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INTRODUCTION
Chapter 1

Introduction

1.1 History and Scope of Fluid Dynamics

The present thesis contributes some problems concern to fluid flow, heat and mass transfer past stretching sheet. These fluid flows determine many real life situations and are formulated into mathematical models and tackled using either Analytical methods or Numerical methods. With respect to above information we are discussing the History and Scope of Fluid Dynamics.

Like most scientific disciplines, fluid mechanics has a history of erratically occurring early achievements, then an intermediate era of steady fundamental discoveries in the eighteenth and nineteenth centuries, leading to the twentieth-century era of “modern practice,” as we self-centeredly term our limited but up-to-date knowledge. Ancient civilizations have enough knowledge to solve certain flow problems. Sailing ships with oars and irrigation systems were both known in prehistoric times. The Greeks produced quantitative information. Archimedes and Hero of Alexandria both postulated the parallelogram law for addition of vectors in the third century B.C. The pioneering recognized work to Fluid dynamics concepts was introduced by the Greek Mathematician Archimedes (285–212 B.C.) who formulated the laws of buoyancy and applied them to floating and submerged bodies, actually deriving a form of the differential calculus as part of the analysis. The Romans built extensive aqueduct systems in the fourth century B.C. but left no records showing any quantitative knowledge of design principles. From the birth of Christ to the Renaissance there was a steady improvement in the design of such flow systems as ships and canals and water conduits
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but no recorded evidence of fundamental improvements in flow analysis. Then Leonardo da Vinci (1452–1519) derived the equation of conservation of mass in one-dimensional steady flow. He was an excellent experimentalist and his notes contain accurate descriptions of waves, jets, hydraulic jumps, eddy formation, and both low-drag (streamlined) and high-drag (parachute) designs. A Frenchman, Edme Mariotte (1620–1684), built the first wind tunnel and tested models in it. Italian Benedetto Castelli (1577-1644) was the first man to publish the continuity principle for fluids. Problems involving the momentum of fluids could finally be analyzed after Isaac Newton (1642–1727) postulated his laws of motion and the law of viscosity of the linear fluids now called Newtonian also determined the inertia, resistance, free jets. The theory first yielded to the assumption of a “perfect” or frictionless fluid, and eighteenth-century mathematicians (Daniel Bernoulli, Leonhard Euler, Jean d’Alembert, Joseph-Louis Lagrange, and Pierre-Simon Laplace) produced many beautiful solutions of frictionless-flow problems. Euler developed both the differential equations of motion and their integrated form, now called the Bernoulli equation. Also Daniel Bernoulli and Leonhard Euler together contributed the Energy and Momentum equations for the flow. D’Alembert used them to show his famous paradox: that a body immersed in a frictionless fluid has zero drag. These beautiful results amounted to overkill, since perfect-fluid assumptions have very limited application in practice and most engineering flows are dominated by the effects of viscosity. Engineers began to reject what they regarded as a totally unrealistic theory and developed the science of hydraulics, relying almost entirely on experiment. Such experimentalists as Chézy, Pitot, Borda, Weber, Francis, Hagen, Poiseuille, Darcy, Manning, Bazin, and Weisbach produced data on a variety of flows such as open channels, ship resistance, pipe flows, waves, and turbines. All too often the data were used in raw form without regard to the fundamental physics of the flow. In Eighteenth century Fluid Properties are introduced in Engineering by Riche de Prony (1755-1839), He was the First man to introduce the Integral Calculus and systematic theory into the World of Engineering. Further work is carried by Antonie Chezy (1718-1798), Luis Narier (1788- 1836), Henny Darey (1803-1858). In the Mid of Nineteenth century Physician Jean Poiseuille (1799-1869) has done the experiment on flow in capillary tubes and the difference between Laminar and Turbulent flow in pipes has given by Gotthilf Hagen (1797-1884) . At the end of the nineteenth century, unification between experimental hydraulics and theoretical hydrodynamics
finally began. William Froude (1810–1879) and his son Robert (1846–1924) developed laws of model testing, Lord Rayleigh (1842–1919) proposed the technique of dimensional analysis, and Osborne Reynolds (1842–1912) published the classic pipe experiment in 1883 which showed the importance of the dimensionless Reynolds number named after him. Meanwhile, viscous-flow theory was available but unexploited, since Navier (1785–1836) and Stokes (1819–1903) had successfully added newtonian viscous terms to the equations of motion. The resulting Navier-Stokes equations were too difficult to analyze for arbitrary flows. Then, in 1904, a German engineer, Ludwig Prandtl (1875–1953), published perhaps the most important paper ever written on fluid mechanics. Prandtl pointed out that fluid flows with small viscosity, e.g., water flows and airflows, can be divided into a thin viscous layer, or boundary layer, near solid surfaces and interfaces, patched onto a nearly inviscid outer layer, where the Euler and Bernoulli equations apply. Boundary-layer theory has proved to be the single most important tool in modern flow analysis. The twentieth century foundations for the present state of the art in fluid mechanics were laid in a series of broad-based experiments and theories by Prandtl and his two chief friendly competitors, Theodore von Kármán (1881–1963) and Sir Geoffrey I. Taylor (1886 – 1975). The Scope of the Fluid dynamics is unbounded and touches every human endeavor because the earth is 75 percent covered with water and 100 percent covered with air. The sciences of meteorology, physical oceanography, and hydrology are concerned with naturally occurring fluid flows, as are medical studies of breathing and blood circulation. All transportation problems involve fluid motion, with well-developed specialties in aerodynamics of aircraft and rockets and in naval hydrodynamics of ships and submarines. Almost all our electric energy is developed either from water flow steam flow through turbine generators. All combustion problems involve fluid motion, as do the more classic problems of irrigation, flood control, water supply, sewage disposal, projectile motion, and oil and gas pipelines.

1.2 Applications

Few common Engineering applications of fluid mechanics give some idea about why it is very important to learn. Fluid mechanics perhaps exploring little bit and keep our interest in topic. Most of the common applications specially in Mechanical Engineering are
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• Furnaces, Air Conditioners and ducting systems.

• Industrial pumps and piping system also involve many principles of fluid mechanics. We are selecting a pump to get desire amount of flow, sizing the piping.

• Wide range of applications find in transportation sector Aircraft, Automobiles and Ships. In the Aircraft Aerodynamic flow of the wings which keeps Aircraft a last, there also fluid flow through the turbines Engine, they provides trust in moving the Aircraft forward. Air dynamic Drag is also important consideration for Ships.

• Fluid dynamic problems are evacuees in the field of electrical power generations. Coal fire power plant, Gas fire power plant and Nuclear power plant.

• Fluid flow through Turbines, Condensers, Nuclear reactors.

• A common everyday application is Electronic cooling of computer applications this involves Fan blowing air over heat circumstances.

• Hot area for Engineers at the moment is Nuclear Energy, Fluid Dynamics principles are fundamental to design these systems such as Solar collectors, Wind turbines, Hydro power Which is all about Air dynamics.

• it is also important in designing the Hydro electric power systems.

• For the Bio-medical Engineering students, it is important to learn Angiogram in fluid mechanics to understand blood flow and circulation in Cardiovascular systems, Biological pumps.

• Many chemical engineering process like those in metallurgy and polymer extrusion process involves cooling of melton liquid by converting into cooling liquid. While converting it is sometimes stretched, The stretched impacts a unidirectional orientation to the extrude thereby improving its fluid mechanical properties. Some of these are listed below

• Continuous stretching, rolling, manufacturing of polymer sheets

• Drawing, annealing, finning of coper wires.
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- Cooling of an infinite metallic plate in a cooling path.
- Boundary layer along a liquid film in condensation process.
- Manufacturing of materials by extrusion process and heat treated materials traveling between a feed and wind up rolls or conveyer belt.
- Glass blowing, paper production, crystal glowing.

