The output voltage is given by,

\[ V_o = \frac{V_i}{1-d} \]  

(3.1)

The inductor and capacitor values of the Boost converter are derived by having the same assumption as that of the Buck converter. Now the critical value of the inductor \( L_C \) which decides the condition for the continuous current mode of the operation is given by,

\[ L_C = \frac{d(1-d)R}{2f_s} \]  

(3.2)

where \( f_s \) is the switching frequency. The inductor value is determined by using the following equation,

\[ \Delta I_L = \frac{V_{sd}d}{f_sL} \]  

(3.3)

Similarly the capacitor value can be determined by assuming appropriate ripple voltage and by substituting the necessary values in the following equation,

\[ \Delta V = \frac{I_{sd}d}{f_sC} \]  

(3.4)

In this type of converter, a very high peak current flows through the switch. It is very difficult to attain the stability of this converter due to high sensitivity of the output voltage to the duty cycle variations. When compared to Buck converter the inductor and capacitor sizes are larger since high RMS current would flow through the filter capacitor.

Based on the above discussion the parameters designed for Boost Converter is shown in Table 3.1.
Table 3.1 Design values of Boost converter

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Parameters</th>
<th>Design Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input Voltage</td>
<td>24 V</td>
</tr>
<tr>
<td>2</td>
<td>Output Voltage</td>
<td>50 V</td>
</tr>
<tr>
<td>3</td>
<td>Inductance, L</td>
<td>72 µH</td>
</tr>
<tr>
<td>4</td>
<td>Capacitance, C</td>
<td>216.9X10^{-6} F</td>
</tr>
<tr>
<td>5</td>
<td>Load Resistance, R</td>
<td>23 Ω</td>
</tr>
<tr>
<td>6</td>
<td>Switching frequency, ( f_s )</td>
<td>20 kHz</td>
</tr>
</tbody>
</table>

The design details of Boost converter are perceived above and using the designed values the open loop response of the Boost converter is obtained and shown in the Figure 3.2, where the peak overshoot and steady error are found to be maximum. The voltage ripples are also observed which requires the design of closed loop control.

![Figure 3.2 Open loop response of Boost converter](image)

Figure 3.2 Open loop response of Boost converter
The differential equations describing the Boost converter can be explained in the following section by assuming two modes of operation. The inductor current, $i_L$ and the capacitor voltage, $V_o$ are the state variables. The semiconductor Switch is in on condition for the time interval, $0 \leq t \leq T_{on}$ and hence the inductor $L$ gets connected to the supply and stores the energy. Since the diode is in off condition, the output stage gets isolated from the supply. Here the inductor current flows through the inductor and completes its path through the source. The equivalent circuit for this mode is shown in the Figure 3.3.

![Equivalent circuit of Boost converter for mode 1](image)

**Figure 3.3 Equivalent circuit of Boost converter for mode 1**

Applying Kirchoff’s laws, the following equations describing mode 1 are obtained as,

$$\begin{align*}
\frac{dV_o}{dt} &= -\frac{V_o}{RC} \\
\frac{di_L}{dt} &= \frac{V_s}{L}
\end{align*}$$

(3.5)

Now the coefficient matrices for this mode are obtained as,

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}$$

(3.6)

$$B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

(3.7)
During the time interval $T_{on} \leq t \leq T$, the diode is in on state and the switch is in off state and hence the energy from the source as well as the energy stored in the inductor is fed to the load. The inductor current flows through the inductor $L$, the capacitor $C$, the diode and the load. The equivalent circuit for this mode is shown in Figure 3.4.

![Figure 3.4 Equivalent circuit of Boost converter for mode 2](image)

Applying Kirchoff’s laws, the following equations describing mode 2 are obtained,

$$\begin{align*}
\frac{d}{dt}i_L &= \frac{V_s}{L} - \frac{V_o}{L} \\
\frac{d}{dt}V_o &= \frac{i_L}{C} - \frac{V_o}{RC}
\end{align*}$$

(3.8)

The coefficient matrices for this mode is defined as follows,

$$A_2 = \begin{bmatrix} 0 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$$

(3.9)

$$B_2 = \begin{bmatrix} 1 \\ \frac{1}{L} \end{bmatrix}$$

(3.10)

The output voltage $V_o(t)$ across the load is expressed as,

$$V_o(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

(3.11)
Thus the design and modelling of the Boost converter has been done which further leads to the design of Observer controller. In the next section, the derivation of the state feedback matrix for the Boost converter is carried out which is the first step for the design of Observer controller transfer function. By substituting the values of $L$ and $C$ thus designed, the state coefficient matrices for the Boost converter is obtained as follows:

$$A = \begin{bmatrix} 0 & -6944.33 \\ 6944.33 & -200 \end{bmatrix}$$  \hspace{1cm} (3.12)$$

$$B = \begin{bmatrix} 13.8887 \times 10^3 \\ 0 \end{bmatrix}$$  \hspace{1cm} (3.13)$$

$$C = [0 \hspace{0.5cm} 1]$$  \hspace{1cm} (3.14)$$

$$D = [0]$$  \hspace{1cm} (3.15)$$

### 3.3 DERIVATION OF STATE FEEDBACK MATRIX FOR THE BOOST CONVERTER

The general derivation of the state feedback matrix for the DC-DC converter has already been discussed in detail in chapter 2. Now in this section, the state feedback gain matrix for the Boost converter is explained as follows. The root locus of the Boost converter under continuous time is drawn as shown in the Figure 3.5. The open loop poles of the Boost converter is shown by cross in the figure. The desired poles are arbitrarily placed in order to obtain the state feedback matrix.
Figure 3.5 Root locus of Boost converter in s-domain

The derivation of the state feedback matrix for the Boost converter is carried out in the same manner as that for the Buck converter using pole placement technique which has already been explained in the second chapter. Now, the state feedback matrix for the Boost converter is derived as follows:

**Step 1:** The characteristic polynomial to find the unknown values of \([k_1 \quad k_2]\) is formed as follows:

\[
|sI - (A - Bk)| = s^2 + (200 + 13.8887 \times 10^3 k_1)s + 2.7777 \times 10^6 k_1 + 96.4470 \times 10^6 k_2 + 48.2237 \times 10^6 = 0 \quad (3.16)
\]

**Step 2:** The desired characteristic equation is formed by arbitrarily placing the poles as follows:

\[
s^2 + 14.2042 \times 10^3 s + 57.97459 \times 10^6 = 0 \quad (3.17)
\]

By equating the like powers of \(s\) in the Equations (3.16) and (3.17), the state feedback matrices are obtained as \(k_1 = 1.008\) and \(k_2 = 0.07206\).
In order to check the robustness of the control law, the step input is used and the output response has been illustrated in the Figure 3.6. From the Figure 3.6, it is very well understood that the system settles down faster and the state feedback matrix is capable enough to realize the stability of the Boost converter.

\[ G = \begin{bmatrix} 0.9405 & -0.3386 \\ 0.3386 & 0.9308 \end{bmatrix} \]  \hspace{1cm} (3.18)

\[ H = \begin{bmatrix} 0.6806 \\ 0.1190 \end{bmatrix} \]  \hspace{1cm} (3.19)

\[ C_d = [0 \hspace{0.5cm} 1] \]  \hspace{1cm} (3.20)

\[ D_d = [0] \]  \hspace{1cm} (3.21)
As per the Nyquist criterion which states that sampling frequency of the analog signal should be at least twice the maximum signal frequency, the sampling frequency is assumed as 1 MHz. The digital state feedback matrix can be derived in the same method as that for the continuous domain excepting the domain considered here is $z$. The root locus of Boost converter in $z$ domain is drawn as shown in the Figure 3.7. The desired poles are chosen arbitrarily in order to find the digital state feedback matrix. The open loop poles are shown by cross in the figure. The stability of the converter is obtained in such a way by moving the poles towards the left half of the $z$-plane. More the poles are moved towards the left hand side, more stability of the converter is obtained.

