[ R_e = \frac{vd}{v} \quad \text{or} \quad \frac{\rho vd}{\mu} \]

a. If ‘Re’ is small viscous forces are pre-dominant and the effect of viscosity will be felt in the flow region.

b. If ‘Re’ is large then the inertia forces are pre-dominant and in that case the effect of viscosity is confined to a narrow region near the solid boundary known as boundary layer region.

Reynold’s Number:

\[ R_e = \frac{\text{Inertia force}}{\text{Viscous force}} \]

\[ R_e = \frac{\text{Mass} \times \text{Acceleration}}{\text{Shear stress} \times \text{Cross-sectional area}} \]

\[ R_e = \frac{(\text{Volume} \times \text{density}) \times \left( \frac{\text{Velocity}}{\text{time}} \right)}{\text{Shear stress} \times \text{Cross-sectional area}} \]

\[ R_e = \frac{(\text{Cross-sectional area} \times \text{linear dimension}) \times \text{density} \times \left( \frac{\text{Velocity}}{\text{time}} \right)}{\text{Shear stress} \times \text{Cross-sectional area}} \]

\[ R_e = \frac{(\text{linear dimension}) \times \text{density} \times \left( \frac{\text{Velocity}}{\text{time}} \right)}{\text{Shear stress}} \]

\[ R_e = \frac{(\text{Velocity})^2 \times \text{density}}{\text{Shear stress}} = \frac{V^2 \times \rho}{\mu \frac{du}{dy}} = \frac{V^2 \rho}{\mu \frac{V}{L}} \]

\[ R_e = \frac{VL\rho}{\mu} = \frac{VL}{\mu} \]

\[ \because v = \frac{\mu}{\rho} \]

\[ R_e = \frac{VL\rho}{\mu} = \frac{VL}{\nu} \]
Note: 1. velocity = cross-sectional × linear dimension = \( (l^2 \times l) = l^3 \)

1. \( t = \frac{\text{linear dimension}}{\text{velocity}} = \frac{l}{v} \)

2. \( A = \text{linear dimension} \times \text{linear dimension} = L^2 \)

3. Length(l)=Linear dimension=L

4. The critical Reynold’s number based on boundary layer thickness is

\[ \text{Re}_{(cr)} = 2800 \quad \text{or} \quad \text{Re}_{(cr)} = 5 \times 10^5 \quad [\text{Transition point}] \]

5. If ‘Re’ is very large the flow (turned) becomes to be a laminar and turbulent.

6. Flow in a circular pipe , Re = 2300

For flow in pipes:

7. If Re < 2000 , the flow is laminar .Ex: flow in a pipe

8. If Re > 4000 , The flow is turbulent.

9. If 2000 < Re < 4000, The flow is “Transional flow” (may be laminar or turbulent)

   Ex: Tap water

10. In general the flow is assumed to be turbulent if Reynold’s number

   Re>2300

For flow on plates:

12. If Reynolds number Re < 3 \times 10^5 , the flow is laminar

13. If Re > 5 \times 10^5 the flow is turbulent.

14. If 3 \times 10^5 < Re < 5 \times 10^5 , the flow may be laminar of turbulent.
Application of Reynolds Number

1 Flow through orifices and mouth pieces

2 Turbulant flow in a conduits

3 Resistance experienced by sub-morines, fully immersed bodies

4 Air planes where the speed is subsonic.

(II) Prandtl’s Number: It is defined as the ratio of viscous force and the thermal force.

\[ \text{Pr} = \frac{\text{viscous force}}{\text{Thermal force}} = \frac{F_v}{F_t} \]

OR

It is the ratio of kinematic viscosity to the thermal diffusivity.

\[ \text{Pr} = \frac{\text{kinematic viscosity}}{\text{Thermal diffusivity}} = \frac{\nu}{k} \]

\[ \text{Pr} = \frac{\mu/\rho}{k/\rho C_p} = \mu \times \frac{\rho C_p}{k} = \frac{\mu C_p}{k} \]

\[ \text{Pr} = \frac{\mu C_p}{k} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}} \]

Where \( \mu \) = co-efficient of dynamic viscosity

\( C_p \) = specific heat at the constant pressure.

\( k \) = conductivity

Note: 1. It is a measure if the relative importance of heat conduction and viscosity of the fluid.
2. It throws light on the relative importance of viscous dissipation to the thermal dissipation.