In the polymer extrusion process the fluid mechanical properties used for the ultimate outcome depending mainly on rate of cooling, the liquid which is basically meant to cool. The stretching sheet serves an important role in determining the property sought for to consider two important aspects in this physically interesting problem.1) Proper choice of cooling liquid and 2) regulation of the flow of the cooling liquid. In early days water was used for the coolant but this has draw back of rapidly quenching the heat leading to sudden solidification of the stretching sheet that is water does not seem to be a ideal cooling liquid. Thus a cooling liquid which helps in quenching the heat from the sheet at a desire rate must be chosen and an optimum stretching rate has to be achieved as sudden stretching may result in spoiling the desire characteristics of the product. A careful examination of the needs in the system suggests the at it is advantageous to have a controlled cooling system. An electrically conducting Newtonian or non-Newtonian liquid seems to be a good solution for such an application because its flow can be regulated by externally through magnetic field. A week electrically conducting liquid can be opted as coolant as it is induces a magnetic field negligibly small strength. It is clear that magneto hydrodynamics of the cooling liquid mediate in controlling the heat transfer from the stretching sheet. Variable properties of the cooling liquid also contribute in controlling the dynamics. A material property such as viscosity and thermal conductivity are tools to vary with temperature gradient, especially in the boundary layer region. Variable fluid properties have significant impact on the heat transfer characteristics of the cooling liquid. Heat generation and absorption may become very necessary in week electrically conducting cooling liquid due to the non isothermal situation where they are in and also due to the presence of cation or anion salts dissolving in them. Usually heat sinks are useful as they help in quenching the heat from the sheet.
1.3 Continuum Hypothesis

The continuum hypothesis means the following: at each point of the region of the fluid it is possible to construct one volume small enough compared to the region of the fluid and still big enough compared to the molecular mean free path. Why is that important? Because of two things. First, since the volume we can build at each point is very small compared to the size of the region of the fluid, we can think of the volume as located at a point instead of considering it as a collection of points.

1.4 Classification of fluids

The fluid can be classified as Ideal fluid and real fluid. Ideal fluid are also known as perfect fluids, when they are in motion two contacting layers experiencing no tangential force but act on each other with normal force only. Ideal fluids offer no internal force to change in shape. In ideal fluids pressure at every point equals in all directions whether the fluid is at rest or in motion. In nature ideal fluids do not exist and such fluids are imaginary fluids because of having the property of low viscosity. Real fluids are also known as viscous fluids, which have viscosity, surface tension, density and compressibility. Viscous fluids are those when they are in motion, two contacting layers of those fluid experience the tangential and normal forces. In viscous fluid internal friction plays vital role during the motion of the fluid. Viscous fluids are classified into following two categories.

- Newtonian liquids and

- Non-Newtonian liquids

Distinction between the Newtonian and non-Newtonian fluids can be readily illustrated when the velocity gradient $\frac{\partial u}{\partial y}$ is plotted against the viscous shear stress. Fluids for which the viscosity is independent of velocity gradient are Newtonian fluids. For these fluids the plot between shear stress and velocity gradient is a
straight line passing through the origin. Slope of the line is equal to the coefficient of viscosity.

\[ \mu = \frac{\tau}{\frac{\partial u}{\partial y}} \]  

(1.4.1)

Fluids represented by curves (a) and (b) are Newtonian fluids. Fluid represented by curve (a) is more viscous than that represented by curve (b). Fluids like air, water, kerosene and thin lubricating oils are essentially Newtonian in character under normal working conditions. Fluids such as Human blood, thick lubricating oil and certain suspensions for which the viscosity coefficients depends up on velocity gradient are referred to as non-Newtonian fluids. The viscous behavior of the non Newtonian fluid may be prescribed by the power equation

\[ \tau = k \left( \frac{\partial u}{\partial y} \right)^n \]  

(1.4.2)

Where \( k \) is the consistency index and \( n \) is the flow behavior index. For a Newtonian fluid, the consistency index \( k \) becomes the dynamic viscosity coefficient \( \tau \) and the flow behavior index \( n \) assumes a unity values. Fluids for which the flow behavior index \( n \) is less then unity are called Pseudo-plastic. Viscosity coefficient is smaller at greater rates of velocity gradient and curve becomes flatter as the shear rate (i.e., velocity gradient) increases (curve c). Examples of pseudo plastic fluids are the milk, blood clay and liquid cement. Fluids for which the index \( n \) is greater then unity are called Dilatant. Viscosity coefficient is more at greater rates of viscosity and the flow curve steeps with increasing shear rate (curve d). Concentrated solution of sugar and aqueous suspension of rice starch are examples of dilatants fluids. An Ideal plastic substance indicate no deformation when stretched up to a certain point (yield stress) and behind that it behaves like a Newtonian fluid and hence is represented by line (e). For certain substances, there is finite deformation for a given load, i.e., rate of deformation is zero. These material plot as ordinates (curve f) and are called Elastic materials or ideal solids. Actual solid deform slightly when subjected to shear stress of larger magnitude and hence plot as a straight line almost vertical (g). A fluid for which shear stress is zero (even if there is velocity gradient) is the ideal fluid and it plots as abscissa (h). Fluids which shows an apparent increase in viscosity with time are called Thixotropic. Conversely if the apparent viscosity decreases with time, the fluid is called Rheoplectic. Further
we can classify time independent and time dependent fluids as follows. Time independent fluids are Bingham plastics, Pseudo plastic and Dilatant fluids and time dependent fluids are Thixotropic and Rheopexic fluids.

1.4.1 Visco-elastic fluids

There are several systems of fluids, which have characterized as both fluids and solids and showing partial elastic retrieval property after deformation. These fluids are known as visco-elastic fluids. Visco-elastic material exhibits both viscous and elastic properties. The simplest type of such a material is one which obeys the Hooke’s law for the elastic part and also by the Newtonian law of viscosity and it can be written as

\[ \gamma = \frac{\tau}{\mu_0} + \frac{\tau}{\lambda} \]  

(1.4.3)

where \( \lambda \) is a rigidity modulus. Under steady flow this fluid behaves like a simple Newtonian fluid. However, if the shear stress is changed, an elastic effect is observed. Maxwell first proposed the above equation in the form

\[ \tau + \left( \frac{\mu_0}{\lambda} \right)^{-1} \gamma = \mu_0 \gamma \]  

(1.4.4)

And liquid which obey this law are known as Maxwell’s fluids. The constant
\((\frac{\mu_0}{\lambda})^{-1}\) is known as relaxation time constant for exponential decay of stress at a constant strain. If the motion is stopped the stress relaxed as \(e^{-\frac{t}{\tau}}\). In elastic materials, stress depends on strain only i.e., the stress is function of strain. Thus we apply certain stress on an elastic material, the material undergoes some deformation and when this stress is removed the material returns to original stage. So we can say that the elastic material have memory, that is, it is capable of recognizing original shape. Thus there are some fluids like soap solution, polymer solution, which have some elastic properties besides having fluid properties, such fluids are the examples of visco-elastic fluids.

### 1.4.2 Walter’s Model:

A detail theoretical investigation has begun for the incompressible elastic-viscous prototype specified as liquid \(B'\) by Walter’s. Equations for liquid \(B'\) can be expressed in the following form

\[
\tau_{ij} = -P\delta_{ij} + \tau_{ij} \tag{1.4.5}
\]

Where \(\tau_{ij}\) is the stress tensor, \(P\) is the arbitrary hydrostatic pressure.

\[
\tau_{ij}(x, t) = 2 \int_{-\infty}^{t} \psi(t, \tilde{t}) \frac{\partial x^i}{\partial \tilde{x}^m} \frac{\partial x^j}{\partial \tilde{x}^r} e^{(l)mr}(\tilde{x}, \tilde{t}) \, d\tilde{t} \tag{1.4.6}
\]

and

\[
\psi(t, \tilde{t}) = \int_{0}^{\infty} \frac{N(\tau)}{\tau} \exp \left( - \left( \frac{t - \tilde{t}}{\tau} \right) \right) d\tau \tag{1.4.7}
\]

Where \(N(\tau)\) is the distribution function of relaxation times

\[
\tau_{ij}(x, t) = 2 \int_{-\infty}^{t} \psi(t, \tilde{t}) \frac{\partial x^i}{\partial x^m} \frac{\partial x^j}{\partial x^r} e^{(l)mr}(\tilde{x}, \tilde{t}) \, dt \tag{1.4.8}
\]
where $x^i$ and $x^k$ are the rate of strain tensor.

The fluid specified as liquid $B'$ (by Oldroyd), with equation

$$
\left(1 + \lambda_1 \frac{d}{dt}\right) \tau^{ji} = 2\eta_0 \left(1 + \lambda_2 \frac{d}{dt}\right) \epsilon^{(ij)}
$$

(1.4.9)

In a peculiar case of liquid $B'$ incurred by

$$
N (\tau) = \eta_0 \left(\frac{\lambda_2}{\lambda_1}\right) \delta (\tau) + \eta_0 \left(\frac{\lambda_1 - \lambda_0}{\lambda_1}\right) \delta (\tau - \lambda_1)
$$

(1.4.10)

In equations (1.4.5) to (1.4.10) where prime denotes a function of Dirac delta and which is defined as

$$
\delta (x) = 0 \quad \text{when} \ x \neq 0
$$

(1.4.11)

In above equations $\frac{d}{dt}$ shows convicted differentiation of tensor quantity. For contravariant tensor $h^{ik}$,

$$
\frac{d}{dt} \left(h^{ik}\right) = \frac{\partial}{\partial t} \left(h^{ik}\right) + v^m \frac{\partial h^{ik}}{\partial x^m} - \frac{\partial v^k}{\partial x^m} - \frac{\partial v^i}{\partial x^m} h^{mk}
$$

(1.4.12)

Here $v$ is the velocity vector and $\eta_0$ is the Newtonian liquid constant viscosity and can be expressed as

$$
N (\tau) = \eta_0 \delta (\tau)
$$

(1.4.13)

For mathematical formulation, we limit the discussion to fluid with short time memory having the following simplified mathematical form

$$
\tau^{ji} = 2\eta_0 \epsilon^{(ij)} - 2k_0 \frac{d}{dt} \left(\epsilon^{(ij)}\right)
$$

(1.4.14)
Where \( \eta_0 = \int_0^\infty N(\tau)d\tau \) viscosity at small shear rates.

\[ k_0 = \int_0^\infty N(\tau)d\tau \] and terms regarding \( \int_0^\infty \tau^n N(\tau)d\tau \quad (n \geq 2) \) is ignored.

