Chapter - 3

VELOCITY AND THERMAL SLIP EFFECT ON FLOW AND HEAT TRANSFER DUE TO AN EXPONENTIALLY STRETCHING SHEET WITH VISCOUS DISSIPATION AND THERMAL RADIATION

Part of this chapter is published in Advances in Physics Theories and Applications(IISTE), 2016, Vol. 53, pp.53-62
Chapter 3

Velocity and Thermal Slip effect on Flow and Heat Transfer due to an Exponentially Stretching sheet with Viscous Dissipation and Thermal Radiation

3.1 Introduction:

The flow and heat transfer characteristics due to stretching sheet has gained much more interest in past two decades. In view of many industrial applications such as Aero dynamic extrusion of plastic sheets, condensation process of metallic plates in a cooling bath and glass and in polymer industries, many authors have studied on the flow and heat transfer characteristics due to linear and non linear stretching sheets. Some of the authors worked on exponential stretching sheet which are discussed below.

Partha et al. (2005) investigated the mixed convection flow and heat transfer of viscous fluid over an exponentially vertical stretching surface, and used shooting technique with numerical method to obtain solution of their governing boundary value problems and showed the velocity boundary layer thickness increased with increase of both mixed convection and viscous dissipation. Bidin and Biliana and Roslinda (2009) obtained numerical solutions for boundary layer
flow due to an exponentially stretching sheet with thermal radiation. They used Keller box method for obtaining solutions of the problem and analyzed the effect of viscous dissipation and thermal radiation on temperature profile. Ishak (2011) analyzed the effect of magnetic field on viscous flow and thermal radiation on temperature field due to an exponentially stretching sheet. Nadeem et al. (2011) studied the effects of thermal radiation on boundary layer flow of Jeffrey fluid over an exponentially stretching surface and considered two heating processes namely prescribed surface temperature and prescribed wall heat flux. Mukhopadhyay and Gorla (2012) investigated the effects of partial slip on the boundary layer flow due to an exponentially stretching sheet in the presence of thermal radiation and obtained the solutions of governing equations numerically using shooting technique and showed that velocity and temperature decreases with increasing values of slip parameter and temperature is decreasing with increasing values of thermal slip parameter. Kameswaran et al. (2012) studied the effect of radiation on hydro magnetic Newtonian liquid due to an exponentially stretching sheet and obtained the solution of boundary valued problem by both numerical and analytical methods. Nadeem and Lee (2012) analyzed the flow and heat transfer characteristics of Nano fluid due to an exponentially stretching sheet. They analyzed the effects of governing parameters on flow and heat transfer profiles and used the Homotopy analysis method. Mukhopadhyay et al. (2013a) studied the effects of partial slip on chemical reaction in boundary layer flow due to an exponentially stretching sheet. They obtained the solution by numerical method using shooting technique and showed that fluid velocity and temperature profiles decrease with increasing value of slip parameter. They also analyzed the effect of suction on flow, which results in decrease of velocity with increase in suction parameter values. Jat and Chand (2013) investigated the effects of magnetic field, viscous dissipation and radiation parameter on flow and heat transfer due to an exponentially stretching sheet and solved the bvp's numerically. Hayat et al. (2014) studied the magnetic effects on Nano fluid flow in presence of porous medium with convective boundary conditions over an exponentially stretching sheet and solved the bvp's Interns of series solution, i.e., Homotopy analysis method. Kumari and Nath (2014) studied the mixed convection flow of Maxwell's fluid over an exponentially stretching sheet and analyzed the effects of magnetic field and viscous dissipation and Chebyshev finite difference method for solving boundary valued equations. They showed that the Nusselt number is slightly decreasing with increasing values of vis-
cous dissipation. They solved the equations by forth order Runge-Kutta method and interpreted that dual solution exists only for shrinking sheet condition. Bég et al. (2014) have given explicit numerical solution for unsteady hydro magnetic mixed convection of a nano fluid from an exponentially stretching sheet in porous medium. They solved the governing equations using Robust explicit finite difference method. Malik et al. (2014) analyzed the flow of a Casson nano fluid over an vertical exponentially stretching cylinder and solved the governing system of equations by Runge-Kutta Felburg method and analyzed the effect of governing parameters on the flow, temperature and concentration profiles. Nadeem and Hussain (2014) investigated the heat transfer analysis of Williamson’s fluid due to an exponentially stretching sheet and solved the problem with the help of optimal homotopy analysis method, the optimal convergence control parameter is also obtained for Prescribed surface temperature and prescribed wall heat flux. Chaudhary et al. (2015) studied the thermal radiation effects on MHD boundary layer flow over an exponentially stretching surface. Mahmoud (2015) studied the variable fluid properties on MHD fluid flow over an exponentially stretching sheet with mass transfer. Naramgari and Sulochana (2016) obtained dual solution for radiative MHD nanofluid over an exponentially stretching sheet with heat source / sink.

