CHAPTER 2

DESIGN, MODELLING AND IMPLEMENTATION OF
BUCK CONVERTER WITH OBSERVER CONTROLLER

2.1 OVERVIEW

The objective of this chapter is to present the procedures for the
design and implementation of compensation for a Buck converter which is a
single input single output time invariant system. The purpose of the
compensation is to modify the dynamic characteristics of the converter in
order to satisfy the performance specifications of the Buck converter. The
performance specifications of the converter are maximum peak overshoot,
settling time and steady state requirements and should be stated precisely so
that the optimal control of the converter can be obtained. The performance
specifications are achieved by using Observer controller which was designed,
modelled and implemented by using pole placement method which will be
described in detail in the following sections. The first section of this chapter is
about the design and modelling of the Buck converter.

2.2 DESIGN AND MODELLING OF BUCK CONVERTER

The Buck converters convert unregulated DC supply into a
regulated DC voltage and the output voltage is lesser than the input voltage.
The circuit diagram of Buck converter is shown in Figure 2.1. The Buck
converter comprises an inductor \( L \), a capacitor \( C \), a semiconductor switch and
a diode. \( V_i \) denotes the input voltage and \( R \) denotes the load resistance. The
coil non linearities and the noise which are caused mainly due to the oscillations of stray inductors and parasitic capacitors at each switching instants are neglected and the switch is assumed as an ideal one. The Buck converter design details are discussed as follows, where the parameter values are determined.

\[ V_o = dV_s \]  \hspace{1cm} (2.1)

where \( d = \frac{T_{on}}{T} \) is the duty cycle ratio, \( T_{on} \) is the on time of the semiconductor switch and \( T \) is the switching period. To ensure the continuous current mode of conduction the selected value of inductance should be greater than the critical value of the inductance \( L_c \), which acts as a boundary condition for continuous and discontinuous current mode of operation. The critical value of inductance is given by,

\[ L_c = (1 - d) \frac{R}{2f_s} \]  \hspace{1cm} (2.2)

where \( f_s \) is the switching frequency.
The inductor value must be chosen by considering the fact that the magnitude of the ripple current in the output capacitor as well as the load current is determined by the appropriate inductor value. Hence normally a ripple current of 10% to 20% of the average output current is assumed for the design to achieve good performance of the converter. The value of inductor is determined by,

$$\Delta I_L = \frac{V_i T (1-d)}{L}$$  \hspace{1cm} (2.3)

where $T$ is the time period and $\Delta I_L$ is the peak-peak ripple current of the output capacitor.

The capacitor value is determined by assuming the output voltage ripple as 1% to 2% of the output voltage. The capacitor value is determined by,

$$\Delta V = \frac{\Delta I_L}{8 f_s C}$$  \hspace{1cm} (2.4)

where $\Delta V$ is the peak-peak output voltage ripple.

Based on the above discussion the parameters designed for Buck Converter is shown in Table 2.1.

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Parameters</th>
<th>Design Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input Voltage</td>
<td>48 V</td>
</tr>
<tr>
<td>2</td>
<td>Output Voltage</td>
<td>12 V</td>
</tr>
<tr>
<td>3</td>
<td>Inductance, L</td>
<td>720 $\mu$H</td>
</tr>
<tr>
<td>4</td>
<td>Capacitance, C</td>
<td>8.667X10^{-7} F</td>
</tr>
<tr>
<td>5</td>
<td>Load Resistance, R</td>
<td>14.4 $\Omega$</td>
</tr>
<tr>
<td>6</td>
<td>Switching frequency, $f_s$</td>
<td>100 kHz</td>
</tr>
</tbody>
</table>
The design details of Buck converter are witnessed above and using the designed values the open loop response of the Buck converter is obtained and shown in the Figure 2.2, where the steady state error is found to be maximum as well as the voltage ripples are noticed which necessitates the design of closed loop control.

![Figure 2.2  Open loop response of Buck converter](image-url)

The converter is modelled using state space averaging method. The very cause for designing a controller for the DC-DC converters is to drive the semiconductor switch with a duty cycle so that the DC component of the output voltage is equal to the desired reference value. The regulation should be maintained as constant, despite variations in the load or in the input voltage. The major constraint in the design of a controller is mainly imposed on the duty cycle which is bounded between zero and one. The duty cycle problem can be solved by modelling the DC-DC converters using state space averaging technique. By using the above technique, the converter can be described by a single equation approximately over a number of switching cycles. State space averaging method is highly significant for this kind of
converters since PWM converters are special type of non linear systems which is switched between two or more non linear circuits depending upon the duty ratios. Further, control signals include not only the independent voltages and currents but also the duty ratios. The unique feature of this method is that the design can be carried out for a class of inputs such as impulse, step or sinusoidal function in which the initial conditions are also incorporated. This technique is expedient to use but it presents a low frequency estimate of the accurate dynamics where the discontinuous results initiated by switching is disregarded. The averaged model makes the simulation and control design much faster.