The liquid which satisfies the equation of state (1.4.5) and (1.4.13) is refer to as Walter’s liquid \( B' \) and the liquid which satisfies the equation (1.4.5) and (1.4.6) is called Walter’s liquid. For the Oldroyd’s liquid \( B' \), the equation (1.4.9) and (1.4.10) becomes

\[ k_0 = \eta_0 \left[ 1 - (\lambda_1 - \lambda_2) \frac{d}{dt} \right] e^{(i)ij} \quad (1.4.15) \]

Here approximation is equivalent to ignoring second order terms in \( \lambda_1 \) and \( \lambda_2 \).

### 1.4.3 Oldroyd Constitutive Equations

There is another class of fluid studied by Oldroyd (1954,1958) known as Oldroyd fluid. The constitutive equations of Oldroyd fluid is of the following form.

\[ \tau_y' + T_1 \frac{D}{Dt} \tau_y + \mu_1 \tau_y' k e_y - \mu_2 (\tau_y' e_y k + \tau_y' j e_y k) + \nu_1 \tau_y' i e_y k \delta_y = 2\mu (e_y + T_2 \frac{D}{Dt} e_y - 2\mu_3 e_y k e_y j + \nu_2 e_y k \delta_y) \quad (1.4.16) \]

where

\[ \tau_y' = P \delta_y + \tau_y \quad (1.4.17) \]

\[ e_y = \frac{1}{2}(v_y + v_x) \quad (1.4.18) \]

and

\[ \frac{D}{Dt} b_y = \frac{\partial}{\partial t} b_y + v_k \frac{\partial b_y}{\partial x_k} + w_i k b_k j + w_j k b_k j \quad (1.4.19) \]

with \( w_i j = v_i j - v_j i \) (verticity vector) Here \( \mu, T_1, T_2 (< T_1) \) are the co-efficient of viscosity, stress relaxation time and rate of stress retardation time respectively and the constants \( \mu_1, \mu_2, \nu_1, \nu_2 \) have dimensions of time. The elasticity or the memory
of the fluid has been taken into account by relaxation times $T_1$ and $T_2$. The linearity of the Newtonian constitutive equation has been broken up by introducing quadratic terms in the rate of strain components and the product of stress and the rate of strain components.

1.4.4 Nanofluid

A nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. Nanofluids have novel properties that make them potentially useful in many applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid powered engines, engine cooling/vehicle thermal management, domestic refrigerator, chiller, heat exchanger, in grinding, machining and in boiler flue gas temperature reduction. They exhibit enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid. Knowledge of the rheological behavior of nanofluids is found to be very critical in deciding their suitability for convective heat transfer application. Nanofluids also have special acoustical properties and in ultrasonic fields display additional shear-wave reconversion of an incident compressional wave, the effect becomes more pronounced as concentration increases. In analysis such as computational fluid dynamics (CFD), nanofluids can be assumed to be single phase fluids. However, almost all of new academic paper use two-phase assumption. Classical theory of single phase fluids can be applied, where physical properties of nanofluid is taken as a function of properties of both constituents and their concentrations. An alternative approach simulates nanofluids using a two-component model. The spreading of a nanofluid droplet is enhanced by the solid-like ordering structure of nanoparticles assembled near the contact line by diffusion, which gives rise to a structural disjoining pressure in the vicinity of the contact line. However, such enhancement is not observed for small droplets with diameter of nanometer scale, because the wetting time scale is much smaller than the diffusion time scale.
1.4.4.1 Smart cooling nanofluids

Realizing the modest thermal conductivity enhancement in conventional nanofluids, a team of researchers at Indira Gandhi Centre for Atomic Research Centre, Kalpakkam developed a new class of magnetically polarizable nanofluids where the thermal conductivity enhancement up to 300% of basefluids is demonstrated. Fatty-acid-capped magnetite nanoparticles of different sizes (3-10 nm) have been synthesized for this purpose. It has been shown that both the thermal and rheological properties of such magnetic nanofluids are tunable by varying the magnetic field strength and orientation with respect to the direction of heat flow. Such response stimuli fluids are reversibly switchable and have applications in miniature devices such as micro- and nano-electromechanical systems. In 2013, Azizian considered the effect of an external magnetic field on the convective heat transfer coefficient of water-based magnetite nanofluid experimentally under laminar flow regime. Up to 300% enhancement obtained at Re=745 and magnetic field gradient of 32.5 mT/mm. The effect of the magnetic field on the pressure drop was not as significant.

1.4.5 Casson fluid

The rheological equation of state of the flow of an isotropic incompressible Casson fluid is

\[
\tau_{ij} = \begin{cases} 
2 \left( \mu_B + \frac{P_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\
2 \left( \mu_B + \frac{P_y}{\sqrt{2\pi}} \right) e_{ij}, & \pi < \pi_c 
\end{cases}
\]  

(1.4.20)

\[
\tau = \tau_0 + \mu \dot{\varepsilon} \]  

(1.4.21)

Where \(\tau\) is the shear stress, \(\tau_0\) is the casson yield stress, \(\mu\) is the dynamic viscosity, \(\dot{\varepsilon}\) is the shear rate, \(\pi = e_{ij} e_{ij}\) and \(e_{ij}\) is the \((i, j)^{th}\) component of the deformation rate, \(\pi_c\) is the double component of deformation rate, \(\pi_B\) is critical value of this double based on the non-Newtonian model, \(P_y\) is the plastic dynamic viscosity of the non-Newtonian fluid, is the yield stress of the fluid there fore when the shear stress is smaller then the yield stress , fluid exhibits no motion i.e., it acts like a solid and it represents flow characteristics only when shear stress is greater then \(P_y\).
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1.5 Classification of Fluid Flow

In physics, fluid flow is classified as steady or unsteady, compressible or incompressible, viscous or non-viscous, and rotational or irrotational, laminar or turbulent, Subsonic or transonic. Some of these characteristics reflect properties of the liquid itself and others focus on how the fluid is moving.

Fluid flow can get very complex when it becomes turbulent. Physicists haven’t developed any elegant equations to describe turbulence because how turbulence works depends on the individual system.

Fluid Evenness: Steady or Unsteady flow

Fluid flow can be steady or unsteady, depending on the fluid’s velocity. Steady. In steady fluid flow, the velocity of the fluid is constant at any point. Unsteady. When the flow is unsteady, the fluid’s velocity can differ between any two points. For example, suppose we’re sitting by the side of a stream and note that the water flow is not steady: We see eddies and backwash and all kinds of swirling. Imagine velocity vectors for a hundred points in the water, and we get a good picture of unsteady flow, the velocity vectors can be pointing all over the map, although the velocity vectors generally follow the stream’s overall average flow.

Fluid Squeezability: Compressible or Incompressible flow

Fluid flow can be compressible or incompressible, depending on whether we can easily compress the fluid. Liquids are usually nearly impossible to compress, whereas gases (also considered a fluid) are very compressible. A hydraulic system works only because liquids are incompressible that is, when we increase the pressure in one location in the hydraulic system, the pressure increases to match everywhere in the whole system. Gases, on the other hand, are very compressible, even when the bike tire is stretched to its limit, we can still pump more air into it by pushing down on the plunger and squeezing it in.

Fluid Thickness: Viscous or Nonviscous flow

Liquid flow can be viscous or nonviscous. Viscosity is a measure of the thickness of a fluid, and very gloppy fluids such as motor oil or shampoo are called viscous fluids. Viscosity is actually a measure of friction in the fluid. When a fluid flows, the layers of fluid rub against one another, and in very viscous fluids, the friction is so great that the layers of flow pull against one other and hamper that flow. Viscosity usually varies with temperature, because when the molecules of a fluid
are moving faster (when the fluid is warmer), the molecules can more easily slide over each other.