**Figure 3.7** Root locus of the Boost converter in $z$-domain

The digital state feedback matrix can be obtained by substitution method and is explained as follows:

**Step 1:** The characteristic polynomial to find the unknown values of $[k_{d_1} \ k_{d_2}]$ is formed as follows:
\[ |zI - (G - Hk)| = z^2 + (0.6806k_{d1} - 1.8713)z - 0.6335k_{d1} + 0.04029k_{d2} + 0.99007 = 0 \] (3.22)

**Step 2:** The desired characteristic equation is formed by arbitrarily placing the poles as follows:

\[ z^2 - 0.6094z - 0.1442 = 0 \] (3.23)

By equating the like powers of \( s \) in the Equations (3.22) and (3.23), the state feedback matrices are obtained as \( k_{d1} = 1.854 \) and \( k_{d2} = 1 \).

In order to check the robustness of the control law, the step input is used and the output response has been demonstrated in the Figure 3.8. It is clear that the system settles down faster and the digital state feedback matrix is efficient enough to obtain the stability of the Boost converter.

![Step Response](image)

**Figure 3.8** Step response of the Boost converter in discrete time domain

Thus the state feedback matrix for the Boost converter under both continuous time and discrete time domain are derived and also the suitability of these matrices to obtain the stability of the Boost converter is illustrated
clearly in this section. Now the derivation of the observer gain matrix for the
Boost converter under both continuous and discrete time domain has been
dealt in the following section.

3.4 DERIVATION OF OBSERVER GAIN MATRIX FOR BOOST
CONVERTER

The general derivation of full order state observer gain matrix for
DC-DC converter has already been explained in the second chapter. Now, for
the Boost converter this matrix can be derived by the substitution method by
assuming appropriate natural frequency of oscillation and damping ratio as
per the thumb rule. By assuming the damping ratio, \( \zeta = 0.6 \) and the natural
frequency of oscillation, \( \omega_n = 12.02788 \times 10^3 \) rad/sec, the desired
characteristic equation can be obtained as follows,

\[
\lambda^2 + 173.2015 \times 10^3 \lambda + 2.08324 \times 10^{10} = 0 \quad (3.24)
\]

The polynomial equation with unknown values of observer poles is
given by,

\[
\lambda^2 + (200 + g_2)\lambda + (48.2237 \times 10^6 + 6944.33 g_1) = 0 \quad (3.25)
\]

Comparing the Equations (3.24) and (3.25), the observer gain matrix
is obtained. The values are \( g_1 = 2992.9 \times 10^3 \) and \( g_2 = 173.0015 \times 10^3 \).

Since the observer poles are mainly designed to check the
robustness of the control law, it is essential that the estimated state variables
and error variables of the Boost converter should converge at zero from any
non-zero initial value. This ensures the asymptotic stability of the converter
under consideration with the desired pole locations. It is demonstrated in the
Figures 3.9 to 3.12 respectively. All the variables such as \( x_1, x_2, e_1 \) and \( e_2 \)
under consideration attain zero value from any non zero value thereby
ensuring that the observer poles are dynamic and the state feedback control is very strong enough to achieve the stability of the Boost converter.

Figure 3.9 Estimation of state variable 1

Figure 3.10 Estimation of state variable 2
Thus the observer gain matrix for the Boost converter is designed and it also has been checked for its dynamic constancy. Now, by using the Separation principle which has already been explained in chapter 2, the transfer function of the observer controller for the Boost converter under
continuous time domain is obtained by the substitution of the state feedback gain matrix, $k$ and observer gain matrix, $g$ in the appropriate observer controller transfer function given by Equation (2.76) as follows,

$$T(s) = \frac{2.09371 \times 10^6 s + 1.449 \times 10^9}{s^2 + 15453.15s + 14494 \times 10^{10}}$$ \hspace{1cm} (3.26)$$

Similar case can be derived for the discrete time system and it is discussed now. By assuming the damping ratio, $\zeta = 0.5$, sampling time $T_s = 1 \mu s$ and the natural frequency of oscillation, $\omega_n = 192.4461 \times 10^3$ rad/sec, the desired characteristic equation can be obtained as follows,

$$z = e^{-\zeta \omega_n T_s} e^{\pm j \omega_n T \sqrt{1-\zeta^2}} = e^{-(0.5 \times 1924461 \times 10^3 \times 1 \times 10^{-6})}$$

$$\times e^{(\pm j1924461 \times 10^3 \times 1 \times 10^{-6} \times \sqrt{1-0.5^2})}$$ \hspace{1cm} (3.27)$$

The above equation can be simplified as,

$$z^2 - 7.50449 \times 10^{-3} z + 6.6235 \times 10^{-5} = 0$$ \hspace{1cm} (3.28)$$

The polynomial equation with unknown values of observer poles is given by

$$z^2 + (g_{d2} - 1.8713)z + (0.3386 g_{d1} - 0.9405 g_{d2} + 0.990067) = 0$$ \hspace{1cm} (3.29)$$

Comparing the Equations (3.28) and (3.29), the observer gain matrix is obtained. The values are $g_{d1} = 2.2948$ and $g_{d2} = 1.8788$.

By using the Separation principle which has already been explained in chapter 2, the transfer function of the prediction observer controller for the Boost converter under discrete time domain is obtained by substituting the values of digital state feedback control matrix and observer gain matrix in the Equation (2.84) as follows,
\[ T(z) = \frac{6133z-61295}{z^2+1388z+6738} \] (3.30)

3.5 RESULTS AND DISCUSSION

3.5.1 Boost Converter (Continuous Time Domain)

The previous sections clearly explain the derivation of the Observer controller for the Boost converter under both continuous and discrete time domain. Now the simulation results are exemplified and are discussed in detail. The design and the performance of Boost converter is accomplished in continuous conduction mode and simulated using MATLAB/ Simulink. The ultimate aim is to achieve a robust controller for the Boost converter inspite of uncertainty and large load disturbances. The performance parameters of the Boost converter under consideration are rise time, settling time, maximum peak overshoot and steady state error, which are shown in the Table 3.2. It is evident that the converter settles down at 0.015 s and the rise time of the converter is 0.01s. No overshoots or undershoots are evident and no steady state error is observed. The simulation of the Boost converter is also carried out by varying the load, not limiting it to \( R \) load and it is illustrated in the Table 3.3.

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Settling Time (s)</td>
<td>0.015</td>
</tr>
<tr>
<td>2</td>
<td>Peak Overshoot (%)</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Steady State Error (V)</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Rise Time (s)</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>Output Ripple Voltage</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3.3 Output response of Boost converter for load variations (Continuous time domain)

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Load</th>
<th>Reference Voltage (V)</th>
<th>Output Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R(Ω)</td>
<td>L(H)</td>
<td>E(V)</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>1 x 10⁻⁶</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>100 x 10⁻⁶</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>1 x 10⁻⁶</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>50 x 10⁻⁶</td>
<td>5</td>
</tr>
</tbody>
</table>

It is obviously understood that the Boost converter with observer controller is efficient enough to track the output voltage irrespective of the load variations. When the load resistance is varied as 25 Ω and 30 Ω, the converter is able to track the output voltages as 50 V and 50.01 V respectively for the reference voltage of about 50 V. Again when the inductance of 1µH and 100 µH are added to the resistance of 25 Ω, the output thus obtained is of the order of 50 V and 50.05 V respectively. The steady state error observed is of very minimum of about 0.05 V. The simulation is also carried out again using RLE load with a resistance of 20 Ω, two different inductances of 1 µH and 50 µH and an ideal voltage source of about 5 V. The response of the converter is such that the controller is capable to work under all the load transients thereby tracking the voltage as 50.04 V and 50.20 V respectively.