The derivation of the state equations for the DC-DC converters under the continuous time domain is explained as follows. The semiconductor switch is turned on and off by a sequence of pulses with a constant switching frequency, \( f_s \). The inductance currents and capacitance voltages are state variables. For the given duty cycle \( d(k) \) for the \( k^{th} \) period, the systems are illustrated by the following set of state space equations in continuous time domain.

\[
\begin{align*}
\dot{x}(t) &= A_1x(t) + B_1V_S(t), s = 1 \\
\dot{x}(t) &= A_2x(t) + B_2V_S(t), s = 0
\end{align*}
\] (2.5)

where \( s = 1 \) represents the condition at which the switch is conducting and \( s = 0 \) represents the off condition of the switch. Here \( x(t) \) is the state variable vector, \( A_1, B_1, A_2 \) and \( B_2 \) are the system matrices respectively. The significance of state space averaging technique lies in replacing the above two sets of state equations by a single equivalent set described as follows,

\[
\dot{x}(t) = Ax(t) + BV_S(t)
\] (2.6)
The $A$ and $B$ matrices are the weighted averages of actual matrices describing the switched system given by the following equations,

\[
\begin{align*}
A &= dA_1 + (1 - d)A_2 \\
B &= dB_1 + (1 - d)B_2
\end{align*}
\]  

(2.7)

The state equations in continuous time domain are converted into discrete time domain equations in order to design the digital controller. The discrete time system is considered same as that of the continuous system except that the system is sampled with a sampling time. The state space solution is transformed into a sampled system by using the relation $t = kT_s$, where $T_s$ is the sampling time and the corresponding equation is described as,

\[
\chi(kT_s) = e^{AT_s} \chi(0) + \int_{0}^{kT_s} e^{A(kT_s-\tau)} Bu(\tau) d\tau
\]  

(2.8)

where $\tau$ is a variable. With the analog coefficient matrices, discrete counter parts are obtained by using the following relationship,

\[
\begin{align*}
G &= e^{AT_s} \\
H &= \int_{\tau=0}^{T_s} e^{AT_s} d\tau B
\end{align*}
\]  

(2.9)

\[
\begin{align*}
C_d &= C \\
D_d &= D
\end{align*}
\]  

(2.10)

where $G$, $H$, $C_d$ and $D_d$ are the coefficient matrices for discrete systems.
The state equations of the Buck converter can be explained by assuming mode 1 and mode 2 and here the inductor current, \( i_L \) and the capacitor voltage, \( V_o \) are the state variables. During mode 1, semiconductor switch is turned on for a time interval of \( 0 \leq t \leq T_{on} \) and the diode is in the off state and the equivalent circuit of the Buck converter for this mode is shown in the Figure 2.3.

**Figure 2.3 Equivalent circuit of Buck converter for mode1**

The input current, \( i_L \) completes its path through the inductor \( L \), the capacitor \( C \) and the load. During this interval supply terminals get connected across the load and the energy gets stored in the inductor. Since the diode gets reverse biased, the input energy is being fed to the inductor and the load. Therefore by applying Kirchoff’s laws the dynamic equations governing the system during this interval are obtained as follows,

\[
\begin{align*}
\frac{di_L}{dt} &= \left( \frac{V_S - V_o}{L} \right) \\
\frac{dV_o}{dt} &= \frac{i_L}{C} - \frac{V_o}{RC}
\end{align*}
\]

(2.11)

Here \( i_L \) and \( V_o \) are the state variables \( x_1 \) and \( x_2 \) respectively and hence the coefficient matrices for mode 1 are defined as,
During mode 2, (i.e.) during the time interval, $T_{on} \leq t \leq T$, switch is in off state and Diode is in on state. The inductor current $i_L$ starts flowing through the inductor, capacitor and the load. The inductor and capacitor combination act as a low pass filter and hence the voltage fluctuations are very much diminished. It is desirable if the corner frequency of this filter is chosen to be much lesser than the switching frequency in order to dominate essentially the presence of switching frequency ripple in the output voltage. The equivalent circuit for this mode is shown in the Figure 2.4.