**Fluid Spinning: Rotational or Irrotational flow**

Fluid flow can be rotational or irrotational. If the small object spins, the flow is rotational; if the object doesn’t spin, the flow is irrotational. Some flows that we may think are rotational are actually irrotational. On the other hand, flows that have no apparent rotation can actually be rotational. Take a shear flow, for example. In a shear flow, all the fluid is moving in the same direction, but the fluid is moving faster on one side. Suppose the fluid is moving faster on the left than on the right. The fluid isn’t moving in a circle at all, but if we place a small floating object in this flow, the flow on the left side of the object is slightly faster, so the object begins to spin. The flow is rotational.

**Flow pattern: Laminar or Turbulent Flow**

The flow of a fluid moving with a moderate speed has fluid layers moving past other layers as if some sheets are moving over other layers. Such flow of fluids is called Laminar Flow. In Laminar Flow viscous shear stresses act between these layers of the fluid which defines the velocity distribution among these layers of flow. In Laminar Flows the shear stresses are defined by Newton’s equation for shear stress. As the flow speed of the otherwise calm layers increases, these smoothly moving layers start moving randomly, and with further increase in flow velocity, the flow of fluid particles becomes completely random and no such laminar layers exist any more. Shear stresses in the Turbulent Flow are more than those in Laminar Flow. A dimensionless parameter, Reynolds Number, is defined as the ratio of inertial and viscous force to characterize these two types of flow patterns. With increase in flow velocity the initial forces increase so the Reynolds Number. For moderate flows the Reynolds Number is below 2000 and for Turbulent Flows it is well above 2300. For the transition region between the two types the Reynolds Number varies between 2000-4000.

**Subsonic or transonic, supersonic and hypersonic flows**

While many terrestrial flows (e.g. flow of water through a pipe) occur at low mach numbers, many flows of practical interest (e.g. in aerodynamics) occur at high fractions of the Mach Number M=1 or in excess of it (supersonic flows). New phenomena occur at these Mach number regimes (e.g. shock waves for supersonic flow, transonic instability in a regime of flows with M nearly equal to 1, nonequilibrium chemical behavior due to ionization in hypersonic flows) and it is
necessary to treat each of these flow regimes separately.

1.6 Porous Media

The material which consists of pores is called porous medium. Matrix or foam is the term used for skeletal portion. In this, pores contains fluids where the skeletal material is mainly solid. Foam like structure are used for analysis of the concept of porous media. Porosity is the characteristic of porous medium, the respective properties of its constituent are the derivation of the other properties of the porous medium. For example tensile strength, permeability, electrical conductivity. Several natural substances like rocks, soil, biological tissues, cement and ceramic are the few examples of porous medium. In recent days the concept of porous medium is applying in several fields of mechanics, filtration, construction, geosciences, biology and biophysics, material science etc. Now the study of porous medium is becoming great interest has emerged a separate area. Studies of flow through porous medium first engaged the attention of several engineers of the famous corps-despots at Chaussees during the second half of the 19th century. Henry P.G. Darcy (1803-1858) as a director of the public work in Dijon has worked in the design and execution of the municipal water supply system. The law which Darcy discovered known as ‘Darcy law’ states that the rate of flow is proportional to the pressure drop through a bed of fine particle. It is mathematically expressed as

\[
Q' = -\frac{k' \frac{d\rho}{\mu}}{dx}
\]  

(1.6.1)

Where \(k'\) is represents permeability of the material, \(Q\) is the volumetric flow rate per unit cross sectional area. Darcy law is generally accepted as the microscopic equation of motion for Newtonian fluids. The flow governed by this law, in the case of homogeneous isotropic porous medium is of potential type rather then the boundary layer. One of the approximate boundary layer type of equations in a porous medium is the Brinkman Model, Brinkman model consists of viscous term \(\gamma \nabla^2 \vec{q}'\) in addition to the Darcy resistance term \(\frac{\rho}{k'} \vec{q}'\) in the momentum equation. Brinkman model generalizes the fact that when permeability \(k' \to \infty\), we obtain the equation of pure MHD flow and when \(k' \to 0\) tends to Darcy flow through
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porous medium.

Basic equations:
Equation of continuity:

$$\nabla \cdot \vec{q} = 0$$  \hspace{1cm} (1.6.2)

Equation of motion:

$$\rho \left( \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right) = -\nabla p + \rho \vec{a} - \frac{\mu}{k} \vec{q} + \mu \nabla^2 \vec{q}$$  \hspace{1cm} (1.6.3)

Equation of Energy:

$$\rho C_p \left( \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T \right) = k \nabla^2 T$$  \hspace{1cm} (1.6.4)

Equation of concentration:

$$\rho C_p \left( \frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C \right) = k \nabla^2 C$$  \hspace{1cm} (1.6.5)

1.7 Hydrodynamics

Hydrodynamics describes the behavior of liquids in motion. Use of Hydrodynamics principles is the base of Hydraulic engineering and to design certain machines. Engineers use hydrodynamics in planning of water supply system, canals and irrigation system etc.

Laws of hydrodynamics play the fundamental role in designing water turbines. Some water turbines are submerged in rivers and the normal flow of water provides power. Other water turbines are located at the bases of dams.

Laws of hydrodynamics also describe the behavior of flowing fluids. Fluid flow may be steady or unsteady. An unsteady flow results changes in the velocity, temperature or pressure of a fluid. The movement of a fluid around obstructions may also cause an unsteady flow.
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Fluid dynamics including mechanics of liquid and gases, is a part of physics and we can divide the processes of any physical theory into four steps.

- Placing observed data into general physical law, assuming the uniformity of nature.

- Transferring the physical law into mathematical form, thus obtaining a logically consistent system of axioms which usually consists of a syatem of differential equations.

- Drawing the conclusions from differential equations. This is purely mathematical step.

- Check on the results obtained in step 3 by experiments.

It is often assumed that the first two steps are completed for all branches of mechanics.

The above first two steps have been carried out for as the fluids which are concerned in two forms. Firstly a theory of perfect fluid has been developed and then it was suplemented by the theory of viscous fluids.

The theory of perfect fluid(or ideal fluid) was studied by Euler during 1760 and the theory of viscous fluid was given by Navier and Stokes during 1850. In both the cases, we have a definite system of partial differential equations. Now the third step consists of the solution of certain boundary value problems of these partial differential equations.

In 1970, the world wide production of plastic materials was about 150 billion pounds. Based on the rate of growth at that time was estimated that the total volume of polymers used for all purpose would exceed that of metals by early 1980’s. We can see around us to found thousands of many things which are made of polymeric materials that come across daily in our life. Basically all of these materials were made and refined in liquid state( molten or solution). These liquids, are in compound structure and it shows that widely varying non-Newtonian fluids have linear viscous properties.
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Basic equations:

Continuity equation:

\[ \nu^i = 0 \]  \hspace{1cm} (1.7.1)

Where \( \nu^i \) is the velocity vector.

Momentum equation:

\[ \nu^i,_i + \nu^i \nu^j, j = \frac{1}{\rho} T^{ij}, j \]  \hspace{1cm} (1.7.2)

In the absence of external forces and is the density of the fluid.

Energy Equation:

\[ \rho C_p (T_{i, i} + T_{i, j}) = k g^{ij} T_{ij} + T^{ik} E_{kj} \]  \hspace{1cm} (1.7.3)

Where \( k \) is the thermal conductivity of the fluid and \( C_p \) denotes specific heat of the fluid.

Equations of motion for Two dimensional flow
(Cartesian co-ordinate system)

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  \hspace{1cm} (1.7.4)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \nabla^2 u - \frac{1}{\rho} \frac{\partial p}{\partial x} \]
\[ -k_0 \left\{ \left( u \frac{\partial \nu}{\partial x} + v \frac{\partial \nu}{\partial y} \right) \nabla^2 u - \frac{\partial u}{\partial x} \nabla^2 u - \frac{\partial u}{\partial y} \nabla^2 u - 2 \left[ \frac{\partial u}{\partial x} \frac{\partial^2 \nu}{\partial x^2} + \frac{\partial u}{\partial y} \frac{\partial^2 \nu}{\partial y^2} + \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{\partial^2 \nu}{\partial x \partial y} \right] \right\} \]  \hspace{1cm} (1.7.5)

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \gamma \nabla^2 v - \frac{1}{\rho} \frac{\partial p}{\partial y} \]
\[ -k_0 \left\{ \left( u \frac{\partial \nu}{\partial x} + v \frac{\partial \nu}{\partial y} \right) \nabla^2 v - \frac{\partial u}{\partial x} \nabla^2 u - \frac{\partial u}{\partial y} \nabla^2 u - 2 \left[ \frac{\partial u}{\partial x} \frac{\partial^2 \nu}{\partial x^2} + \frac{\partial u}{\partial y} \frac{\partial^2 \nu}{\partial y^2} + \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{\partial^2 \nu}{\partial x \partial y} \right] \right\} \]  \hspace{1cm} (1.7.6)
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Where $\gamma = \frac{2\mu}{\rho}$ is kinematic viscosity and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