Motivated by all the above studies in the present chapter it is studied the combined effect of viscous dissipation and thermal radiation with velocity and temperature jump effects and skin friction due to the effect of slip parameter and wall temperature gradient for various governing parameters.

### 3.2 Flow Analysis:

A flow of viscous fluid is considered past an exponential stretching sheet. The governing continuity and momentum equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.2.1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{3.2.2}
\]
Here $u$ and $v$ are the velocity components along x-axis and y-axis respectively. $v = \frac{\nu}{\rho}$ is kinematic viscosity.

The boundary conditions of considered flow can be written as

$$u = U_0 e^{\frac{x}{L}}, \quad v = 0 \text{ at } y = 0 \text{ and } u \to \infty \text{ at } y \to \infty$$

Equations (3.2.1) and (3.2.2) cannot be solved directly hence they are converted to ODE by using the following similarity transformations

$$u = U_0 e^{\frac{x}{L}} f'(\eta), \quad v = -\sqrt{\frac{\nu U_0}{2L}} e^{\frac{x}{L}} \left\{ f(\eta) + \eta f'(\eta) \right\}$$

Equation (3.2.1) is satisfied identically by using (3.2.4) and equation (3.2.2) take the following form

$$f''' - 2f'' + ff'' = 0$$

With corresponding initial/boundary conditions

$$f(0) = 0, f'(0) = 1 + \alpha' f''(0) \text{ and } f'(\infty) \to 0$$

where $\alpha' = N_1 \sqrt{\frac{U_0 \nu}{2L}}$ is the velocity slip parameter.

### 3.3 Heat Transfer Analysis:

In this segment Prescribed Exponential Surface Temperature (PST) in heat transfer analysis is considered. The governing energy equation is of the following type

$$\rho C_p \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 - \frac{\partial q_\tau}{\partial y}$$

Where $T$ is the temperature of the flow field, $\kappa$ is the thermal conductivity of
the fluid, $\mu$ is the viscosity, $\rho$ is the density, $C_p$ is the constant pressure, $q_r$ is the radiative heat flux and $q_r$ is given (as Roseland approximations) as

$$ q_r = -\frac{4\sigma^*}{3k^*} \left( \frac{\partial T^4}{\partial y} \right) $$  \hspace{1cm} (3.3.2)$$

Where $\rho$ is Stefan-Boltzman constant and $k^*$ is absorption coefficient and $T^4$ is given by

$$ T^4 = 4T_\infty^3 T - 3T_\infty^4 $$  \hspace{1cm} (3.3.3)$$

The PST heating boundary condition is given by

$$ T = T_w + T_\infty e^{\frac{\xi}{k}} \text{ at } t \to 0 \text{ and } T \to \infty \text{ at } y \to \infty \text{ Where } \theta = \frac{T - T_\infty}{T_w - T_\infty} $$  \hspace{1cm} (3.3.4)$$

Using (3.2.4), (3.2.8), (3.2.9) and (3.2.10) in (3.2.7) the following non-dimensional equation for temperature is derived.

$$ \left( 1 + \frac{4R}{3} \right) \theta'' + \Pr \left( f \theta' - f' \theta + E \xi f'' \right) = 0 $$  \hspace{1cm} (3.3.5)$$

Where

$\Pr = \frac{\mu C_p}{k}$ is the Prandtl number, $Ec = \frac{U^2}{\gamma E \xi}$ is the Eckert number, $R = \frac{4\sigma^* T^3}{k^* k}$ is the radiation parameter.