The simulation is also carried out by varying the input voltage and load resistance and the corresponding, input voltage, load resistance, output voltage, inductor current and load current are shown in the Figure 3.13. The input voltage is first set as 24 V until 0.4 s and again varied from 24 V to 22 V upto 0.6 s. Again at 0.6 s it is varied to 24 V and at 0.8 s it has been varied to 26 V respectively.
Simultaneously the load resistance is also varied from 20 Ω to 19 Ω and again to 18.5 Ω respectively at 0.3 s and 0.6 s. The corresponding output response of the Boost converter shows fixed output voltage regulation. Undershoots and Overshoots are not observed and the steady state error is also not apparent. The inductor current and load current are also shown in the Figure 3.13, which shows minimum amount of current ripples. In order to check the dynamic performance of the system, the $L$ and $C$ parameters of the Boost converter are varied and the output response of the system is shown in the Table 3.4.
Table 3.4 Output response of Boost converter with variable converter parameters (Continuous time domain)

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Inductance, L</th>
<th>Capacitance, C</th>
<th>Reference Voltage (V)</th>
<th>Output Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100 µH</td>
<td>220 µF</td>
<td>50</td>
<td>49.85</td>
</tr>
<tr>
<td>2</td>
<td>150 µH</td>
<td>200 µF</td>
<td>50</td>
<td>50.34</td>
</tr>
<tr>
<td>3</td>
<td>200 µH</td>
<td>150 µF</td>
<td>50</td>
<td>49.96</td>
</tr>
<tr>
<td>4</td>
<td>250 µH</td>
<td>100 µF</td>
<td>50</td>
<td>50.10</td>
</tr>
<tr>
<td>5</td>
<td>250 µH</td>
<td>250 µF</td>
<td>50</td>
<td>50.24</td>
</tr>
</tbody>
</table>

The inductance of the Boost converter is varied as, 100 µH, 150 µH, 200 µH and 250 µH and the corresponding capacitor values are set as 220 µF, 200 µF, 150 µF, 100 µF and 250 µF respectively. It is understood from the Table 3.4 that the system is very much dynamic in tracking the reference voltages inspite of the variations in the inductance and capacitance values. The system does not show any overshoots or undershoots and it settles down fast with a settling time of about 0.015 s for all the values. The steady state error thus noticeable ranges between 0.3% and 0.6% which is considered as within the tolerable limits. The load current, output power, losses and efficiency of the Boost converter with Observer controller is determined and is illustrated in Table 3.5.

Table 3.5 Efficiency of the Boost converter with analog observer controller

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>(I_O) (A)</th>
<th>(P_O) (W)</th>
<th>Losses (W)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5455</td>
<td>77.3368</td>
<td>1.9098</td>
<td>97.590</td>
</tr>
<tr>
<td>2</td>
<td>1.9723</td>
<td>98.7136</td>
<td>2.3662</td>
<td>97.659</td>
</tr>
<tr>
<td>3</td>
<td>2.1755</td>
<td>108.8185</td>
<td>3.5067</td>
<td>96.878</td>
</tr>
<tr>
<td>4</td>
<td>2.3750</td>
<td>118.6313</td>
<td>4.2135</td>
<td>96.570</td>
</tr>
<tr>
<td>5</td>
<td>2.5170</td>
<td>126.1017</td>
<td>5.6936</td>
<td>95.680</td>
</tr>
<tr>
<td>6</td>
<td>4.4590</td>
<td>222.8608</td>
<td>12.7787</td>
<td>94.577</td>
</tr>
</tbody>
</table>
The efficiency of the Boost converter remains more or less same with the increase in the load current. It is very well understood that the Boost converter with Observer controller is highly efficient and the highest efficiency is obtained as 97.659% at a load current of about 1.9723 A and the corresponding output power is 98.7136 W. The simulation for Boost converter with prediction observer controller is carried out in the same manner as explained in the above section and is presented in the following section.

### 3.5.2 Boost Converter (Discrete Time Domain)

Simulation has been carried out for the Boost converter with prediction observer controller under discrete time domain and the performance parameters are tabulated in Table 3.6. The system settles down much faster at 0.0075 s and the rise time is only 0.004 s as shown in the Table 3.6. The overshoots and undershoots are not seen and there is no peak overshoot.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Settling Time (s)</td>
<td>0.0075</td>
</tr>
<tr>
<td>2</td>
<td>Peak Overshoot (%)</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Steady State Error (V)</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Rise Time (s)</td>
<td>0.004</td>
</tr>
<tr>
<td>5</td>
<td>Output Ripple Voltage</td>
<td>0</td>
</tr>
</tbody>
</table>

The output response with load variations is shown in the Table 3.7. When the load resistance is varied as 25 Ω and 30 Ω, the Boost converter with
discrete controller is capable to track the output voltages as 50.02 V and 49.95 V respectively for the reference voltage of about 50 V. Again when the inductance of 1 µH and 100 µH are added to the resistances of 25 Ω, the output thus obtained is of the order of 49.99 V and 49.79 V respectively. The steady state error observed is of very lesser of the order of about 0.01 V and 0.21 V respectively. The simulation is also carried out again using RLE load with a resistance of 20 Ω, two inductances of 1 µH and 50 µH and an ideal voltage source of about 5 V. The response of the converter is such that the controller is proficient to work under all the load transients thereby tracking the voltage as 50.03 V and 49.89 V respectively.

Table 3.7 Output response of Boost converter for load variations (Discrete time domain)

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Load</th>
<th>Reference Voltage(V)</th>
<th>Output Voltage (v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(Ω)</td>
<td>L(H)</td>
<td>E(V)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>1x10⁻⁶</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>100x 10⁻⁶</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>1x10⁻⁶</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>50x10⁻⁶</td>
<td>5</td>
</tr>
</tbody>
</table>

The simulation is also carried out by varying the input voltage and load resistance and the corresponding, input voltage, load resistance, output voltage, inductor current and load current are shown in the Figure 3.14. The input voltage is first set as 24 V until 0.01 s and again varied to 22 V up to 0.02 s. Again at 0.02 s it is varied to 24 V and at 0.03 s it has been varied to 26 V up to 0.04 s and again varied to 24 V respectively till 0.05 s. Simultaneously the load resistance is also varied as 22 Ω, 20 Ω and 10 Ω.
respectively and the corresponding output response of the Boost converter thus obtained shows tight output voltage regulation. Undershoots and Overshoots are not observed and the steady state error is also not evident. The inductor current and load current which are also shown, shows no evidence of current ripples. In order to check the dynamic performance of the system, the \( L \) and \( C \) parameters of the Boost converter are varied and the output response of the system is shown in the Table 3.8.