(1.7.7)

Making the usual boundary layer approximations, equations of motions for unsteady flow are derived as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} - k_0 \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial x \partial y^3} + \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y} \right\}$$

(1.7.8)

**Momentum equation**: (Steady case)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \gamma \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} - k_0 \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial x \partial y^3} + \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y} \right\}$$

(1.7.9)

Momentum equation for the flow of visco-elastic fluid due to stretching sheet by boundary layer approximations

$$\left( \frac{\partial p}{\partial x} = 0 \right)$$

(1.7.10)

is given by

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( \frac{\partial^2 u}{\partial y^2} \right) + k_0 \left\{ u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial x \partial y^3} + \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y} \right\}$$

(1.7.11)

**Energy equation**: The relevant two-dimensional equation of energy for unsteady flow is

$$\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \phi$$

(1.7.12)

Boundary layer Energy equation for two dimensional Steady flow is
\[ \rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \phi \]  \hspace{1cm} (1.7.13)

And Boundary layer Energy equation for unsteady incompressible flow is

\[ \rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \phi \]  \hspace{1cm} (1.7.14)

We discuss some of the other constitutive equations as follows

1.8 Magneto Hydrodynamics:

Magneto hydrodynamics is the study of motion of an electrically conducting fluid in the presence of external magnetic field. It is the combinations of branches hydrodynamics and electromagnetism. Magneto hydrodynamics (MHD) is relatively new but an important branch of fluid dynamics. It is concern with the interaction of electrically conducting fluids and electromagnetic fields. When conduction fluid moves through a magnetic field, an electric field and consequently current may be induced and in turn the current interacts with the magnetic field to produce by a body force. The magnetohydrodynamics is the compound word composed of two words magneto which means magnetic and hydro means water and dynamics refers to the movement of an object by force. MHD concern with the dynamics of magnetic field in electrically conducting fluids. MHD interactions occur both in nature and in new man made devices. Such as in the earth interior, the Sun, the ionosphere, Star and their atmosphere to maintain a few area. In the laboratory several latest devises have been made which utilize the MHD directly, such as propulsion units and power generators or which involve fluid electromagnetic field interactions. Such as electron beam dynamics, traveling wave tubes, electrical discharges and many others. The subject of MHD has its origin in the study of magnetism of cosmic problems i.e. problems of the earth interior of Sun, the star, the interstellar space etc. MHD deals with the problems such as cooling of nuclear reactors by liquid sodium in the extraction of electrical energy directly from plasma through a powerful magnetic field. In presence of magnetic field Hart-
mann in 1937, have studied about the motion of electrically conducting fluid. The physicists Ferraro and Chapman were shown great interest to develop the theory magnetic storms. The following information about the early development of MHD is worth mentioning. By combining the Maxwell’s equations with the fundamental equations of hydrodynamics, Alfven in 1940 predicted new type of wave motion, which is presently called as Alfven’s wave. At the same time, Alfven established the theorems of Frozen fluid i.e., in a highly conducting fluid the magnetic lines of force are frozen into the fluid. The basic equations of MHD when the displacement current and free charges neglected are

Maxwell’s Equations:

\[ \text{Curl} \vec{H} = J \quad (1.8.1) \]
\[ \text{Div} \vec{E} = 0 \quad (1.8.2) \]
\[ \text{Curl} \vec{E} = \mu_c \frac{\partial H}{\partial t} \quad (1.8.3) \]
\[ \text{Div} H = 0 \quad (1.8.4) \]
\[ \text{Div} J = 0 \quad (1.8.5) \]
\[ \vec{J} = \sigma \left( \vec{E} + \vec{q} \times \vec{B} \right) \quad (1.8.6) \]

Equation of continuity:

\[ \text{Div} \vec{q} = 0 \quad (1.8.7) \]

Equation of motion:

\[ \frac{d\vec{q}}{dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{q} + \frac{\vec{J} \times \vec{B}}{\rho} \quad (1.8.8) \]

Where \( \vec{q} \) is the velocity of the fluid, \( p \) is the hydrostatic pressure, \( \rho \) is the density. \( \vec{B} = \mu_c \vec{H}, \vec{B} \) is the electromagnetic induction, \( \vec{E} \) is the electric field, \( \vec{J} \times \vec{B} \) is the Lorentz force and \( \frac{\partial}{\partial t} \) the operator.
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Equation of Energy:

\[ \rho C_v \frac{dT}{dt} = k \nabla^2 T + \varphi + \frac{J^2}{\sigma} \quad (1.8.9) \]

Where

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \quad (1.8.10)
\]

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (1.8.11)
\]

Two dimensional form of Lorentz force:
When the conduction current \( \sigma \vec{E} \) is negligible compared to the conduction current, the Ohms law gives \( \vec{J} (\vec{q} \times \vec{B}) \)

If

\[
\vec{q} = q_i + q_j \quad (1.8.12)
\]

\[
\vec{H} = \vec{H}_0 = \pm H_0 \quad (1.8.13)
\]

\[
\vec{B} = \vec{B}_0 = \pm B_0 \quad (1.8.14)
\]

\[
\vec{q} \times \vec{B} = \pm B_0 q_k \quad (1.8.15)
\]

\[
\vec{J} = \pm \sigma B_0^2 q_i \quad (1.8.16)
\]

Hence the Lorentz force reduces to the following form

\[
\vec{J} \times \vec{B} = -\sigma B_0^2 q_i \quad (1.8.17)
\]

Where \( \vec{H}_0 \) is the applied magnetic field and \( i, j, \) and \( k \) are the three orthogonal vectors.
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1.9 Heat Transfer

The science of thermodynamics deals with the amount of heat transfer as system undergoes a process from one equilibrium state to another, without any information concerning the nature of interaction or the time rate at which it occurs. But often we are interested in the rate of heat transfer as well as the temperature distribution within the system at a specified time. The heat transfer is a branch of thermal science which deals with analysis of rate of heat transfer and temperature distribution taking place in a system as well as the nature of heat transfer. The design of boilers, condensers, evaporators, heaters, refrigerators and heat exchangers, requires consideration of the amount of heat to be transmitted as well as at which heat is to be transferred. The successful operation of equipment component such as turbine blades, the walls of combustion chambers etc. depending on the cooling rate, in order to avoid their metallurgical failure. A heat transfer analysis must also be accounted in the design of electronic components, electric machines, transformers and bearings to avoid the overheating and damage of equipments.

Heat is a form of energy in transit due to temperature difference. Heat transfer is transmission of energy from one region to another region as a result of temperature difference between them. Whenever there exist a temperature difference in medium or within media, heat transfer must occur.

1.9.1 Steady and unsteady heat transfer

For analysis of the heat transfer problems, two types of heat transfer is considered: steady state and unsteady state. In case of steady state heat transfer, temperature at any location on the system does not vary with time. The temperature is function of space coordinates, but it is independent of time. Mathematically, for rectangular coordinate system.

\[ T = f(x, y, z) \]  \hspace{1cm} (1.9.1)

During the steady state condition, the heat transfer rate is constant and there is no change of internal energy of the system. For examples, the heat transfer in coolers, heat exchangers, heat transfer from large furnaces etc. In unsteady state heat transfer, the temperature varies with time as well as position. Temperature is function of time and space coordinates. Mathematically for rectangular
coordinates

\[ T = f(x, y, z, t) \]  

(1.9.2)

During the unsteady state or transient heat transfer, rate of heat transfer varies with time due to change in internal energy of the system. Most of the actual heat transfer process are unsteady in nature, but some of them is considered in steady state to simplify them. For examples, heat transfer from hot coffee left in a room, cooling and heating process etc are transient processes. The heat transfer may be one, two or three directional, depending on the configuration of the system considered.

