Chapter 5

Models for disoriented chiral condensates

5.1 Equilibrium and non-equilibrium pictures

Formation of domains below critical temperature is common to physical systems exhibiting second order phase transition. The order parameter having zero average in the symmetric high temperature phase, develops a vacuum expectation value below the critical temperature. It is worth mentioning that the effective field theories governing the second order phase transition are nonlinear in nature, an explicit example of the same and its implications will be seen below. There is a possibility of having domains with the order parameter orienting along different directions, in the broken symmetry phase. The domains in a ferromagnet, below the Curie temperature, are the prime examples of this scenario.

The chiral phase transition in quantum chromodynamics (QCD), provides an opportunity for the realization of the above type of domain formation. This phase transition, originating from the spontaneous breaking of the chiral symmetry, leads to the existence of pseudo scalar mesons e.g., pions. The chiral order parameter in a given domain is disoriented from the zero temperature vacuum direction $\sigma$; therefore these type of space-time
domains are called disoriented chiral condensates (DCC). The eventual decay of these metastable DCC would lead to the production of pions, whose number distributions may be quite different from the expected proportion of 1/3 for each of the pion species in non-coherent production. A key motivation behind the search of DCC has been the Centauro events [1], where the neutral pion fraction $\lambda$, has been observed to be more than the expected value 1/3. There also have been reports of anti-Centauro events [2]. An opportunity for the experimental realization of DCC is provided by the collision of heavy ions at high energy, where there is a possibility of restoring the spontaneously broken chiral symmetry.

In the absence of baryons, the pion dynamics is captured by the $0(4)\sigma$-model with the Lagrangian,

$$L = \frac{1}{2} \left[ (\partial_\mu \sigma)^2 + (\partial_\mu \pi_\alpha)^2 \right] - \frac{\lambda}{4} [\sigma^2 + \pi_0^2 - \nu^2]^2 + H\sigma. \quad (5.1)$$

The last term breaks the $0(4)$ symmetry explicitly and is responsible for the masses of the pions. This model provides the starting point for various scenarios of DCC formation [3], which can be broadly classified into equilibrium and non-equilibrium pictures.

In the baked Alaska model of DCC, proposed initially by Bjorken [4], the high multiplicity ultra-high energy hadronic collisions lead to a rapidly expanding hot partonic shell. In its interior the chiral field may get misaligned from the true vacuum resulting in the formation of DCC. As pointed out earlier, depending on the direction of the order parameter in the domain, the resulting distribution of coherent pions, originating from the decay of the domain, can be substantially different from those of the incoherent pions originating from a regular plasma.

This is the most discussed signature of DCC. Defining the ratio of neutral pions to all pions by

$$f = \frac{n_{\pi_0}}{n_{\pi_0} + n_{\pi\pm}}, \quad (5.2)$$
it has been shown that, the probability distribution for $f$ can take a form

$$P(f) = \frac{1}{2\sqrt{f}}.$$ (5.3)

This distribution arises from the assumption that all points on the manifold $S^3$, representing $\sigma, \pi_1, \pi_2$ and $\pi_3$ fields, are equally likely as initial conditions. The number of pions of a given Cartesian isospin is taken to be proportional to $n'f$, $i$ being the component of the isospin. It should be pointed out that the above distribution is appropriate only when all the pions are produced from a single domain. If pions are incoherently produced then the probability distribution $f$, will be a Gaussian peaked around the value $1/3$. The above distribution differs markedly from the Gaussian one, especially for lower values of $f$. Probability for very small value of $f$ is negligible for incoherent emission of pions, while it can be substantial for the DCC case. Hence, significant departure from the value $1/3$ for $f$ is the cleanest signal for the formation of DCC.