### Table 3.8 Output response of Boost converter with variable converter parameters (Discrete time domain)

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Inductance, ( L )</th>
<th>Capacitance, ( C )</th>
<th>Reference Voltage(V)</th>
<th>Output Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100 ( \mu )H</td>
<td>220 ( \mu )F</td>
<td>50</td>
<td>49.95</td>
</tr>
<tr>
<td>2</td>
<td>150 ( \mu )H</td>
<td>200 ( \mu )F</td>
<td>50</td>
<td>50.04</td>
</tr>
<tr>
<td>3</td>
<td>200 ( \mu )H</td>
<td>150 ( \mu )F</td>
<td>50</td>
<td>49.89</td>
</tr>
<tr>
<td>4</td>
<td>250 ( \mu )H</td>
<td>100 ( \mu )F</td>
<td>50</td>
<td>50.10</td>
</tr>
<tr>
<td>5</td>
<td>250 ( \mu )H</td>
<td>250 ( \mu )F</td>
<td>50</td>
<td>50.06</td>
</tr>
</tbody>
</table>

The inductance and the capacitance values are changed in the same manner as that for the analog controller as shown in the Table 3.8. It is obvious from the Table 3.8 that the system is very much dynamic in tracking the reference voltages inspite of the variations in the inductance and capacitance values. The system does not show any overshoots or undershoots and it settles down fast with a settling time of about 0.01 s for all the values. The steady state error thus noticeable is within the tolerable limits ranging from 0.2% to 0.6%.
The load current, output power, losses and efficiency of the Boost converter is determined and is illustrated in Table 3.9. The efficiency of the Boost converter with Prediction Observer controller remains more or less same with the increase in the load current. It is very well understood that the system is highly efficient and the highest efficiency is obtained as 98.659% at a load current of about 1.342 A and the corresponding output power is 98.674 W.
Table 3.9 Efficiency of the Boost converter with prediction observer controller

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>$I_o$(A)</th>
<th>$P_o$(W)</th>
<th>Losses(W)</th>
<th>Efficiency(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5455</td>
<td>77.04</td>
<td>1.1018</td>
<td>98.590</td>
</tr>
<tr>
<td>2</td>
<td>1.9723</td>
<td>98.674</td>
<td>1.3412</td>
<td>98.659</td>
</tr>
<tr>
<td>3</td>
<td>2.1755</td>
<td>108.775</td>
<td>2.3582</td>
<td>97.878</td>
</tr>
<tr>
<td>4</td>
<td>2.3750</td>
<td>118.798</td>
<td>2.9587</td>
<td>97.570</td>
</tr>
<tr>
<td>5</td>
<td>2.5170</td>
<td>125.850</td>
<td>4.3217</td>
<td>96.680</td>
</tr>
<tr>
<td>6</td>
<td>4.4590</td>
<td>223.396</td>
<td>7.9179</td>
<td>96.577</td>
</tr>
</tbody>
</table>

The efficiency of the Boost converter under both continuous and discrete time domain drawn against the load current is shown compared in the Figure 3.15. The efficiency of the Boost converter with prediction observer controller shows improved results when compared with its analog counterpart.

![Figure 3.15](image)

Figure 3.15 Comparison of efficiencies of Boost converter

The Boost converter with prediction controller has been efficient enough in such a way that it is capable of tracking the reference voltages of 50 V and 60 V inspite of the input voltage variations. The input voltage is
varied as 24 V till 0.01 s and at 0.01 s it is varied as 22 V and again it is varied as 24 V and 26 V at 0.02 s and 0.03 s respectively. Finally it is varied as 24 V at 0.04 s. The reference values are varied as 50 V and 60 V and it is illustrated in the Figure 3.16. In this figure green line represents the set value and the blue line represents the actual output voltage.

![Figure 3.16 Output response of Boost converter for variation in the reference voltages](image)

3.6 CONCLUSION

A state feedback control approach has been designed for the Boost converter under both continuous time domain and discrete time domain using pole placement technique and separation principle. The full order state observer has been designed to ensure robust control for the converter. The separation principle allows designing a dynamic compensator which very much looks like a classical compensator since the design is carried out using simple root locus technique. The mathematical analysis and simulation study show that the Boost converter with both analog and digital controller thus
designed achieves tight output voltage regulation, good dynamic performances and higher efficiency.

   In the next chapter the Observer controller for interleaved Buck converter is designed.
CHAPTER 4

DESIGN, MODELLING AND IMPLEMENTATION
OF INTERLEAVED BUCK CONVERTER WITH
OBSERVER CONTROLLER

4.1 OVERVIEW

The main objective of this chapter is to explain the derivation of Observer controller for the Interleaved Buck converter. The derivation of state feedback gain matrix for the Interleaved Buck converter using pole placement method as well as linear quadratic optimal regulator method is explained as that of Buck converter but with a difference in the order of the system which will be explained in the following sections. The derivation of the full order state Observer is also presented. Finally by combining the state feedback matrix and full order state observer, the Observer controller has been designed using separation principle.

4.2 DESIGN AND MODELLING OF INTERLEAVED BUCK CONVERTER

In this section a detailed discussion is carried out regarding the design and state space modelling of the Interleaved Buck converter which is also a time invariant, non linear but a third order system. When single stage DC-DC converters are used, major limitations are imposed on the switching frequency and efficiency, i.e. the switching losses increase with the increased switching frequencies and therefore the efficiency is reduced. The interleaved
converters provide a feasible solution to this problem. Since in the case of interleaved converters the fundamental frequency is being multiplied by the number of phases \( m \), the transient response would be improved. The other advantages include reduction in input and output rms current of the capacitor, lesser requirement of EMI filters, allowing the integration of number of filter inductors into one magnetic device, improved thermal performance, increased reliability and more over redundancy in power stages.

In Interleaved power converters, high current is always delivered to the loads without going in for high power rating devices. The major constraint in the design of this converter is to control the sharing of current among the parallel connected converters. In order to obtain a stiff output voltage regulation, the converters must be perfectly identical to each other. The schematic diagram of the Interleaved Buck converter is shown in Figure 4.1. Here only two phases are considered, i.e. two single stage Buck converters are connected in parallel to form an Interleaved Buck converter.

![Figure 4.1 Schematic diagram of interleaved Buck converter](image)

Here \( V_s \) is the input voltage, \( L_1 \) and \( L_2 \) are the two identical inductors, \( S_1 \) and \( S_2 \) are the semiconductor switches, \( D_1 \) and \( D_2 \) are the diodes, \( C \) is the output capacitor and \( R \) is the load resistance. The design of inductance and capacitance values are explained in detail as follows.
4.2.1 Inductor Selection

Inductor selection in the case of interleaved converters is vital in deciding the efficiency of the converter modules and transient response of the system. The inductor is mainly selected depending on the value of the allowable inductor ripple. The load current amplitude and switching frequency also play a major role in the inductor selection. Hence the inductor values are chosen in such a way that the inductor ripple current must be assumed a higher value as 40% of the maximum current flowing through one particular channel. Since the two converter modules differ in phase by 180°, the inductor current ripple in the two converter modules cancel each other thereby allowing a small current ripple to flow through the output capacitor. The output current ripple frequency is doubled and hence resulting in the requirement of smaller output capacitor for the same ripple voltage requirement. The inductor value can be obtained from the following expression,

\[ \Delta I_o = \frac{2V_o(1-d)T}{L} \frac{|1-2d|}{|1-2d|+1} \]  

(4.1)

where \( \Delta I_o \) is the inductor ripple current, \( d \) is the duty cycle, \( T \) is the time period and \( L \) is the inductor. Here it is assumed as \( L_1=L_2 \). Substitution of the necessary values in the above equation gives the required value of the inductor.
4.2.2 Capacitor Selection

The exact capacitance value, the Equivalent Series Resistance (ESR) and Equivalent Series Inductance (ESL) are the key factors that should be taken into account while designing the capacitor value. The major issues in considering the above are mainly due to the fact that these parameters affect the transient response, output ripple voltage and overall stability of the system. The capacitor value can be determined by the estimation of output ripple voltage given by,

$$\Delta V_o = \frac{\Delta I_o T}{8mC} + \Delta I_o \times ESR$$

where $m$ is number of phase, $C$ is the capacitor and $\Delta V_o$ is peak-peak ripple voltage on the capacitive components of $C$ and the second term $\Delta I_o \times ESR$ represents the ripple voltage developed across the ESR of the output capacitor.