With the corresponding boundary conditions

$$ \theta (0) = 1 + \beta \theta' (0) \text{ at } \eta = 0 \text{ and } \theta (\eta) \to 0 \text{ as } \eta \to \infty $$  \hspace{1cm} (3.3.6)$$

and $\beta' = D_1 \sqrt{\frac{U^2}{2\nu L}}$ is the temperature jump parameter.
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3.4 Numerical Solution:

The governing ordinary differential equations (3.2.5) and (3.3.5) with the boundary conditions (3.2.6) and (3.3.6) are converted into first order system of differential equations with corresponding initial conditions. The missing boundary condition \( f''(0) \) and \( \theta'(0) \) are obtained using the Newton-Raphson scheme. Further the system of differential equations are solved by Runge-Kutta forth order integration scheme by MATLAB (ode45). Various numerical values of \( f''(0) \) and \( \theta'(0) \) for different set of parameters are calculated and tabulated. For brevity detailed explanation of method of solution is ignored.

3.5 Discussion of Results:

In this Chapter an investigation of the combined effect of thermal radiation and viscous dissipation in presence of slip velocity and thermal jump and various numerical values of skin friction co-efficient and temperature gradient are obtained and tabulated.

Figures 3.1 and 3.2 are plotted for horizontal and vertical flow velocity profiles for different values of velocity slip parameter and figures 3.3 to 3.7 are plotted for temperature profiles for different values of governing parameters \( \alpha', \beta', Pr, Ec, \text{and} R \). Further figure 3.8 is plotted for \( f''(0) \) for different values of velocity slip parameter \( \alpha' \). Figure 3.9 to 3.16 are plotted for temperature gradient for different governing parameters.

Figure 3.1 shows the effect of velocity slip parameter on horizontal velocity flow with increasing values of slip velocity \( \alpha' \), it is observed from the figure that velocity decreases initially and after certain distance from the sheet it increases.

Figure 3.2 shows the effect of velocity slip parameter on vertical velocity profile, on observing this the velocity profile decreases with increasing values of \( \alpha' \). In presence of slip, the flow velocity close to sheet is no longer equal to the stretching velocity of the sheet. With increase of velocity slip fluid velocity decreases because of slip, the pulling of the stretching sheet can be partially transmitted to the fluid.

Figure 3.3 shows the effect of Eckert number on temperature profile, on observing this graph the thermal boundary layer thickness enhances with increase in values of Ec. The effect of increasing the values of Ec is to increase temperature distribution in the flow region. This enhancement happens because of heat energy
stored in the fluid due to fractional heating.

Figure 3.4 depicts the effect of thermal radiation $R$ on the temperature profile which shows that temperature profile increases with increase in values of $R$. This is due to the physical fact that thermal boundary layer thickness increases with increasing $R$.

Figure 3.5 is drawn to show the effect of Prandtl number $Pr$ on temperature profile and it is observed that thermal boundary layer thickness decreases with increasing values of $Pr$. The graph reveals the fact that the increase of $Pr$ results in decrease of temperature distribution, this is because of decrease of thermal boundary layer thickness with increasing values of slow rate of thermal diffusion. The wall temperature is unity at the wall.

Figure 3.6 explains the effect of velocity slip parameter $\alpha'$ on temperature profile, which shows that temperature profile increases with increase in values of velocity slip parameter. Whereas Figure 3.7 shows the effect of thermal slip parameter $\beta'$ on temperature profile and reveals the fact that the thermal boundary layer thickness increases with increasing value of $\beta'$. These slip effects on temperature profile, the less is transmitted to fluid from sheet, but as temperature gradient increases with increasing value of slip parameters $\alpha'$ and $\beta'$.

Figure 3.8 shows the missing boundary condition profile for velocity $f''(0)$ for different values of slip parameter $\alpha'$ and depicts that it increases with increasing values of $\alpha'$.

Figure 3.9-3.12 show the missing boundary condition profiles of temperature gradient for different values of $\beta'$, $Pr$, $Ec$ and $R$ respectively. It is observed from figure that wall temperature gradient enhances with increasing values of $\beta'$, $Pr$, $Ec$ and $R$ respectively. Similarly Figures 3.13 to 3.16 represent the graphs of temperature gradient $\theta'(0)$ with respect to $\beta'$ for different values of $\alpha'$, $Ec$, $Pr$ and $R$ respectively. From the figures it is seen that temperature gradient increases with increasing values of respective parameter except $Pr$. Figure 3.15 shows opposite results of other, i.e., temperature gradient decreases with increasing values of $Pr$.