1.9.2 Modes of heat transfer

1. Conduction

2. Convection and

3. Radiation

1.9.2.1 Conduction

The conduction occurs usually in the stationary mediums. It is the mode of heat transfer in which energy exchange takes place from a region of high temperature to that of low temperature by direct molecular interactions and by the drift of electrons. The heat conduction may be viewed as the transfer of energy from more energetic molecules to adjacent less energetic molecules of a substance. When a fast moving molecules from a region of high temperature to collides with slower moving molecules from a region of lower temperature, the heat energy transfer takes place between them. The low energy molecules absorbs energy and thus gain temperature and the temperature of high energy molecules is lowered. The thermal energy in the solids may be conducted by two mechanism, migration of free electrons and lattice vibrations. These two effects are additive, but in general the transport due to free electron is more effective then transport due to vibrational energy in the lattice structure. In pure solids, the flow of electron is very powerful, thus make them good conductor. A good electrical conductor is always a good conductor of heat due to presence of large number of free electrons, which
may thermal energy from high temperature region to low temperature region as they may transport electrical charge. In non-metals the energy transfer is due to lattice vibrations only. The conduction heat transfer in liquids and gases occurs due to collision and diffusion of molecules during their random motion. However, the nature is much more complex. Most of the liquids accept water becomes poorer conductors at higher temperatures. For gases, the distance between molecules is related to the pressure. However, unless at very high pressure, the molecules in gases are at large distance and therefore, thermal conduction is almost negligible. The temperature gradient is the potential for heat conduction. If a body in any phase exists a temperature gradient, will definitely have the conduction heat transfer.

1.9.2.2 Convection

Convection is the mode of heat transfer in which the energy is transported by moving fluid particles. The convection heat transfer comprises of two mechanism. First is transfer of energy due to random molecular motion(diffusion) and second is the energy transfer by bulk or macroscopic motion of the fluid(advection). The molecules of the fluid are moving collectively or as aggregates thus carry energy from high temperature region to low temperature region. Thus the faster the fluid motion, greater the convection heat transfer. In absence of any bulk fluid motion, the heat transfer occurs by pure conduction. Hence heat transfer rate increases in presence of temperature gradient. The convection heat transfer is due to superposition of energy transfer by random motion of the molecules and by bulk motion of the fluid. Therefore, the convection is not fundamentally different mode of heat transfer, it consists of sum of Conduction from surface to adjacent layer of fluid, Energy transfer due to mass transfer and Conduction to adjacent layer of fluid to receiving surface. Convection heat transfer may be classified according to nature of fluid flow. If the fluid motion is artificially induced by a pump, fan or a blower, that forces the fluid over a surface to flow, the heat transfer is said to be forced convection. If the fluid motion is set up by buoyancy effects, resulting from density difference caused by temperature difference in the fluid, the heat transfer is said to be by the free or natural convection. For example, a hot plate vertically suspended in stagnant cool air, causes a motion in air layer adjacent to the plate surface because of temperature difference in air gives rise to density gradient which
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in turn setup the air motion.

1.9.2.3 Radiation

Thermal radiation is the energy emitted by a substance because of its temperature. The radiation energy emitted by a body is transmitted in the space in the form of electromagnetic waves according to Maxwell wave theory or in the form of discrete photons according Max Plank’s theory. Both concepts have been used in analysis of radiation heat transfer. Regardless of the form of substance (solid, liquid or gas) the emission of energy due to change in electron configuration of the constituent molecules. While the transfer of energy by conduction or convection requires the presence of material medium, radiation does not. In fact, the radiation heat transfer is more efficient in vacuum. Thermal radiation occurs in the region of wavelengths 0.1\(\mu m\) to 100\(\mu m\) on electromagnetic spectrum.

1.9.2.4 Thermal Conductivity

Thermal conductivity is the property of the material and is defined as ability of the material to conduct the heat through it. It is also interpreted as the rate of heat transfer through a unit thickness of material per unit area per unit temperature difference. The thermal conductivity of the material is a measure of how fast heat will flow in the material. The large value of thermal conductivity indicates that the material is a good heat conductor and low value indicates that the material is poor heat conductor or insulator. Thermal conductivity measured in Watts per meter per degree Celsius or Watt per meter per Kelvin, when heat flow rate expressed in Watts. The thermal conductivity of a substance is highest in solid phase and lowest in gaseous phase. The value of the thermal conductivity depends upon the manner in which energy is transferred. The pure metals allow faster transmission of heat energy through the vibrations of the crystal lattices. Therefore, a metal in pure state has the maximum thermal conductivity and is a good conductor. The thermal conductivity decreases with increasing amount of impurities in the metals. Most non-metals are poor conductor of heat transfer, therefore, have low values of thermal conductivity and are called thermal insulators. In gases, the faster the molecules move, the faster they will transport energy. Therefore, the thermal conductivity of gases depends on the square root of absolute temperature.
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1.9.2.5 Variation in Thermal Conductivity

The temperature is a measure of kinetic energies of molecules of a substance. Thus the thermal conductivity is a function of the temperature. It may also change with pressure in fluids. The heat energy may be conducted in metals by two mechanism: migration of free electrons and lattice vibrations. These two effects are additive. In general, the presence of electron gas (large free electrons) in metals, makes it good conductor of heat, but the conduction also takes place due to vibrational energy in lattice structure. The flow of free electrons in metal results in an increase in value of thermal conductivity several times. But at the same time, due to increase in the temperature, the vibrations of the molecules in the metal becomes violent and they obstruct the flow of free electrons and contribution to the heat conduction by free electrons decreases. Thus it may result in net decrease in the flow. Hence, for most of metals, the value of the thermal conductivity decreases as temperature increase. In non-metals due to absence of free electrons, the most heat conduction is only due to lattice vibrations. As temperature increases, the number of collagen per unit time increases, hence the rate of heat flow increases in non-metals. Thus the thermal conductivity of non-metals increases with increase in temperature. For most of liquids, the thermal conductivity decreases with increase in temperature. But water and glycerine are the exceptional cases. The thermal conductivity of liquids is independent of pressure. As a general rule, the thermal conductivity of liquids decreases with increasing molecular weight. For the gases, the molecules are in continuous random motion. As temperature increases, velocities of the molecules become higher then in some lower temperature region. The molecules move from high temperature region to a region of low temperature and give up its energy through collisions to lower energy molecules. Thus the thermal conductivity of gases increases with increases in temperature and it is proportional to square root of the absolute temperature. It is also affected by change in pressure and humidity. The thermal conductivity $k$ can be defined by Fourier law as follows

$$ k = \frac{Q}{A} \frac{dx}{dt} $$

(1.9.3)
1.10 Mass Transfer and Concentration Boundary Layer Equations

In general, transfer of mass is dealt in subject of mechanics. The mass transfer associated with convection, occurs on a microscopic level due to change in momentum is turned as mass (fluid) transport and is usually treated in subject of fluid mechanics. When system contains two or more species (components) whose concentration vary from point to point, there is natural tendency for mass to be transferred for minimizing the concentration difference within the system. This transport of mass on a microscopic level as a result of diffusion from a region of high concentration to that of the lower concentration is called mass transfer.

1.10.1 Governing equations of Mass Transfer

The equations which govern the boundary layer conditions of convective mass transfer are

For species A

\[ \nabla \cdot n_A + \frac{\partial \rho_A}{\partial t} - r_A = 0 \]  \hspace{1cm} (1.10.1)

For species B

\[ \nabla \cdot n_B + \frac{\partial \rho_B}{\partial t} - r_B = 0 \]  \hspace{1cm} (1.10.2)

The continuity equation of the mixture is

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \nu) = 0 \]  \hspace{1cm} (1.10.3)

In order to obtain the concentration profiles, the flux, \( n_A \) is given as

\[ n_A = -\rho D_{AB} \nabla m_A + m_A (n_A + n_B) \]  \hspace{1cm} (1.10.4)
or

\[ n_A = -\rho D_{AB} \nabla m_A + \rho_A \nu \]  

(1.10.5)

Substituting the equations 5 in 1 we obtain

\[ -\nabla \cdot \rho D_{AB} \nabla m_A + \nabla \cdot \rho_A \nu + \frac{\partial \rho_A}{\partial t} - r_A = 0 \]  

(1.10.6)

Equation (1.11.6) is the general equation for describing a mass concentration profile in a general diffusing system. For system with constant \( \rho \) and \( D_{AB} \), equation 6 becomes

\[ -D_{AB} \nabla^2 \rho_A + \rho_A \nabla \cdot \nu + \nu \cdot \nabla \rho_A + \frac{\partial \rho_A}{\partial t} - r_A = 0 \]  

(1.10.7)

Defining each term of equation 6 by the molecular weight of A, we get

\[ -D_{AB} \nabla^2 \rho_A + C_A \nabla \cdot \nu + \nu \cdot \nabla C_A + \frac{\partial C_A}{\partial t} - R_A = 0 \]  

(1.10.8)

Equation (1.11.8) for an incompressible flow \( (\nabla \cdot \nu = 0) \) without any chemical reaction (production term \( R_A = 0 \) ) takes the form

\[ \frac{\partial C_A}{\partial t} + \nu \cdot \nabla C_A = D_{AB} \nabla^2 \rho_A \]  

(1.10.9)

Rewriting equation (1.11.9) as

\[ \frac{\partial C_A}{\partial t} + u \frac{\partial C_A}{\partial x} + \nu \frac{\partial C_A}{\partial y} + w \frac{\partial C_A}{\partial z} = D_{AB} \left( \frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} \right) \]  