The output ripple current and voltage approach zero, when the duty cycle ratio is equal to the critical value given by,

$$d_{critical} = \frac{i}{m}, i = 1, 2, \ldots \ldots m - 1$$

Thus by assuming appropriate values for the peak to peak ripple voltage the capacitor value can be determined by substituting in the Equation (4.2).

Based on the above discussion the parameters designed for Interleaved Buck Converter is shown in Table 4.1.
Table 4.1. Design Values of Interleaved Buck Converter

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Parameters</th>
<th>Design Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input Voltage</td>
<td>48 V</td>
</tr>
<tr>
<td>2</td>
<td>Output Voltage</td>
<td>12 V</td>
</tr>
<tr>
<td>3</td>
<td>Inductances, L1=L2</td>
<td>720 µH</td>
</tr>
<tr>
<td>4</td>
<td>Capacitance, C</td>
<td>8.667X10^{-7} F</td>
</tr>
<tr>
<td>5</td>
<td>Load Resistance, R</td>
<td>14.4 Ω</td>
</tr>
<tr>
<td>6</td>
<td>Switching frequency, f_s</td>
<td>100 kHz</td>
</tr>
<tr>
<td>7</td>
<td>Sampling time</td>
<td>1 MHz</td>
</tr>
</tbody>
</table>

The design details of Interleaved Buck converter are explained above and using the designed values the open loop response of the Interleaved Buck converter is obtained and shown in the Figure 4.2, where the peak overshoot and steady error are found to be maximum. The voltage ripples are also observed which requires the design of closed loop control.

Figure 4.2  Open loop response of interleaved Buck converter
The state space modelling of the Interleaved Buck converter is explained as follows. During continuous conduction mode of operation, diodes $D_1$ and $D_2$ are always in complementary switching states with the switches $S_1$ and $S_2$ respectively, that is when $S_1$ is on, $D_1$ is off and vice versa. Similarly when $S_2$ is on, $D_2$ is off and vice versa. Accordingly four modes of switching states are possible and the corresponding state equations are explained as follows. The inductor currents $i_{L1}$ and $i_{L2}$ and the capacitor voltage $V_o$ are considered as the state variables. The four modes of operation is discussed now.

During mode 1 both the switches $S_1$ and $S_2$ are on, where as the diodes $D_1$ and $D_2$ are in the off condition. The corresponding equivalent circuit for this mode is shown in Figure 4.3.

![Figure 4.3 Equivalent circuit of interleaved Buck converter for mode 1](image)

Applying Kirchoff’s laws to the above circuit, the equations describing this converter for mode 1 can be obtained as follows,

\[
\frac{d i_{L1}}{dt} = \frac{V_S - V_o}{L_1} \quad (4.4)
\]

\[
\frac{d i_{L2}}{dt} = \frac{V_S - V_o}{L_2} \quad (4.5)
\]
where, \( i_L = i_{L1} + i_{L2} \) is the sum of the current flowing through the inductances \( L_1 \) and \( L_2 \) respectively.

The coefficient matrices for this mode is defined as follows,

\[
A_1 = \begin{bmatrix}
0 & 0 & \frac{-1}{L_1} \\
0 & 0 & \frac{-1}{L_2} \\
\frac{1}{C} & \frac{1}{C} & \frac{-1}{RC}
\end{bmatrix}
\]

(4.7)

and

\[
B_1 = \begin{bmatrix}
\frac{1}{L_1} \\
0 \\
0
\end{bmatrix}
\]

(4.8)

During mode 2, the switch \( S_1 \) is on and \( S_2 \) is off and the respective diodes \( D_1 \) is off and \( D_2 \) is on and the equivalent circuit for this mode is shown in Figure 4.4.

![Equivalent circuit of interleaved Buck converter for mode 2](image)

Figure 4.4  Equivalent circuit of interleaved Buck converter for mode 2

Applying Kirchoff’s laws to the above circuit, the equations describing this converter for mode 2 can be obtained as follows,
The coefficient matrices for this mode is defined as follows,

\[
\frac{dI_{L1}}{dt} = \frac{V_s - V_o}{L_1} \quad (4.9)
\]

\[
\frac{dI_{L2}}{dt} = -\frac{V_o}{L_2} \quad (4.10)
\]

\[
\frac{dV_o}{dt} = \frac{i_{L2}}{C} - \frac{V_o}{RC} \quad (4.11)
\]

In mode 3, switch \( S_1 \) is off and \( S_2 \) is on and the corresponding diodes \( D_1 \) is on and \( D_2 \) is in off condition. The corresponding equivalent circuit of the Interleaved Buck converter for this mode of operation is shown in the Figure 4.5.

![Figure 4.5 Equivalent circuit of interleaved Buck converter for mode 3](image)
Applying Kirchoff’s laws to the above circuit, the equations describing this converter for mode 3 can be obtained as follows,

\[
\frac{dL_1}{dt} = -\frac{V_o}{L_1} \tag{4.14}
\]

\[
\frac{dL_2}{dt} = \frac{V_s - V_o}{L_2} \tag{4.15}
\]

\[
\frac{dV_o}{dt} = \frac{i_s}{C} - \frac{V_o}{RC} \tag{4.16}
\]

The coefficient matrices for this mode is defined as follows,

\[
A_3 = \begin{bmatrix}
0 & 0 & -\frac{1}{L_1} \\
0 & 0 & -\frac{1}{L_2} \\
\frac{1}{C} & \frac{1}{C} & -\frac{1}{RC}
\end{bmatrix} \tag{4.17}
\]

and

\[
B_3 = \begin{bmatrix}
0 \\
\frac{1}{L_2} \\
0
\end{bmatrix} \tag{4.18}
\]

During mode 4 both the switches \(S_1\) and \(S_2\) are in the off condition and the both the diodes \(D_1\) and \(D_2\) are in the on condition. The equivalent circuit of the Interleaved Buck converter for this mode of operation is shown in the Figure 4.6.
Applying Kirchoff’s laws to the above circuit, the equations describing this converter for mode 4 can be obtained as follows,

\[
\frac{d \mathbf{i}_1}{dt} = \frac{-V_O}{L_1}
\]

(4.19)

\[
\frac{d \mathbf{i}_{L2}}{dt} = \frac{-V_O}{L_2}
\]

(4.20)

\[
\frac{d V_O}{dt} = \frac{1}{C} \left( \frac{\mathbf{i}_{L1}}{R} - \frac{V_O}{RC} \right)
\]

(4.21)

The coefficient matrices for this mode is defined as follows,

\[
A_4 = \begin{bmatrix}
0 & 0 & -1 \\
0 & 0 & \frac{-1}{L_2} \\
\frac{1}{L_1} & \frac{1}{C} & \frac{-1}{RC}
\end{bmatrix}
\]