The numerical values of $\theta'(0)$ for different values of various parameters encountering in the problem are tabulated in Table-3.1. Wall temperature gradient increases with increasing values of $Ec$, $R$, $Pr$, $\alpha'$, $\beta'$ and decreases with increasing values of $Pr$. Table-3.2, shows the comparison of results of $\theta'(0)$ for different values of $Pr$. On observing those values, we can conclude that our results are in good agreement with earlier results.
3.6 Conclusions

The important conclusions drawn from the study are
1) Radiative effect has the capacity to increase the momentum boundary layer thickness.
2) Velocity profile decreases with increasing values of velocity slip parameter.
3) Temperature distribution is more due to the increasing parametric values of all governing parameters except Pr.
4) Exponential parameter improves the heat transfer rate and velocity distribution.

Table 3.1: The numerical values of $-\theta'(0)$ for different values of various parameters

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<th>$Ec$</th>
<th>$R$</th>
<th>$Pr$</th>
<th>$\alpha'$</th>
<th>$\beta'$</th>
<th>$-\theta'(0)$</th>
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Table 3.2: Comparison of results of $\theta'(0)$ for different values of Pr

<table>
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<tr>
<th>Pr</th>
<th>Bidin and Nazer with Ec=0 for two values of radiation parameter R</th>
<th>Nadeem et al. for PST case with Ec=0, $\lambda = 0$, S=0, $f\omega=0$ for 2 values of R</th>
<th>Swati and Gorla with $\lambda = 0$, $\delta = 0$, S=0 for two Values of R</th>
<th>Present results $\alpha' = 0$, $\beta' = 0$, $f(0) = 0$ for two values of R</th>
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Figure 3.1: Flow profiles $f$ for different values of velocity slip parameter $\alpha'$ with $Pr=2$, Radiation parameter $R = 0.5$, $Ec = 0.5$

Figure 3.2: Velocity profile $f'$ for different values of velocity slip parameter $\alpha'$ with $Pr=2$, Radiation parameter $R = 0.5$, $Ec = 0.5$
Figure 3.3: Temperature profiles for different values of Eckert number $Ec$ with $R = 0.5$, $Pr = 1.0$, Velocity slip $\alpha' = 0.2$, Thermal slip $\beta' = 0.2$

Figure 3.4: Temperature profiles for different values of radiation parameter $R$ with $Ec = 0.5$, $Pr = 1.0$, Velocity slip parameter $\alpha' = 0.2$, Thermal jump parameter $\beta' = 0.2$
Figure 3.5: Temperature profiles for different values of prandtl number Pr with $Ec = 0.5, R = 1.0, \text{Velocity slip parameter } \alpha' = 0.2, \text{ Thermal jump parameter } \beta' = 0.2$

Figure 3.6: Temperature profiles for different values of velocity slip parameter $\alpha'$ with $Ec = 0.5, R = 1.0, Pr = 1.0, \beta' = 0.2$
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Figure 3.7: Temperature profiles for different values of Thermal slip parameter $\beta'$ with $E_c = 0.5$, $R = 0.5$, $Pr = 1.0$, $\alpha' = 0.2$

Figure 3.8: Skin friction $f''(0)$ for different values of velocity slip parameter $\alpha'$
Figure 3.9: Temperature gradient $\theta'(0)$ for different values of velocity slip parameter $\alpha'$ with $\beta' = 0.25, 0.5, 1.0$

Figure 3.10: Wall temperature $\theta'(0)$ for different values of velocity slip parameter $\alpha'$ with $Pr = 0.5, 1.0, 1.5$
Figure 3.11: Temperature gradient $\theta'(0)$ Versus velocity slip parameter $\alpha'$ with varying $Ec = 0.5, 1.0, 1.5$

Figure 3.12: Temperature gradient $\theta'(0)$ Versus velocity slip parameter $\alpha'$ with Radiation parameter $R = 0.5, 1.0, 1.5$
Figure 3.13: Temperature gradient $\theta'(0)$ Versus Thermal slip parameter $\beta'$ with $\alpha' = 0.25, 0.5, 1.0$

Figure 3.14: Temperature gradient $\theta'(0)$ for different values of thermal slip parameter $\beta'$ with $Ec = 0.5, 1.0, 1.5$
Figure 3.15: Temperature gradient $\theta'(0)$ V/s thermal slip parameter $\beta'$ with $Pr = 0.5, 1.0, 1.5$

Figure 3.16: Temperature gradient $\theta'(0)$ V/s thermal slip parameter $\beta'$ with different values of $R = 0.5, 1.0, 1.5$