The fact that pions have finite mass leads to a small correlation length and hence small domains, of the order of $m^{-1}$, in the equilibrium picture of DCC formation. This has led to the nonequilibrium, quenching picture of DCC formation by Rajagopal-Wilczek [5] in the relativistic heavy ion collisions. In this scenario, the expansion of highly relativistic debris from the heavy ion collisions lead to the quenching of high temperature field configurations and therefore to the growth of long wavelength modes in time. These then give rise to large correlated DCC domains, leading to coherent low energy pions. Here the DCC domains after getting detached from the heat bath evolve according to the zero temperature equations of motion [3, 5, 6]. Here dramatic signals like the above mentioned fluctuations in the ratio of the neutral pions to all pions may not arise. An annealing scenario for DCC formation has also been proposed [7]. The role of fluctuations above $T_G$ (Ginzburg temperature), in destabilizing the DCC in the conventional picture has also been analyzed [8].
A first order transition which may be a possibility in the chiral phase transition, can also give rise to DCC domains [9]. Recently in Ref. [10], the authors have demonstrated that a first order transition naturally leads to a quench like scenario, which is conducive to the growth of DCC domains.

The ability to detect DCC depends on their lifetime. The perfect non-equilibrium quenching scenario of Rajagopal and Wilczek, leads to the dissipation of the condensate. This is due to the possible energy exchange between different degrees of freedom, after the chiral condensate detaches from the heat bath, and evolves according to the zero temperature equations of motion. This problem has been recently studied [11, 12, 13, 14] for ascertaining various type of signals of DCC. Interestingly, a recent experiment, involving heavy-ions has hinted at the possibility of DCC formation [15, 16].

In the non-equilibrium picture of DCC formation in heavy ion collisions, one looks for extended solutions of the classical equations of motion, originating from the $\sigma$-model Lagrangian given above. In the linear $\sigma$-model, the potential responsible for the spontaneous breaking of chiral symmetry, introduces nonlinearity in the equations of motion. Because of the nonlinearity, it is difficult to obtain analytical solutions for the time evolution of the field configurations in $(3 + 1)$-dimensions. Hence, one starts with the idealized Heisenberg-type boundary conditions [17].

The thin disc representing Lorentz-contracted nuclei at the time of collision is assumed to be infinite in extent in the transverse directions. Assuming the fields to be independent of the transverse directions, one can simplify the problem to a $(1 + 1)$-dimensional field theory [6]. To obtain boost invariant solutions, which can lead to DCC formation, we resort to numerical methods. It should be pointed out that the existence of a central-plateau structure in the rapidity distribution of particles produced in cosmic ray events [18] and $pp$ or $pp$ collisions [19] has led to the assumption of an approximate $(1 + 1)$- Lorentz invariance.
5.2 Boost invariant solutions

Recently, Z. Wang et al. [20, 21] have studied the cluster structure of DCC, in the nonequilibrium picture, by solving the equations of motion corresponding to linear sigma model numerically. We will follow their approach in our simulations. After reproducing their results, we will investigate the scenario in which quenching takes the field to low enough a temperature, where the magnitude of the order parameter is constant, leaving the phase as the dynamical variable. In order to validate the initial conditions more appropriate to relativistic heavy ion collisions, they have neglected the transverse dimensions, and have assumed that $\Phi$ is only a function of $t$, and $z$.

In order to look for boost invariant solutions, the proper time $\tau = \sqrt{t^2 - z^2}$, and rapidity $\eta = \frac{1}{2} \ln \frac{t + z}{t - z}$, variables are introduced. We note that

$$\left[ \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \right] \rightarrow \left[ \frac{1}{\tau} \frac{\partial}{\partial \tau} \left( \tau \frac{\partial}{\partial \tau} \right) - \frac{1}{\tau^2} \frac{\partial^2}{\partial \eta^2} \right].$$