(4.22)

and

\[
B_4 = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

(4.23)
The coefficient matrix for the interleaved converter is defined as,

\[
[A] = A_1d_1 + A_2d_2 + A_3d_3 + A_4d_4 \quad \text{and} \quad [B] = B_1d_1 + B_2d_2 + B_3d_3 + B_4d_4 , \quad [U] = V_S \quad \text{and the duty cycle ratio is given by} \quad d_1 + d_2 + d_3 + d_4 = 1. \quad \text{The output equation is defined as follows,}
\]

\[
y(t) = [0 \quad 0 \quad 1] \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ V_O \end{bmatrix}
\]

(4.24)

By substituting the values of \( L \) and \( C \) thus designed in the state equations derived in the previous section for this converter, the state coefficient matrices are obtained as follows,

\[
A = \begin{bmatrix} 0 & 0 & -2.7767 \times 10^3 \\ 0 & 0 & -2.7767 \times 10^3 \\ 2.3049 \times 10^6 & 2.3049 \times 10^6 & -320.128 \times 10^3 \end{bmatrix}
\]

(4.25)

\[
B = \begin{bmatrix} 694.1668 \\ 694.1668 \\ 0 \end{bmatrix}
\]

(4.26)

\[
C = [0 \quad 0 \quad 1]
\]

(4.27)

\[
D = [0]
\]

(4.28)

Thus the state model for the Interleaved Buck converter has been derived and in the following sections the derivation of the Observer controller for the Interleaved Buck converter is discussed. The derivation of Observer controller for this converter also involves two steps. In the first step state feedback matrix is derived using pole placement technique and in the second step the full order observer gain matrix is derived. In this type of converter, a special case has been considered in which the state feedback gain matrix for this converter has also been derived using Linear quadratic optimal regulator.
method to obtain an optimal solution. And finally the state feedback matrices obtained by using the above said methods are individually combined with the observer gain matrix and two different Observer transfer functions are derived using the same separation principle which has already been explained in the second chapter.

4.3 DERIVATION OF STATE FEEDBACK MATRIX FOR INTERLEAVED BUCK CONVERTER

4.3.1 Pole Placement Method

This section clearly explains the derivation of the state feedback matrix for the Interleaved Buck converter using pole placement method. The procedure for this third order system is same as that explained for the second order systems which were considered and elucidated in the previous chapters. The root locus of the Interleaved Buck converter is drawn as shown in the Figure 4.7. The desired poles are arbitrarily placed in order to obtain the state feedback matrix.

Figure 4.7 Root locus of interleaved Buck converter
The state feedback matrix can be obtained by substitution method and is explained as follows,

**Step 1:** The characteristic polynomial to find the unknown values of state feedback matrix, \([k_1 \quad k_2 \quad k_3]\) are formed as follows,

\[
|sI - (A - Bk)| = s^3 +
\]

\[
(222.22 \times 10^6k_1 + 222.22 \times 10^6k_2 + 400.76)s^2 +
\]

\[
(11.1319 \times 10^6k_1 + 11.1319 \times 10^6k_2 + 255.992 \times 10^6k_3 + 127.9918 \times 10^6)s + 3.09211 \times 10^{11}k_1k_2 = 0 \quad (4.29)
\]

**Step 2:** The desired characteristic equation is formed by arbitrarily placing the poles as follows,

\[
s^3 + 28.17 \times 10^3s^2 + 121.9723 \times 10^6s + 7.7303 \times 10^{10} = 0 \quad (4.30)
\]

By equating the like powers of \(s\) in the Equations (4.29) and (4.30), the state feedback matrix is obtained as \(k_1 = 0.98985\), \(k_2 = 29.845\) and \(k_3 = 0.885\).

In order to check the robustness of the control law, the step input is used and the output response has been illustrated in the Figure 4.8. It is very well understood that the system settles down faster and the state feedback matrix is capable enough to realize the stability of the Interleaved Buck converter.
Figure 4.8 Step response of the interleaved Buck converter

Again the state feedback matrix is derived using Linear quadratic optimal regulator method and is explained in the following section.

4.3.2 Linear Quadratic Optimal Regulator Method

In this section the importance and necessity of the Linear quadratic optimal regulator method has been explained. Also the application of this method to the Interleaved converters have been detailed. In order to make the system insensitive to parameter variations and disturbances and to achieve tight output voltage regulation, it is desirable to determine the state feedback gain matrix optimally. The linear quadratic regulation method is emphatic, in such a way that the dominant closed loop poles are assigned close to the desired locations and the remaining poles are non dominant. This method assures insensitivity to plant parameter variations by choosing appropriate performance index. This method is very useful to predetermine the control configuration in such a way that an optimum solution can be obtained. Especially in the case of a linear time invariant closed loop system, optimal solution can be achieved by state feedback control. Such problems are defined
as the Linear Quadratic Optimal control problems which are most widely used in industries.

Now the time invariant systems such as Interleaved Buck and Interleaved Boost converters are considered and the optimal control law is derived as discussed as follows:

The ultimate objective in deriving the optimal solution is to minimize the performance index defined as,

$$ f = \int_{0}^{\infty} (x^T Q x + u^T R u) \, dt $$  \hspace{1cm} (4.31)

With the initial condition being, $x(0) \triangleq x^0$. Here $Q$ is $n \times n$, real, positive definite matrix and $R$ is $p \times p$, real, positive definite, symmetric, constant matrix. The right side of the equation 4.31, contributes to the spending of energy of the control signals. The relative importance of the error and the usage of the energy of the control signals are determined by the matrices $Q$ and $R$. It is assumed that the control law $u(t) = -k x(t)$ is unconstrained. Now with the optimal control law the system equation can be defined as,

$$ \dot{x} = (A - Bk)x $$  \hspace{1cm} (4.32)

The solution can be derived as follows,

It is assumed that the matrix $(A-Bk)$ is stable. Substitution of the control law in the Equation (4.31) gives,

$$ f = \int_{0}^{\infty} (x^T Q x + x^T k^T R k x) $$  \hspace{1cm} (4.33)
The above equation can be rewritten as follows,

\[
J = \int_0^\infty x^T (Q + k^T R k) x \, dt \tag{4.34}
\]

Let,

\[
x^T (Q + k^T R k) x = \frac{d}{dx} (x^T P x) \tag{4.35}
\]

On differentiation we get,

\[
x^T (Q + k^T R k) x = -\dot{x}^T P x - x^T \dot{P} \dot{x}
\]

\[
= -x^T [(A - B k)^T P + P (A - B k)] x \tag{4.36}
\]

On investigation of the left hand side and right hand side of the above equation, it is understood that this equation holds good for any value of \(x\). Hence the following equation is required and it is defined as,

\[
(A - B k)^T P + P (A - B k) = -(Q + k^T R k) \tag{4.37}
\]

In order that \((A-Bk)\) should be a stable matrix, the positive definite \(P\) matrix must exist which satisfies the Equation (4.31). Hence it is required to determine the elements of \(P\) matrix. The performance index \(J\) can be analyzed as follows:

\[
J = \int_0^\infty x^T (Q + k^T R k) x \, dt = -[x^T P x]_0^\infty \tag{4.38}
\]

On simplification it can be written as,

\[
J = -x^T (\infty) P x (\infty) + x^T (0) + P x (0) \tag{4.39}
\]
Since \((A-Bk)\) is assumed stable or have negative real parts, \(x(t) \rightarrow 0\). Therefore it can be written as,

\[ J = x^T(0)Px(0) \quad (4.40) \]

Hence the value of \(J\) is acquired in terms of the initial condition \(x(0)\) and \(P\). The solution to the quadratic optimal control problem is obtained as follows,

