CHAPTER 3

STEADY STATE STABILITY ANALYSIS

3.1 INTRODUCTION

The steady state stability analysis using state space technique of the LCL, LCC and LCL-T Resonant Converters are present in this chapter. From this analysis it is concluded that LCL Resonant converter has better stability margin compared to LCC and LCL-T converters.

3.1.1 Steady State Stability Analysis Using State – Space Analysis

The following assumptions are made while doing the state space analysis of the LCL, LCC and LCL-T Resonant Converters.

- The switches, diodes, inductors, and capacitors used are ideal.
- The effects of snubber capacitors are neglected.
- Losses in the tank circuit are neglected.
- DC supply used is assumed to be smooth.
- Only fundamental components of the waveforms are used in the analysis.
- Ideal high frequency transformer with turns ratio n =1.
- The state equation describes the state during the period $t_{p-1} < t < t_p$
Where $t_{p-1}$ is the starting instant of continuous conduction mode and $t_p$ is the instant when the continuous conduction mode ends.

![Figure 3.1 Equivalent circuit model of LCL resonant converter](image)

The equivalent circuit shown in Figure 3.1 is used for the analysis. The state space equation for the converter is

$$\dot{x} = Ax + Bu$$

(3.1)

The state space equation for LCL converter is obtained from Figure 3.1 and given as

$$\frac{di_{i1}}{dt} = \frac{m}{L_1}V - \frac{n}{L_2}V_o - \frac{1}{L_1}V_C$$

(3.2)

$$\frac{dV_C}{dt} = \frac{1}{C}i_{i1}$$

(3.3)

$$\frac{di_{i2}}{dt} = \frac{n}{L_2}V_o$$

(3.4)

From the above equations, we get
The sum of the zero input response and the zero state response for LCL RC is given by

\[
X(t) = [\Phi(t)X(0)] + L^{-1}[(\Phi(s)B[U(s)])]
\]  

(3.6)

Solving equation (3.6), the current and voltage components can be related

\[
\begin{bmatrix}
i(t) \\
V(t) \\
i_2(t)
\end{bmatrix} = \begin{bmatrix}
\cos(\omega(t-t_{m})) & \frac{\sin(\omega(t-t_{m}))}{L} & 0 \\
-\frac{\sin(\omega(t-t_{m}))}{C} & \cos(\omega(t-t_{m})) & V(t_{m,1}) \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
i(t_{m,1}) \\
v(t_{m,1}) \\
i_2(t_{m,1})
\end{bmatrix} + \begin{bmatrix}
mV - nV_s \\
mV - nV_s[1 - \cos(\omega(t-t_{m,1}))] \\
nV_s
\end{bmatrix}
\]

(3.7)

From the above equation, it can be concluded that the output voltage is not dependent on the load resistance and converter gain follows a sine function. In order to maintain the output voltage constant at desired value against the variations in the load resistance and supply voltage, the pulse width has to be changed in a closed-loop manner. However, the required change in pulse width is very small.
3.2 DESIGN OF THE LCL RESONANT CONVERTER

In order to pursue the analysis, a converter with following specification is designed.

Power output = 133W
Minimum input voltage = 125V
Minimum output voltage = 100V
Maximum load current = 1.33A
Maximum overload current = 4A
Inductance ratio (K_L) = 1

The high frequency transformer turns ratio is assumed to be unity.

The input RMS voltage to the diode bridge

\[ (D1-D4) (V_{L2}) = \frac{2\sqrt{2}}{\pi} V_o \] \hspace{1cm} (3.8)

The input RMS current at the input of the diode bridge

\[ I_d = \frac{\pi I_o}{2\sqrt{2}} \] \hspace{1cm} (3.9)

The load impedance \[ Z_L = \frac{100}{1.33} = 75.18 \Omega \]

The reflected ac resistance on the input side of the diode bridge is

\[ R_{ac} = \frac{V_{L2}}{I_d} \] \hspace{1cm} (3.10)
\[ \frac{V_{L_2}}{I_d} = \frac{2V_o \sqrt{2}}{\pi d_0 / 2\sqrt{2}} = \frac{8V_o}{\pi^2} \quad (3.11) \]

\[ \frac{\sqrt{L_1}}{\sqrt{C}} = \frac{8Z_{L_1}}{\pi^2} = \frac{8 \times 75}{\pi^2} = 60.8 \Omega \quad (3.12) \]

Since the switching frequency is 50 KHz

\[ \frac{1}{2\pi \sqrt{L_1 C}} = 50 \times 10^3 \quad (3.13) \]

\[ \sqrt{L_1} = 60.8 \sqrt{C} \]

\[ \sqrt{C} = \frac{\sqrt{L_1}}{60.8} \]

\[ \frac{1}{2\pi \sqrt{L_1 \times L_1 / 60.8}} = 50 \times 10^3 \quad (3.14) \]

\[ \frac{60.8}{2\pi L_1} = 50 \times 10^3 \]

\[ f_o = \frac{1}{2\pi \sqrt{(L_1 + L_2)C}} \quad (3.15) \]