3. Following the Pr values for different fluids
   i. For air $Pr = 0.7$
   ii. For water $Pr = 0.7$ at $68^\circ F$ temperature
   iii. For liquid metal mercury, $Pr = 0.044$
   iv. For Glycerine $Pr = 7250$ (due to high viscosity, $\mu$)
   v. For oils it of the order of 1000 due to large values of viscosity.

(III) **Eckert’s Number (Ec):** It is defined as the ratio of square of the velocity to the product of specific heat at constant pressure and temperature.

Or

$$Ec = \frac{\text{kinetic energy of the flow}}{\text{Boundary layer enthalpy difference}}$$

$$Ec = \frac{v^2}{C_p \cdot T_0} \quad \text{or} \quad Ec = \frac{U_0^2}{C_p \cdot T_0} = (v - 1) Ma^2$$

Where $v =$ velocity of the fluid.

$C_p =$ specific heat difference between the wall and the fluid at a large distance from the body.

$$v = \frac{C_p}{C_v} = \text{adiabatic constant or adiabatic index}$$

$Ma =$ Mach number.

(IV) **Nusselt’s Number (Nu):** It is defined as the ratio of product of heat transfer coefficient and space co-ordinate ‘x’ to the thermal conductivity.
\[ \text{Local Nusselt's Number (} \bar{N}_u\text{:}) \text{It is denoted and defined by } \bar{N}_u = \frac{h_x}{k} \quad \text{Or} \]

which can be expressed in terms of wall heat flux and temperature difference as

\[ \bar{N}_u = \frac{q_w x}{k(T_w - T_\infty)} \quad \text{where } q_w = \text{heat flux per unit area} \]

\( k = \text{conductivity, } T_w = \text{temperature of the wall} \)

\( T_\infty = \text{temperature if the fluid outside the boundary layer} \)

\[ \text{Average Nusselt's Number (} \bar{N}u\text{:}) \text{It is denoted and defined by} \]

\[ \bar{N}u = \frac{\bar{h} x}{k} \text{ where } \bar{h} = \frac{1}{x} \int_0^1 h_x dx = 2h \quad \text{or} \]

\[ \bar{N}u = \frac{\bar{h} L}{k} = 2N_u = L, \quad \bar{h} = \text{average heat} \]
(V) **Biot’s Number (Bi):** It is defined as the ratio of product of heat transfer coefficient and a characteristic dimension to the thermal conductivity.

\[ Bi = \frac{hs}{k} \quad \text{or} \quad Bi = \frac{\text{Internal resistance to heat conduction}}{\text{External resistance to heat conduction}} \]

Where:
- \( h = \) heat transfer coefficient
- \( s = \) Characteristic dimension
- \( k = \) Thermal conductivity

Note: 1. Biot number compares the relative magnitude of the surface convection and the internal conduction resistances to heat transfer.

2. A very low value of the Biot (number) modulus means that internal conduction resistance is negligible in comparison with surface convection resistance. This in turn implies that the temperature will be nearly uniform throughout the solid and its behavior may be approximated by the lumped capacity method of analysis.

(VI) **Schmidt’s Number (Sc):** It is defined as the ratio of kinematic viscosity to the diffusion coefficient.

\[ Sc = \frac{\nu}{D} = \frac{\text{Kinematic viscosity}}{\text{Diffusion coefficient}} \]

\[ Sc = \frac{\mu}{\rho D} \]

\[ Sc = \frac{\mu}{\rho D} = \frac{\nu}{D} \]

Note: 1. It is important in problems where both convection and mass transfer are important.

2. Schmidt plays a role similar to Prandtl number in convection heat transfer problems.

**Sherwood Number (Sh):**

It is defined as the ratio of mass transfer coefficient to the diffusion coefficient.
\[ Sh = \frac{\text{mass transfer co-efficient}}{\text{Diffusion co-efficient}} \]

\[ Sh = \frac{Kx}{D} = \frac{Kd}{D} \]

(VII) Knudsen Number (Kn):

It is defined as the ratio of mean free path to the characteristic body dimension.

\[ Kn = \frac{\text{Mean free path}}{\text{Characteristic body dimension}} \]

\[ Kn = \frac{\lambda}{L}, \quad \lambda = \text{mean free path}, \quad L = \text{length of characteristic dimension} \]

(VIII) Thermal conductivity (k): [From Fourier Law]: According Fourier, we say the energy is transferred by conduction and that the heat transfer rate per unit area is proportional to the normal temperature gradient.

\[ \frac{q}{A} \alpha \frac{\partial T}{\partial \lambda} \Rightarrow q = -kA \frac{\partial T}{\partial x} \]

This is called Fourier Law of heat transfer.