The equations of motion read

$$\left[ \frac{1}{\tau} \frac{\partial}{\partial \tau} \left( \tau \frac{\partial}{\partial \tau} \right) - \frac{1}{\tau^2} \frac{\partial^2}{\partial \eta^2} \right] \sigma = -\lambda \sigma (\sigma^2 + \pi^2 - v^2) + H \quad (5.5)$$

$$\left[ \frac{1}{\tau} \frac{\partial}{\partial \tau} \left( \tau \frac{\partial}{\partial \tau} \right) - \frac{1}{\tau^2} \frac{\partial^2}{\partial \eta^2} \right] \sigma = -\lambda \sigma (\sigma^2 + \pi^2 - v^2). \quad (5.6)$$

The simplest case to solve Eqs. (5.5) and (5.6) is when the system has Lorentz-boost invariance i.e.,

$$\phi(\tau_0, \eta) = \phi_0, \quad (5.7)$$

where $\phi_0$ is independent of $\eta$. Since the starting point is the symmetric phase from where quenching is done, one takes the initial conditions as $\phi(\tau_0) = 0$. The other boundary condition taken by the above authors is $\partial \pi / \partial \tau = (1, 5, 0, 0) \text{ MeV/fm}$, at $\tau_0 = 1 \text{ fm/c}$. The general feature of the solution is that the $\sigma$ field grows from zero and takes about $1 \text{ fm/c}$ propertime
to reach the true vacuum expectation value \( <\sigma> \approx f_\pi \). While the \( \Phi \) field oscillates around zero rather slowly and eventually tends to zero when the proper time gets large.

### 5.3 DCC and sine-Gordon equation

It is of deep interest to enquire, as to what happens when the field configuration in the symmetric phase is quenched to a temperature, such that the magnitude of the order parameter has taken a constant value, leaving the phases as the dynamical variables. We assume that the field configuration evolves in the \( \sigma-\pi_3 \) plane, same as in the above scenario. In this case the explicit symmetry breaking term responsible for the pion mass affects the dynamics. The relevant equation is the sine-Gordon equation given by

\[
\partial_\mu \partial^\mu \theta(x,t) + a \sin \theta(x,t) = 0,
\]

(5.8)
Figure 5.2: Proper time evolution of $\sigma$, and $\pi_3$ field following a Lorentz boost invariant initial condition at $\tau_0 = 1 fm/c$, under SG equation.

where $a = \frac{f}{f_{\pi}}$. The field variables are $\Phi = (a, \pi_3, 0, 0) = (\cos 0(z, t), \sin \theta(x, t), 0, 0)$. We have performed a numerical simulation with the boost invariant evolution of the initial field configuration as given in Ref. [20]. It was found, as indicated in Fig. (5.2), that the $\sigma$ field grows from zero and takes a longer propertime as compared to the previous case to reach the true vacuum expectation value $<\sigma> \sim f_\pi$. While the $\phi$ field oscillates around zero rather slowly and eventually tends to zero when the proper time gets large.

Below we describe a soliton picture of DCC [22]. In this case, the solution does not preserve boost invariance. It is well-known that the $(1+1)$ sine-Gordon equation, possesses stable solitary wave solutions [23]. For example, the well-known kink and anti-kink solutions are: $\theta(\xi) = 4 \arctan[\exp \pm \gamma \xi]$. These domains can be potential candidates for DCC. It is interesting to point out that domains of Bose-Eintein condensates have been successfully modelled as the solitons and soliton trains of the relevant order parameter equation in $(1+1)$ dimensions. As long as the
expansion is one dimensional the solitons are completely stable. Eventually, when the expansion becomes three dimensional, they decay into low-energy coherent pions, since no conservation law prevents it and these solutions have higher energy, as compared to the homogeneous vacuum solution. In the accompanying figure (Fig. 5.4), we depict the results of the classical evolution of a coherent structure in (2 + 1)-dimensions. It clearly shows that, after sufficient time, the coherent structure breaks up into plane waves, representing pions. Below we give the numerical algorithm to solve the SG equation in (2+1)-dimension, by the symmetry it can be extended to the full space.