From the above equation, the values of \( L_1, L_2 \) and \( C \) are obtained as,

\[ L_1 = L_2 = 185 \mu H \]

\[ C = 0.052 \mu F \]
3.2.1 Transfer Function of LCL Resonant Converter

From Figure 3.2, applying Kirchhoff voltage law, the transfer function of the LCL Resonant converter is obtained as,

\[
\frac{nV_o(S)}{mV(S)} = \frac{Z_iCS}{S^2(L_1 + L_2)C + Z_iCS + 1} \tag{3.16}
\]

Apply the values \(L_1, L_2\) and \(C\) in equation (3.16)

\[
\frac{nV_o(S)}{mV(S)} = \frac{(3.9 \times 10^{-6})S}{S^2(9.62 \times 10^{-12}) + (3.9 \times 10^{-6})S + 1} \tag{3.17}
\]

3.2.2 Transfer Function of LCC Resonant Converter

The equivalent circuit of the LCC and LCL-T Resonant Converters are shown in Figures 3.2 and 3.3 respectively. The mathematical modeling using the state space technique can be obtained assuming all the components to be ideal. The transfer function of the LCC and LCL-T resonant converters are given below from Figures 3.4 and 3.5 applying Kirchhoff voltage law in Figure 3.2, the following equation is obtained.

![Figure 3.2 Equivalent circuit model of LCC resonant converter](image)

**Figure 3.2** Equivalent circuit model of LCC resonant converter
\[ \frac{nV_o(S)}{mV(S)} = \frac{C_1}{S^2 C_1 C_2 L + C_1 + C_2} \]  

(3.18)

Apply the values of \( C_1 = C_2 = 0.052 \times 10^{-6} \) and \( L = 185 \times 10^{-6} \) in equation (3.18), the following equation is obtained.

\[ \frac{nV_o(S)}{mV(S)} = \frac{0.052 \times 10^{-6}}{S^2 (0.5 \times 10^{-18}) + 0.104 \times 10^{-6}} \]  

(3.19)

### 3.2.3 Transfer Function of LCL-T Resonant Converter

Applying Kirchhoff Voltage Law in Figure 3.3.

![Figure 3.3 Equivalent circuit model of LCL-T resonant converter](image)

The transfer function is obtained as,

\[ \frac{nV_o(S)}{mV(S)} = \frac{Z_L}{S^3 CL_1 L_2 + S^2 CL_2 Z_L + S (L_1 + L_2) + Z_L} \]  

(3.20)

Apply the values \( L_1, L_2 \) and \( C \) in equation (3.20)

\[ \frac{nV_o(S)}{mV(S)} = \frac{75}{S^3 (0.178 \times 10^{-18}) + S^2 (7.22 \times 10^{-12}) + S (3.7 \times 10^{-6}) + 75} \]  

(3.21)
The transfer function of LCL and LCC resonant converters are of order two and that of LCL-T is of order three. From the principles, if order of the system increases, the system will go to unstable region.

3.2.4 Stability Analysis Using Root Locus

For the transfer function equations (3.16) to (3.21), the nyquist and root locus plots are prepared using MATLAB coding. Figure 3.4, the locus path is lying left half region of the S-plane. So the LCL Resonant converter is stable system. In Figure 3.5, the locus path is lying in the imaginary axis of the S-plane and hence the LCC Resonant converter is marginally stable system. In Figure 3.6, the locus paths of LCL-T are lying in the real axis of the S-plane and right half of the S-plane. Hence the LCL-T resonant converter could be considered as unstable system.

Figure 3.4 Root locus plot for LCL resonant converter
Figure 3.5 Root locus plot for LCC resonant converter

Figure 3.6 Root locus plot for LCL-T resonant converter
3.2.5 Stability Analysis Using Nyquist Plot

The stability of LCL resonant converter can be proved from nyquist plot in Figure 3.7 where the locus of roots does not encircle the -1+j0. But for nyquist plot for LCC shown in Figure 3.8 encircles the -1+j0 which is one of the criteria for unstable system. The nyquist plot of LCL-T converter in Figure 3.9 encircles the -1+j0 twice, hence the system is said to be unstable.

![Nyquist Plot for LCL Resonant Converter](image)

*Figure 3.7 Nyquist plot for LCL resonant converter*

![Nyquist Diagram for LCC RESONANT CONVERTER](image)

*Figure 3.8 Nyquist plot for LCC resonant converter*
Figure 3.9. Nyquist plot for LCL-T resonant converter
3.3 SUMMARY

In this chapter, a detailed analysis using state-space technique was done on a 133W, 50 KHz, LCL, LCC and LCL-T resonant converters for optimal parameter selection.

For the transfer function equations, the nyquist and root locus plots were prepared using MATLAB coding. Results obtained from the plots, the steady state stability criteria with LCL rather than LCC or LCL-T resonant converters were briefed to project the superiority of the LCL over LCC and LCL-T resonant converters for their better stability margin.