(1.10.10)

Equation for the concentration boundary layer can now be obtained from equations (1.11.4) and (1.11.10) by incorporating in them certain approximations and special conditions, characterizing the boundary layer as steady, constant, incompressible and non-Newtonian fluids. The following boundary layer approximations
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may be involved.

\[
\frac{\partial C_A}{\partial x} < \frac{\partial C_A}{\partial y}
\]  \hspace{1cm} (1.10.11)

With this implication the concentration boundary layer equations (1.11.3) and (1.11.10) reduce to

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0 \hspace{1cm} (1.10.12)
\]

\[
u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial y} = D_{AB} \left( \frac{\partial^2 C_A}{\partial y^2} \right) \hspace{1cm} (1.10.13)
\]

1.10.2 Modes of Mass Transfer

The mechanism of mass transfer process can be classified as :

1. Diffusion

2. Convection and

3. Change of phase

1.10.2.1 Mass transfer by diffusion:

the molecular diffusion is the transfer of mass on a microscopic level as a result of concentration gradient of one or more constituents (species) in the system. The diffusion of mass transfer a constituent occurs through a layer of stagnant fluid, and it may be occurred due to concentration gradients, temperature gradient or pressure gradient. The various examples of molecular diffusion can be taken from our day by day experiences. Some of these are

- Diffusion of sugar or salt in the hot water and its uniform diffusion throughout.
- Evaporation of water from a pond to increase the humidity of passing air stream.
- Evaporation of petrol in engine carburetor.
• Evaporation of moisture during drying the cloths or wood.

• Humidification of air in air coolers, cooling towers.

• Spread of fragrance of perfume or flowers in surroundings.

• Diffusion of smoke through a tall chimney into atmosphere.

When one of the diffusion fluid is in turbulent motion, the eddy diffusion takes place. Due to turbulence, the rate of diffusion increases, resulting in an increase in mass transfer. An example of eddy diffusion process is dissipation of smoke from a smoke stack.

1.10.2.2 Mass transfer by convection:

the mass transfer between a surface and a moving fluid or between two immisible moving fluids is referred as convective mass transfer. The convective mass transfer depends on transport properties of fluid and dynamic characteristics (laminar or turbulent) of flowing fluid. The evaporation of water by moving air a desert cooler is an example of convective mass transfer.

1.10.2.3 Mass transfer by change of phase:

A change of phase of a fluid can also cause mass transfer, such mass transfer is a combination of processes of diffusion and convection. Some examples of mass transfer by phase change are: Hot flue gases leaving the chimney rise by convection and then diffuse into atmosphere air and mass transfer from boiling ware into air.

1.10.3 Convective Mass Transfer

The convective mass transfer involves the diffusion of a fluid with bulk momentum of other fluid or diffusion between two immiscible moving fluid. It is the situation analogous to convective heat transfer. The convective mass transfer can also be classified free convection and forced convection. In free mass convection, the concentration gradient changes the density of the fluid, which may be acted upon
by buoyancy force. The evaporation of alcohol, gasoline are the examples of free mass convection. In forced mass convection, one fluid is moving with appreciate velocity over the other. The evaporation of water in ocean, cooling tower or exhaust gas of an automobile is forced to pass over a catalytic converter are some examples of forced mass convection. The fluid may be laminar or turbulent. If the fluid flow is laminar, then all of the transport between a surface and moving fluid will be by molecular means. On the other hand, if fluid flow is turbulent, there will be a physical movement of lumps of fluid across streamlines, transported by eddies present. The properties of these eddies may be ten times larger then the molecular values. Therefore, mass transfer by forced convection is larger then the free convection.

1.11 Comparison between Heat and Mass Transfer

The mass can be transferred by random molecular motion in fluids at the rest or it can be transferred from a surface into a moving fluid, aided by dynamic characteristics of floc. These two distinct mode of heat transfer, molecular mass diffusion and convective mass transfer are analogous to conduction heat transfer and convection heat transfer. There are following similarities between heat and mass transfer:

- The heat transfer takes place, when a temperature gradient exist in a medium and it continues until the entire medium attains a uniform temperature. Similarly, mass transfer takes place, when a concentration gradient of one or more chemical species exists and it continues until the concentration of all species become uniform over the entire volume.

- The conduction heat transfer is a molecular phenomenon and respective thermal conductivity is a material property. Similarly, the diffusion mass transfer is a molecular phenomenon and respective diffusion coefficient is a property of particular combination of species and medium.

- Convective heat transfer coefficient is a flow property depending on fluid flow properties and geometry. Similarly, the convective mass transfer coefficient
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also depend on flow and geometrical parameters and they can be described by similar governing equations.

There are some significant difference between heat and mass transfer. These are:

- Heat transfer is a one way phenomenon. It always flows from high temperature region to low temperature region. But the mass transfer is multiway phenomenon. When two locations A and B are at different concentration within a system of species 1, 2, 3, etc. the different species diffuse simultaneous between A and B, depending on the concentration gradient. Some species may diffuse from A to B while some other may diffuse from B to A and their diffusion rates will also be different.

- Except at low concentrations, the concentration change affects the momentum and continuity equations and they cannot solved independently.

- In heat transfer, the boundary conditions may involve specified temperature, specified heat flux, convection or radiation boundary, and their treatment rather simple. On the other hand, in mass transfer, the boundary may involve different phases, interphase such as condensation, evaporation, adsorption, chemical reaction etc. this makes the boundary condition much more complex.

- The mass diffusion rate depends on molecular size of diffusing species compared to medium size in heat transfer.

- The heat conduction in its pure state, does not deal with any medium velocity. But pure mass diffusion deals with bulk velocity of the medium.

1.12 Thermal and Concentration Boundary Layers

Thermal and concentration boundary layers are very similar to the velocity boundary layers discussed before, except we focus our attention to the temperature and concentration profiles instead of velocity profiles. Instead of the growth of velocity from zero to a free stream value, thermal or concentration boundary layers track
the changes in temperature decay or concentration decay. We may discuss all
these boundary layers by using similarity principles. Consider a flow of polluted
river water that is brought into a tank for purification. As the flow enters the bed
it may be considered a flow over a flat plate. We would pan heat into the water
to kill germs and apply other methods of pollutant control. The velocity profile,
temperature profile, and concentration profile on the flat plate are all shown in
the figure below

In this diagram the velocity boundary layer is shown to grow with distance
x (marked by $\delta_{vel}(x)$), where as the growth of the temperature and concentration
profiles are marked as $\delta_{temp}(x)$ and $\delta_{conc}(x)$. Depending on the transport
characteristics of velocity, vorticity, temperature, and concentration these boundary
layers may grow at different rates. When we speak about velocity changes in the
boundary layer the fluid property that influences them is viscosity, whereas for
temperature and concentration boundary layers, the corresponding properties are
the convective heat transfer and mass transfer coefficients. The governing equa-
tions for velocity boundary layers, thermal boundary layers, and concentration
boundary layers all follow similar patterns. Rather than deriving the thermal and
concentration boundary layer equation we simply present them below. For a fluid
such as air that may be treated as an ideal, incompressible gas or, for an incom-
pressible liquid such as water,

$$
\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \mu \Phi + \dot{q} \quad (1.12.1)
$$

and

$$
\frac{\partial C_A}{\partial x} + \frac{\partial C_A}{\partial y} = \frac{\partial}{\partial x} \left( D_{AB} \frac{\partial C_A}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_{AB} \frac{\partial C_A}{\partial y} \right) + \dot{N}_A \quad (1.12.2)
$$

In the first equation $k$ represents the thermal conductivity of a homogeneous solid,
$\dot{q}$ represents the rate of heat generation per unit volume and $\mu \Phi$ represents the
rate of viscous dissipation per unit volume, given by
\[
\mu \Phi = \mu \left[ \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2 \left\{ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right\} - \frac{2}{3} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right] \tag{1.12.3}
\]

Similarly in the equation (B), \( D_{AB} \) represents the binary diffusion coefficients and \( \dot{N}_A \) represents the rate of generation of the concentration \( C_A \). In deriving the above relations some additional constitutive relations must be recalled. For example if the fluid is an ideal gas, the gas law gives
\[
p = \rho \bar{R}T \text{ or } p = C_A RT
\]
where, Specific gas constant \( \bar{R} = \frac{R}{M_A} \)
\( R = \) Universal gas constant
\( M_A = \text{Molecularweight} \left( \frac{k_g}{\text{kmol}} \right) \) of gas, \( A \).