Now \(R\) matrix is written as,

\[ R = T^TP \quad (4.41) \]

where \(T\) is a non singular matrix. Here \(P\) is assumed as a positive definite matrix and it is the necessary condition for the existence of the optimal solution. Then the Equation (4.37) can be written as,

\[ (A^T - k^TB^T)P + P(A - Bk) + Q + k^T T^TPk = 0 \quad (4.42) \]

The above equation can be rewritten as,

\[ A^TP + PA + [Tk - (T^T)^{-1}B^TP] * [Tk - (T^T)^{-1}B^TP] - PB^{-1}B^TP + Q = 0 \quad (4.43) \]

The minimization of \(J\) with respect to \(k\) requires the minimization of the equation,

\[ x^T[Tk - (T^T)^{-1}B^TP] * [Tk - (T^T)^{-1}B^TP]x \quad \text{with respect to } k. \]

This equation is non negative and the minimum occurs when it is zero or when,

\[ Tk = (T^T)^{-1}B^TP \quad (4.44) \]
Hence,

\[ k = T^{-1}(T^T)^{-1}B^TP = R^{-1}B^TP \]  \hspace{1cm} (4.45)

This Equation (4.45) yields the optimal matrix \( k \). Thus the performance index given by the Equation (4.31) is linear and the optimal control law is given by,

\[ u(t) = -kx(t) = -R^{-1}B^TPx(t) \]  \hspace{1cm} (4.46)

The above equation is defined as the reduced matrix Riccati equation. The design of the regulator is carried out in two steps, defined as follows,

a) The positive definite Riccati matrix, \( P \) is determined which should satisfy the following reduced Riccati matrix equation given by,

\[ A^T \ast P + PA - PB R^{-1}B^T \ast P + Q = 0 \]  \hspace{1cm} (4.47)

For the appropriate \( P \) value \((A-Bk)\) should be asymptotically stable.

b) Substitution of Riccati matrix in the equation described below results in the optimal \( k \) value.

\[ k = -R^{-1}B^T \ast P \]  \hspace{1cm} (4.48)

The control law thus designed using Linear Quadratic optimal regulator method for the Interleaved Buck converter is discussed now.
The positive definite matrices $Q$ and $R$ for this converter are assumed as,

$$Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad (4.49)$$

$$R = [2] \quad (4.50)$$

and

$$P = \begin{bmatrix} 6.9162 \times 10^4 & -6.9162 \times 10^4 & 0 \\ -6.9162 \times 10^4 & 6.9162 \times 10^4 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.51)$$

The $k$ values can be obtained for this converter by substituting the above matrices in Equation (4.48). The values designed for this converter are, $k = [0.8941 \ 0.8941 \ 0.0001]$.

### 4.4 DERIVATION OF OBSERVER GAIN MATRIX FOR INTERLEAVED BUCK CONVERTER

The above section discusses the derivation of state feedback matrix for the Interleaved Buck converter derived using linear quadratic optimal regulator method. Now, for the Interleaved Buck converter observer gain matrix can be derived by the substitution method which was already dealt in the previous chapters by assuming appropriate natural frequency of oscillation and damping ratio as per the thumb rule. By assuming the damping ratio, $\zeta = 0.5$ and the natural frequency of oscillation, $\omega_n = 1.8974 \times 10^6$ rad/sec, the desired characteristic equation can be obtained as follows,

$$\lambda^3 + 2.8461 \times 10^6 \lambda^2 + 5.4 \times 10^{12} \lambda + 3.4153 \times 10^{18} = 0 \quad (4.52)$$
The polynomial equation with unknown values of observer poles is given by,

\[ \lambda^3 + (320.128 \times 10^3 + g_3) \lambda^2 + (2.3049 \times 10^6 g_1 + 2.3049 \times 10^6 g_2 + 1.27999 \times 10^{10}) \lambda = 0 \]  
(4.53)

Comparing the Equations (4.52) and (4.53), the observer gain matrix is obtained. The values are \( g_1 = g_2 = 1.68641 \times 10^6 \) and \( g_3 = 2.5249 \times 10^6 \). Finally by combining the observer gain matrix and the state feedback matrix which has already been derived in the above sections using both the pole placement and Linear quadratic optimal regulator methods, the transfer function for the Observer controller can be determined by substituting the values of \( k \) matrix and \( g \) matrix in the Equation (2.76).

The transfer function obtained using the pole placement method is as follows,

\[ T(s) = \frac{-3.827 \times 10^7 s^2 - 1.699 \times 10^{13} s - 6219}{s^3 + 2.824 \times 10^6 s^2 + 5.336 \times 10^{12} s + 254.2} \]  
(4.54)

Similarly, the transfer function obtained using the linear quadratic optimal regulator method is as follows,

\[ T(s) = \frac{2.09 \times 10^6 s^2 + 657 \times 10^{11} s - 3086}{s^3 + 2.846 \times 10^6 s^2 + 5404 \times 10^{12} s + 3086} \]  
(4.55)

4.5 RESULTS AND DISCUSSION

In this section the simulation results for the Interleaved Buck converter with Observer controller using pole placement method and Linear Quadratic optimal regulator method are carried out using MATLAB/Simulink. Extensive simulation has been done and the results thus obtained
are shown compared with both the above discussed methods. The Interleaved Buck converter specifications under consideration are rise time, settling time, maximum peak overshoot and steady state error which are shown in Table 4.2. It is obvious that the Interleaved Buck converter with Observer controller designed using pole placement technique settles down at 0.015 s with a rise time of 0.01 s. There are no indications of peak overshoot and output voltage ripples. Similarly Interleaved Buck converter with Linear Quadratic optimal Regulator settles at 0.012 s little bit earlier than the previous case with the rise time of about 0.008 s. In this case also there are no indications of peak overshoot and output voltage ripples. In both the cases the steady state error of about 0.02 V is observed which is of very negligible order. The results thus obtained are in concurrence with the mathematical calculations. The simulation is also carried out by varying the load not limiting to $R$ load and it is illustrated in Table 4.3.

### Table 4.2 Comparison of the performance parameters of interleaved Buck converter

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Controller</th>
<th>Settling Time (s)</th>
<th>Peak Overshoot (%)</th>
<th>Steady State Error (V)</th>
<th>Rise Time (s)</th>
<th>Output Ripple Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Observer Controller (Pole Placement Method)</td>
<td>0.015</td>
<td>0</td>
<td>0.02</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Linear Quadratic optimal regulator</td>
<td>0.012</td>
<td>0</td>
<td>0.02</td>
<td>0.008</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 4.3 Output response of interleaved Buck converter for load variations

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>R (Ω)</th>
<th>L (mH)</th>
<th>E (V)</th>
<th>Reference Voltage (V)</th>
<th>Output Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>12</td>
<td>12.02</td>
</tr>
<tr>
<td>2</td>
<td>14.4</td>
<td>-</td>
<td>-</td>
<td>12</td>
<td>12.02</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>-</td>
<td>-</td>
<td>12</td>
<td>12.04</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>100</td>
<td>-</td>
<td>12</td>
<td>12.02</td>
</tr>
<tr>
<td>5</td>
<td>14.4</td>
<td>50</td>
<td>-</td>
<td>12</td>
<td>12.07</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
<td>100</td>
<td>-</td>
<td>12</td>
<td>12.09</td>
</tr>
<tr>
<td>7</td>
<td>14.4</td>
<td>50</td>
<td>5</td>
<td>12</td>
<td>12.10</td>
</tr>
<tr>
<td>8</td>
<td>22</td>
<td>100</td>
<td>10</td>
<td>12</td>
<td>12.05</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>100</td>
<td>8</td>
<td>12</td>
<td>12.10</td>
</tr>
</tbody>
</table>