![Figure 2.4 Equivalent circuit of Buck converter for mode 2](image)

Applying Kirchoff’s laws to the above circuit, the following differential equations are obtained,

\[
\begin{align*}
\frac{di_L}{dt} &= -\frac{V_O}{L} \\
\frac{dv_C}{dt} &= \frac{i_L}{C} - \frac{v_C}{RC}
\end{align*}
\]

(2.14)
The coefficient matrices for mode 2 are defined as,

\[
A_2 = \begin{bmatrix}
0 & -1 \\
\frac{1}{C} & \frac{1}{RC}
\end{bmatrix}
\] (2.15)

\[
B_2 = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\] (2.16)

The output voltage \( V_o(t) \) across the load is expressed as,

\[
V_o(t) = [0 \ 1] \ x(t)
\] (2.17)

By substituting the values of \( L \) and \( C \) in the state equations, the state coefficient matrices for the Buck converter is obtained as follows,

\[
A = \begin{bmatrix}
0 & -1388.333 \\
115.246 \times 10^3 & -8.0032 \times 10^3
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
347.0833 \\
0
\end{bmatrix}
\]

\[
C = [0 \ 1]
\]

\[
D = [0]
\] (2.18)

The modelling of the converters which were discussed above leads to the designing of the controller. The designing of the controller involves three steps, initially the state feedback gain matrix is obtained using Pole Placement technique. Secondly the observer gain matrix is obtained and finally these two matrices are combined using Separation principle to obtain Observer controller. The design of state feedback matrix using Pole Placement technique is explained in the next section for the DC-DC converters.
2.3 ROBUST DESIGN OF STATE FEEDBACK MATRIX BY POLE PLACEMENT TECHNIQUE

The robust design comprises the derivation of state feedback gain matrix based on control law defined as $u = -kx(t)$ for the DC-DC converters. The constant reference input, $r(t)$ is taken as unit step input and the output variable is $y(t)$ which should track the reference input $r(t)$. The root loci of the DC-DC converter topologies are drawn from which the desired closed loop poles are chosen. The poles are arbitrarily placed in $s$ plane for continuous time system and in $z$ plane for discrete time system, so that the output variable $y$ tracks the reference value. The necessary condition for pole placement is that the system should be completely state controllable. The controllability of a control system ensures the subsistence of a complete solution of the system.

The systems described by the state equations are said to be state controllable at time, $t = t_0$, if it is possible to construct an unconstrained control signal that will transfer an initial state to any final state in a finite time interval $t_0 \leq t \leq t_f$. If every state is controllable, then the system is said to be completely state controllable. Without the loss of generality, it is assumed that the final state is the origin of the state space and that the initial time is zero or $t_0 = 0$. The solution of the state equation describing the system is given by,

$$x(t) = e^{At}x(0) + \int_{0}^{t} e^{A(t-\tau)}Bu(\tau)\,d\tau \quad (2.19)$$

By applying the definition of complete state controllability, it is defined as,
Let

$$
\beta_k = \int_0^{t_1} \alpha_k(\tau) u(\tau) d\tau
$$

(2.21)

Therefore Equation (2.20) becomes,

$$
\begin{bmatrix}
\beta_0 \\
\vdots \\
\beta_1 \\
\vdots \\
\beta_{n-1}
\end{bmatrix} = -\begin{bmatrix}
B & AB & \cdots & A^{n-1}B
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\vdots \\
\beta_1 \\
\vdots \\
\beta_{n-1}
\end{bmatrix}
$$

(2.22)

To satisfy the condition, the rank of the $n \times n$ matrix

$$
\begin{bmatrix}
B & AB & \cdots & A^{n-1}B
\end{bmatrix}
$$

should be $n$. This matrix is commonly called as controllability matrix. Now the state equations can be transformed into controllable canonical form. The transformation matrix is defined as,

$$
T = MW
$$

(2.23)

where $M$ is the controllability matrix given by,

$$
M = \begin{bmatrix}
B & AB & \cdots & \cdots & A^{n-1}B
\end{bmatrix}
$$

(2.24)

and

$$
x(0) = -\int_0^{t_1} e^{-A\tau}Bu(\tau)d\tau = -\sum_{k=0}^{n-1} A^kB \int_0^{t_1} \alpha_k(\tau) u(\tau) d\tau
$$

(2.20)
Now the state vector takes the form as,

\[ x = T\bar{x} \quad (2.26) \]

Since the system is completely state controllable then the inverse of matrix \( T \) exists and the system dynamic equation can be modified as follows,

\[ \dot{x} = T^{-1}A^T\bar{x} + T^{-1}Bu \quad (2.27) \]