Fourier’s law of heat conduction:
Heat flux,
\[
q'' = -k \frac{\partial T}{\partial y} \bigg|_{y=0} \tag{1.12.4}
\]
in the \( y \)-direction where, \( k \) is Thermal conductivity of the wall. But heat con vected into the fluid is given by the Newton’s law of cooling
\[
q'' = h(T_s - T_\infty) \tag{1.12.5}
\]
where, \( h = \) heat transfer coefficient (or, coefficient of heat convection)
Surface temperature \( T_s = T(y)|_{y=0} \)
Temperature of the ambient fluid = \( T_\infty \) Thus, \( \dot{Q} = q'' A_S = \)
Rate of heat flow into fluid
\[
= -k A_S \frac{\partial T}{\partial y} \bigg|_{y=0} = h(T_s - T_\infty) \tag{1.12.6}
\]
where, \( A_s = \) surface area through which heat flows. Therefore the heat transfer coefficient may be expressed as
\[h = \frac{-k \frac{\partial T}{\partial y}}{T_s - T_\infty}\]  

(1.12.7)

Similar to the heat transfer case the mass transfer constitutive relations are given by Fick’s law, which specifies molar flux, \(N'_{m_A}\) as

\[N''_A = -D_{AB} \frac{\partial C_A}{\partial y} \bigg|_{y=0}\]  

(1.12.8)

where, \(D_{AB}\) = Binary diffusion coefficient

But the molar flux coefficient may also be expressed as

\[N''_A = h_m (C_{A,S} - C_{A,\infty})\]  

(1.12.9)

where \(h_m\) = convective mass transfer coefficient \(C_{A,S}\) = Concentration of A at the surface \(C_{A,\infty}\) = Concentration of A in the ambient fluid

Therefore the convective mass transfer coefficient may be expressed as

\[h_m = -D_{AB} \frac{\partial C_A}{\partial y} \bigg|_{y=0} \frac{1}{C_{A,S} - C_{A,\infty}}\]  

(1.12.10)

Remember that \(h\) and \(h_m\) are variables defined by the above laws. For a finite size flat plate we may define (similar to the overall skin friction coefficient, \(C_f\) discussed in the the velocity boundary layers)

\[\bar{h} = \frac{1}{A_s} \int_{A_s} h \, dA_s\]  

(1.12.11)

and
\[ h_m = \frac{1}{A_S} \int_{A_S} h_m \, dA_S \quad (1.12.12) \]

The mean flux, \( n''_A \) may be related to the molar flux, \( N''_A \) yielding

\[ n''_A = M_A N''_A = h_m (\rho_{A,S} - \rho_{A,\infty}) \quad (1.12.13) \]

where, \( M_A \) = Molecular weight of A and, \( \rho_{A,S} = M_A C_{A,S} \), etc.

The above law shows striking similarity between the velocity boundary layer, thermal boundary layer, and the concentration boundary layer.

### 1.13 Momentum, Heat and Mass Transfer Analogies:

There is a marked similarity between the law governing the boundary layer growth of the three transport phenomena of momentum, heat and mass. These equations of laminar boundary layer flow over a flat plate are

**Momentum transfer:**

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( \frac{\partial^2 u}{\partial y^2} \right) \quad (1.13.1) \]

**Heat transfer:**

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k \left( \frac{\partial^2 T}{\partial y^2} \right) \quad (1.13.2) \]

**Mass transfer:**

\[ u \frac{\partial C_A}{\partial x} + v \frac{\partial C_A}{\partial y} = D_{AB} \left( \frac{\partial^2 C_A}{\partial y^2} \right) \quad (1.13.3) \]
1.13.1 Similarity Rules of Boundary Layers

If we recall the work related to Prandtl’s analysis in the velocity boundary layer was derived starting from a non-dimensionalization of the governing equations. The critical parameter to analyze the velocity boundary layer was the Reynolds number. Similar relations may be derived in cases of thermal boundary layer and concentration boundary layer. We shall omit the derivations here. However the set of critical parameters resulting from these operations must be noted carefully. For engineers, design solutions are influenced by these numbers encountered everyday. A thorough understanding of these numbers and their physical significance are essential.

1.14 Dimensionless Parameters

Every physical problem involve some physical quantities, which can be measured in different units. Dimensionless analysis of any problem provides information on qualitative behavior of the physical problem. The dimensionless parameter helps us to understand the physical significance of a particular phenomenon associated with the problem. There are usually two general methods for obtaining dimensionless parameters. i) The inspectional analysis ii) The dimensionless analysis 

In this thesis second method has been used, in this method the basic equations are made dimensionless using certain dependent and independent characteristics values. Some of the dimensionless parameter used in this are defined as follows.

Reynolds number:

The Reynolds number is defined as the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions.[5] Reynolds numbers frequently arise when performing scaling of fluid dynamics problems, and as such can be used to determine dynamic similitude between two different cases of fluid flow. They are also used to characterize different flow regimes within a similar fluid, such as laminar or turbulent flow.

\[
Re = \frac{Viscous\, force}{Energi\, force} = \frac{vL}{\nu}
\]

Where \( v \) is the maximum velocity of the object relative to the fluid, \( L \) is the characteristic length and \( \nu \) is the kinematic viscosity.
Skin friction:

When the boundary layer equations are integrated the velocity distribution
can be obtained and the position of the point of separation can be determined,
this in turn permits us to calculate the viscous drag and is known as shear stress
or skin friction. The shearing stress at the wall is calculated as

\[ \tau_0 = \mu \left( \frac{\partial u}{\partial y} \right) at y = 0 \]

Nusselt number:

The important dimensionless parameter associated with heat transfer prob-
lems is the Nusselt number and which is expressed as

\[ N_u = \frac{-h}{T_w - T_\infty} \left( \frac{\partial T}{\partial y} \right) at y = 0 \]

Where h is the some characteristic length and , \( T_w - T_\infty \) is the difference between
wall temperature and the fluid.

Magnetic parameter:

The Stuart number (M), also known as magnetic interaction parameter, is a
dimensionless number of fluids, i.e. gases or liquids. It is defined as the ratio of
electromagnetic to inertial forces, which gives an estimate of the relative impor-
tance of a magnetic field on a flow. The Stuart number is relevant for flows of
conducting fluids, e.g. in fusion reactors, steel casters or plasmas.

\[ M = \frac{B^2 L \sigma}{\rho U} \]

Where B is magnetic induction, L is characteristic length, \( \sigma \) is electric conductivity, \( U \) characteristic velocity scale and \( \rho \) is density.

Visco-elastic parameter:

The dimensionless visco-elastic parameter measure of the relative importance
of elastic and viscous effects and it is designated as \( k_1 \) expressed as

\[ k_1 = \frac{k_0 C}{\gamma} \]

Where \( \gamma \) kinematic coefficient of viscosity, \( k_0 \) is the Elastic coefficient and C is
the linear stretching rate respectively.
Porous parameter:

The dimensionless porous parameter is defined as the measure of viscous force to the Darcy resistance and mathematically

\[ k_2 = \frac{\gamma}{k'b} \]

Where \( k' \) is the permeability of the porous medium, \( \gamma \) is the kinematic coefficient of viscosity of the fluid and \( b \) is the boundary sheet.

Prandtl number:

In convection problem, the mechanism of release of thermal energy associated with a rich variety of phenomenon is exhibited by non-newtonian fluid linear convection. This verity steps primarily from the dependence of the motion on the prandtl number defined as

\[ Pr = \frac{\text{Viscous force}}{\text{Thermal force}} = \frac{\mu C_p}{K} \]

Eckert number:

The dimensionless quantity Eckert number is denoted by Ec and defined as

\[ Ec = \frac{b^2 l^2}{AC_p} \]

Where \( C_p \) is the specific heat at constant pressure, \( l \) be the characteristic length and \( b \) be the stretching rate.

Heat source/sink parameter:

The dimensionless parameter Heat source/sink parameter is defined as

\[ Q = \frac{Q_0}{\rho C_p} \]

Where \( Q_0 \) represents the heat source when \( Q_0 > 0 \) and heat sink when \( Q_0 < 0 \), is the specific heat at constant pressure.

Radiation parameter:

The radiation parameter is defined as

\[ R = \frac{KK^*}{4\sigma T_\infty^3} \]
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Hers K stands for thermal conductivity, $K^*$ is the absorption coefficient, $\sigma$ is the constant of Stefan-Boltzmann and $T_\infty$ shows the temperature away from the wall.

**Schmidt number:**

It is the ratio of momentum diffusivity to the mass diffusivity. It is designated as $Sc$ and expressed as

$$Sc = \frac{Momentumdiffusivity}{Massdiffusivity} = \frac{\nu}{D_{AB}}$$

The role of Schmidt number in convective mass transfer is analogous to role of Prandtl number in convective heat transfer I compares the relative thickness of velocity and concentration boundary layer.