To solve the sine-Gordon equation in (2 + 1)-dimensions, we use the centered finite difference method to discretize the space and time variables. For this, we take \( x = iAx; \ y = jAy; \ t = kAt \) and the solution as

\[
 u^k_{i,j} = u(i\Delta x, j\Delta y, k\Delta t).
\]

The adjoining figure corroborates our insights. Then the finite difference
Models for disoriented \textit{chiral} condensates

Figure 5.4: Classical evolution of a domain. The four frames correspond to times 100, 500, 800, and 1200 in appropriate units.
SGE takes the form

\[
\begin{align*}
    u^{k+1}_{i,j} &= -u^{-1}_{i,j} + 2[1 - 2(\frac{\Delta t}{\Delta x})^2]u^k_{i,j} + (\frac{\Delta t}{\Delta x})^2[u^k_{i+1,j} + u^k_{i-1,j} + u^k_{i,j+1} + u^k_{i,j-1}] \\
    u^k_{i-1,j} + u^k_{i,j+1} + u^k_{i,j-1}] - (\Delta t)^2 \sin[\frac{1}{4}(u^k_i + 1, j + u^k_{i-1,j} + u^k_{i,j+1} + u^k_{i,j-1})]
\end{align*}
\] (5.9)

5.4 Traveling wave solutions of 0(4) sigma model

It is natural to ask about the possibility of solitary wave solutions in the O(4)-sigma model itself. These can be localized solitons, or periodic ones. It has been shown that [22], the σ-model Lagrangian, in the presence of an additional isospin violating terms quadratic in the field variables, e.g., \( \mu \pi_3^2 \) (without the explicit symmetry breaking term) leads to stationary solutions of the type

\[
\begin{align*}
    \pi_1 &= \text{Lsech}\sqrt{2\mu} \frac{\xi}{f_\pi}, \\
    \pi_2 &= \text{msech}\sqrt{2\mu} \frac{\xi}{f_\pi}, \\
    \pi_3 &= \text{ntanh}\sqrt{2\mu} \frac{\xi}{f_\pi}, \\
    \text{and} \quad \sigma &= \text{Lsech}\sqrt{2\mu} \frac{\xi}{f_\pi},
\end{align*}
\] (5.10)

where \( \alpha^2 = 1 + \frac{\mu}{2\lambda} \), and \( l^2 + m^2 + n^2 = 1 - \frac{\mu}{2\lambda} \). These type of inhomogeneous solutions can model DCC and the corresponding neutral pion fraction will differ significantly from the expected 1/\( \sqrt{7} \) distribution.

We now show that, in the absence of the additional quadratic term taken above, we can still find exact propagating wave solutions of the equations of motion for the O(4) Lagrangian in (1 + 1) dimensions. Like in the above case, we assume that the fields are decoupled from heat bath and pointing along \( \sigma \), and \( \pi_3 \) directions. Then we find that the traveling wave solutions are given in terms of the cnoidal functions.
The equations of motion for the $\sigma$, and the $\pi_3$ fields in (1 + 1)-dimensions can be written as

$$\frac{d^2 \sigma}{d\xi^2} + a\sigma^3 + a\sigma\pi_3^2 - a\sigma = 0,$$

$$\frac{d^2 \pi_3}{d\xi^2} + a\pi_3^3 + a\sigma^2\pi_3 - a\pi_3 = 0.$$

where, $\xi = x - vt$, and $a = -\frac{4\lambda}{f_{s2}(1-v^2)}$. Then we find that

$$\sigma = \left[1 + \frac{\alpha^2(1 - 2m)}{a}\right]^{1/2}\text{sn}(\alpha\xi, m),$$

and

$$\pi_3 = \left[1 + \frac{\alpha^2}{a}\right]^{1/2}\text{cn}(\alpha\xi, m),$$

where, $m$ is the modulus parameter. These are periodic excitations of the order parameter. We hope that these type of excitations will be observed experimentally. It is interesting to note that in a single component BEC, these type of soliton trains have been seen experimentally.
References


References