Here the load resistance is varied as 10 Ω, 14.4 Ω and 22 Ω for which the output is tracked as 12.02 V, 12.02 V and 12.04 V respectively for the reference of 12 V. Again when the inductances of 100 mH and 50 mH are added to these resistors, still the converter is capable of tracking the voltages with very minimal value of steady state error. The steady state error thus observed ranges from 0.2% to 0.6% which very well lies inside the allowable range. In order to ensure the dynamic performance the load is varied as RLE, thereby connecting ideal voltage sources of 5 V, 10 V and 8 V respectively with the above mentioned RL series load values. The response thus observed demonstrates that the controller is robust. In order to ensure the dynamic performance, the output voltage is shown compared for both the methods in Figure 4.9. Both the methods show tight output regulation with much lesser settling time, no steady state error without any undershoots or overshoots and here green coloured line shows the output voltage for Observer controller by
pole placement method and blue coloured line represents the output voltage for Linear quadratic optimal regulator method. It is evident that the optimal solution for control law thus obtained shows slighter improvement in the performance parameters when compared with pole placement method as listed in Table 4.2.

![Comparison of output voltage (Observer controller Vs LQR)](image)

**Figure 4.9 Comparison of output voltage (Observer controller Vs LQR)**

The simulation is carried out by varying the input voltage and load resistance simultaneously and the corresponding output voltage, inductor currents and load currents are observed and is shown in the Figure 4.10. The input voltage is varied as 44 V from 0 s to 0.014 s and at 0.014 s, it is varied as 46 V till 0.028 s. At 0.028 s, 46 V is again varied as 48 V and 50 V until 0.038 s and 0.05 s respectively. Simultaneously the load resistance is also varied as 14 Ω, 10 Ω and 5 Ω respectively at 0 s, 0.014 s and 0.028 s. Inspite of these input as well as load transients the interleaved Buck converter with observer controller is capable of tracking the output voltage for the reference
of about 12 V. It is also obvious that the parallel connected converters have good current sharing among them and the load current shows lesser ripple. This ensures the robustness of the controller. The same analysis has been carried out for the Interleaved Buck Converter with Linear quadratic optimal regulator also and the output response is illustrated in the Figure 4.11. It is evident that despite input and load variations the converter along with the optimal regulator is very efficient enough to obtain tight output voltage regulation.

![Simulation results of interleaved Buck converter with observer controller using pole placement method](image)

**Figure 4.10** Simulation results of interleaved Buck converter with observer controller using pole placement method

(V_s – input voltage, V_o-Output Voltage, I_o-Load Current, IL1, IL2 – currents through Inductors L1 and L2)
Simulation has also been carried out in two modes. In mode 1 the inductances are chosen as $L_1 = L_2$ and in mode 2 inductances are chosen as $L_1 = 2L_2$. The efficiency of this converter has been calculated for the load variations and is tabulated in the Table 4.4. The added advantage is that the efficiency is higher even with high input to output ratios. From Table 4.4 it is very well understood that the control scheme offers an influential control and good current sharing among the converters. Figure 4.12 shows the efficiency as a function of output load current and it is seen that the state feedback control method achieves higher efficiency for a wide range of load variations and the maximum efficiency achieved is 96.6% at a load current of 0.8 A.

![Figure 4.11 Simulation results of interleaved Buck converter with Linear Quadratic Optimal Regulator](image)

*Figure 4.11 Simulation results of interleaved Buck converter with Linear Quadratic Optimal Regulator*  
($V_s$ – input voltage, $V_o$-Output Voltage, $I_o$-Load Current, $I_{L1}$, $I_{L2}$ – currents through Inductors L1 and L2)
Table 4.4 Performance calculations for the interleaved Buck converter

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Mode</th>
<th>Vref (V)</th>
<th>Vout (V)</th>
<th>I_o (A)</th>
<th>I_in (A)</th>
<th>Vs (V)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I</td>
<td>12</td>
<td>12.02</td>
<td>0.8432</td>
<td>0.55</td>
<td>48</td>
<td>96.6258</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td></td>
<td></td>
<td>0.8471</td>
<td>0.53</td>
<td>48</td>
<td>96.6257</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>16</td>
<td>15.99</td>
<td>1.1084</td>
<td>0.6825</td>
<td>48</td>
<td>96.6257</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td></td>
<td></td>
<td>1.1123</td>
<td>0.6650</td>
<td>48</td>
<td>96.6256</td>
</tr>
<tr>
<td>3</td>
<td>I</td>
<td>20</td>
<td>19.88</td>
<td>1.3741</td>
<td>0.8117</td>
<td>48</td>
<td>96.6241</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td></td>
<td></td>
<td>1.3788</td>
<td>0.7944</td>
<td>48</td>
<td>96.6239</td>
</tr>
<tr>
<td>4</td>
<td>I</td>
<td>24</td>
<td>24</td>
<td>1.6413</td>
<td>0.9375</td>
<td>48</td>
<td>96.6234</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td></td>
<td></td>
<td>1.6466</td>
<td>0.9203</td>
<td>48</td>
<td>96.6235</td>
</tr>
<tr>
<td>5</td>
<td>I</td>
<td>28</td>
<td>28.01</td>
<td>1.92</td>
<td>1.0625</td>
<td>48</td>
<td>96.6226</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td></td>
<td></td>
<td>1.9149</td>
<td>1.05</td>
<td>48</td>
<td>96.6235</td>
</tr>
</tbody>
</table>

The inductor currents and corresponding duty cycle ratios are shown in the Figure 4.13 and Figure 4.14 for mode 1 and mode 2 respectively.
It is evident from the current waveforms that the controller provides an effective current sharing among the converter modules irrespective of the values of the inductances.

![Figure 4.13 Inductor current and duty ratio for mode 1](image1)

![Figure 4.14 Inductor current and duty ratio for mode 2](image2)

The Interleaved Buck converter with observer controller has been efficient enough in such a way that it is capable of tracking the reference
voltages of 10 V and 12 V inspite of the input voltage variations. The input voltage is varied as 44 V, 46 V and 48 V till 0.157 s, 0.35 s and at 0.5 s respectively. The reference values are set as 10 V and 12 V and the converter very well tracks those values which are illustrated in the Figure 4.15.

![Figure 4.15](image)

**Figure 4.15** Output response of interleaved Buck converter for variation in the reference voltages

Thus the extensive simulation of the Interleaved Buck converter has been carried out and the results are discussed in this section and the conclusion of this chapter is presented in the next section.

4.6 CONCLUSION

A state feedback control approach has been designed for the Interleaved Buck converter in continuous time domain using pole placement technique. The load estimator has been designed by deriving full order state observer to guarantee robust and finest control for the converter. The Separation Principle allows designing a dynamic compensator which very
much looks like a classical compensator since the design is carried out using simple root locus technique. The mathematical analysis and the simulation study shows that the controller thus designed achieves good current sharing among the converters, tight output voltage regulation and good dynamic performances and higher efficiency. Further the state feedback control is optimized using Linear quadratic optimal regulator method. The results thus obtained are compared against each other and it is seen that the Linear quadratic optimal regulator method confirms a better performance in terms of performance specifications.

In the next chapter the design of Observer controller for the Interleaved Boost converter is discussed in detail.