For the DC-DC converters, if the Eigen values in general are assumed as \( \mu_1, \mu_2, \mu_3, \ldots \ldots, \mu_n \), then the desired characteristic equation for finding the state feedback matrix is derived now. For the continuous time systems, the desired characteristic equation is defined as,

\[ s^n + \alpha_1s^{n-1} + \ldots \ldots + \alpha_{n-1}s + \alpha_n = 0 \quad (2.28) \]

Let,

\[ kT = [\delta_n \delta_{n-1} \ldots \ldots \delta_1] \quad (2.29) \]

Thus the characteristic equation of the system is defined as follows,

\[ |sI - T^{-1}A^T + T^{-1}BkT| = 0 \quad (2.30) \]

The above equation takes the same form as that of the characteristic equation of the system, when \( u = -kx \) is used as a control signal. With further simplification in the controllable canonical form, the characteristic equation of the system attains the form as described as follows,
\[ s^n + (a_1 + \delta_1)s^{n-1} + \cdots + (a_{n-1} + \delta_{n-1})s + (a_n + \delta_n) = 0 \quad (2.31) \]

By equating the like powers of \( s \), the state feedback matrix is obtained which takes the form as follows,

\[ k = [\delta_n \quad \delta_{n-1} \quad \cdots \quad \cdots \quad \cdots \quad \delta_1]^{-1} \quad (2.32) \]

Thus all the Eigen values can be arbitrarily placed by choosing the matrix \( k \) using the above equation. The control scheme for the DC-DC converters is shown in the Figure 2.5.

**Figure 2.5 Control scheme for DC-DC converter**

Here \( N \) represents the scalar feed forward gain. Similar equations can be derived for the discrete time systems which are discussed now.

The state feedback control law can be derived for the discrete time system in the same manner as that of the continuous time system. The discrete time systems described in the Equations (2.9) and (2.10) with the control law take the form as follows,

\[ \dot{x}(k) = (G - Hk_d)x(k) \quad (2.33) \]

With the necessary and sufficient condition that the system should be completely state controllable, if all the Eigen values of \( (G-Hk_d) \) are placed
in the left half plane, the closed loop system thus considered is asymptotically stable. If the desired Eigen values in general are assumed as \( \mu_1, \mu_2, \mu_3, \ldots, \mu_n \) then the desired characteristic equation is defined as,

\[
z^n + \alpha_1 z^{n-1} + \cdots + \alpha_{n-1} z + \alpha_n = 0 \quad (2.34)
\]

The transformation matrix \( T_d \) is given by the following equation,

\[
T_d = M_d W_d \quad (2.35)
\]

where \( M_d \) is the controllability matrix given by,

\[
M_d = [H : GH : \cdots : G^{n-1}H] \quad (2.36)
\]

and \( W_d \) is same as that of \( W \) as described by the Equation (2.25).

The system dynamic equation can be modified as,

\[
\hat{x}(k+1) = T^{-1}G T \hat{x}(k) + T^{-1}H u(k) \quad (2.37)
\]

Assuming the following equation,

\[
k_d T = [\delta_n, \delta_{n-1}, \ldots, \delta_1] \quad (2.38)
\]

The characteristic equation is given by,

\[
|z I - T^{-1}G T + T^{-1}H k_d T| = 0 \quad (2.39)
\]

The Equation (2.39) takes the same form as that of the characteristic equation of the system, when \( u = -k_d x(k) \) is used as a control signal. Further simplification in the controllable canonical form, the characteristic equation of the system attains the form as follows,
\[ z^n + (a_1 + \delta_1)z^{n-1} + \cdots + (a_{n-1} + \delta_{n-1})z + (a_n + \delta_n) = 0 \quad (2.40) \]

By equating the like powers of \( z \), we can obtain the digital state feedback matrix. Thus all the Eigen values can be arbitrarily placed by choosing the matrix \( k_d \) using the above equation. The control scheme for the discrete time systems is same as that shown in Figure 2.5, excepting the analog state feedback matrix is replaced by the digital state feedback matrix.

2.4 DERIVATION OF STATE FEEDBACK MATRIX FOR BUCK CONVERTER

The designing of state feedback control law for Buck converter is discussed and the root locus approach is a graphical method which determines the locations of all closed loop poles from the location of the open loop poles and zeros by varying the gain. The root locus of the Buck converter under continuous time domain is drawn and the open loop poles are represented by cross in the Figure 2.6. In practice, merely by adjusting the gain, the desired performance cannot be achieved. It is necessary to reshape the root locus of the converter in order to obtain the performance specifications of the converters. The actual root locus of the DC-DC converter is reshaped in order to meet the desired characteristics. It is desirable to place the open loop poles on the left half of the \( s \) plane in order to obtain the stability of the Buck converter. Such poles are arbitrarily placed and the state feedback gain matrix is obtained. Thus the root locus can be modified by designing a suitable compensation to achieve the stability by pole placement technique and is discussed in the following sections.
Figure 2.6 Root locus of Buck converter in s-domain