Where \( q = \text{heat transfer rate}, \ A = \text{area}, \ \frac{\partial T}{\partial x} = \text{the temperature gradient in the direction of heat flow}, \ k = \text{positive constant, thermal conductivity of material}, \)

Note: 1. Minus sign is taken because heat flows from higher region (hotter region) to lower region (cooler region)

2. \( q = -kA \frac{\partial T}{\partial x} \Rightarrow k = \frac{-q}{A \frac{\partial T}{\partial x}} \)

3. The three dimensional heat conduction equation is

\[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau} \]
\[
\frac{1}{\alpha} = \frac{\rho c}{k} \Rightarrow \alpha = \frac{k}{\rho c} \rightarrow \text{Thermal diffusivity of the material}.
\]

Larger the value of \( \alpha \) the faster heat will diffuse through the material

(IX) **Thermal diffusivity:** It is the ratio of the thermal conductivity to the product of density and specific heat

\[
\alpha = \frac{\text{Thermal conductivity}}{\text{density} \times \text{specific heat}} = \frac{k}{\rho c}
\]

1. Unit of \( \alpha \) is square meters per second.

2. The larger the value of \( \alpha \) the faster heat will diffuse through the material.

3. A high value of \( \alpha \) could result either from a high value of the thermal conductivity, which would indicate a rapid energy transfer rate, or from a low value of the thermal heat capacity \( \rho c \)

4. A low value of heat capacity would mean the less of the energy moving through the material would be absorbed and used to rise the temperature of the material, thus more energy would be available for further transfer.

(X) **NON-UNIFORM HEAT SOURCE/SINK PARAMETER:**

\[
q'''' = \left( \frac{kb}{U} \right) \left[ A^* (T_w - T_\infty) \frac{U}{U_w} + B^* (T - T_\infty) \right].
\]

Where \( A^* \) and \( B^* \) are parameters of space- and temperature-dependent internal heat generation/absorption. It is to be noted that \( A^* > 0 \) and \( B^* > 0 \) correspond to internal heat generation while \( A^* < 0 \) and \( B^* < 0 \) correspond to internal heat absorption, \( T_w \) is the temperature of sheet and \( T_\infty \) is the constant temperature for away from the sheet.
(XI) THERMAL RADIATION PARAMETER:

\[ N = \frac{Kk^*}{4\sigma T_\infty^3} \]

Here, \( \sigma \) is the Stefan-Boltzmann constant and \( k^* \) is the absorption coefficient, \( T_\infty \) is the temperature far away from the wall; \( K \) is the thermal conductivity of the fluid.

1.6: NOMENCLATURE:

- \( u, v \): Velocity components along x and y directions
- \( x, y \): Distance along and perpendicular to the surface respectively
- \( A, D \): Prescribed constants
- \( C_p \): Specific heat at constant pressure
- \( B_o \): Magnetic field strength
- \( k_0 \): Elastic parameter
- \( k_1 \): Viscoelastic parameter
- \( k \): Thermal conductivity
- \( k^* \): Mean absorption coefficient
- \( q_r \): Radiative Heat Flux
- \( u, v \): Velocity components along x and y direction
- \( T \): Fluid temperature of the moving sheet
- \( T_w \): Wall temperature
- \( M \): Confluent hyper geometric function
- \( Pr \): Prandtl number
- \( Mn \): Magnetic parameter
Nr  Thermal Radiation Parameter

$T_\infty$  Temperature far away from the plate

$q''$  Rate of internal heat generation/absorption

$A^*$  Space-dependent internal heat generation/absorption

$B^*$  Temperature-dependent internal heat generation/absorption

$Cf_x$  Local Skin Friction

$Nu_x$  Local Nusselt Number

$Re_x$  Local Reynolds Number

$l$  Characteristic length

$A, A_0, A_1, B, \quad$ Constants

$E, E_0, E_0 \quad$ Constants

GREEK SYMBOLS

$\theta$  Dimensionless temperature

$\alpha$  Real root of cubic equation

$\eta$  Dimensionless space variable

$\nu$  Kinematic viscosity

$\rho$  Density

$\sigma$  Electric conductivity

$\gamma$  wall temperature parameter

$\mu$  Co-efficient of viscosity

$\psi$  Dimensionless stream function