The state feedback matrix can be obtained by substitution method and is explained as follows:

**Step 1:** The characteristic polynomial to find the unknown values of \([k_1 \ k_2]\) is formed as follows:

\[
|sI - (A - Bk)| = s^2 + (8.0031999 \times 10^3 + 347.083389k_1)s + 2.77777 \times 10^6k_1 + 40 \times 10^6k_2 + 159.9999 \times 10^6 = 0
\]  
(2.41)

**Step 2:** The desired characteristic equation is formed by arbitrarily placing the poles as follows:

\[
s^2 + 28.00142 \times 10^3s + 199.999 \times 10^6 = 0
\]  
(2.42)

By equating the like powers of \(s\) in the Equations (2.41) and (2.42), the state feedback matrices are obtained as \(k_1 = 57.6179\) and \(k_2 = -3.0012\).

In order to check the robustness of the control law, the step input is used and the output response has been illustrated in the Figure 2.7. From the
figure, it is very well understood that the system settles down faster and the state feedback matrix is capable enough to realize the stability of the Buck converter.

![Step Response](image)

**Figure 2.7 Step response of the Buck converter in continuous time domain**

The digital state feedback matrix can be derived in the same manner as that of the continuous time domain for the Buck converter under discrete time domain. The state equations for the Buck converter under discrete time domain is defined as,

\[
G = \begin{bmatrix}
0.9999 & -0.00138 \\
0.1148 & 0.9919 \\
\end{bmatrix}
\]

\[H = \begin{bmatrix}
0.00035 \\
0.0000199 \\
\end{bmatrix}\]  \hspace{1cm} (2.43)

\[C_d = [0 \ 1] \]  \hspace{1cm} (2.44)

\[D_d = [0] \]

As per the Nyquist criterion which states that the sampling frequency of the analog signal should be atleast twice the maximum signal
frequency, the sampling frequency is assumed as 1 MHz. The digital state feedback matrix can be derived in the same method as that for the continuous domain excepting the domain considered here is \( z \). The root locus of this converter in \( z \) domain is drawn as shown in the Figure 2.8. Here also the crosses represent the open loop poles. The desired poles are chosen arbitrarily in order to find the digital state feedback matrix.

![Root Locus](image)

**Figure 2.8 Root locus of the Buck converter in \( z \)-domain**

The digital state feedback matrix can be obtained by substitution method and is explained as follows:

**Step1:** The characteristic polynomial to find the unknown values of \([k_{d1}, k_{d2}]\) is formed as follows:

\[
|zI - (G - Hk_d)| = z^2 + (3.471 \times 10^{-4}k_{d1} + 1.995 \times 10^{-5}k_{d2} - 1.9917)z - 3.4428 \times 10^{-4}k_{d1} + 3.9847k_{d2} + 0.98351 = 0 \quad (2.45)
\]

**Step 2:** The desired characteristic equation is formed by arbitrarily placing the poles as follows:
\[ z^2 - 1.6z + 0.6375 = 0 \] \hspace{1cm} (2.46)

By equating the like powers of \( s \) in the Equations (2.45) and (2.46), the state feedback matrices are obtained as \( k_{d1} = 1079.3 \) and \( k_{d2} = 861.6 \).

In order to check the robustness of the control law, the step input is used and the output response has been demonstrated in the Figure 2.9. From the Figure, it is very well understood that the system settles down faster and the digital state feedback matrix is efficient enough to realize the stability of the Buck converter.

![Step Response](image)

**Figure 2.9  Step response of the Buck converter in discrete time domain**

The steady state matrix is designed and implemented which is the initial stage of the controller design. Next the design and implementation of full order state gain matrix which will be discussed in the next section.
2.5 DESIGN AND IMPLEMENTATION OF FULL ORDER STATE OBSERVER GAIN MATRIX

In most of the practical cases, all the state variables are not available for feedback and cannot be measured at all times and hence there is a need to estimate the unmeasurable state variables. The full order state observer gain matrix which is termed as an Observer is used to estimate the unmeasurable state variables. In addition, to ensure the robustness of the control law and for the load estimation, Observer is mainly derived. The major benefit of the Observer method is the simplest design procedure and thus a better dynamic performance is obtained. The Observer is designed in the same way as that of the Kalman like filter and added as an outer loop featuring a correcting integral action. This can be used to avoid the complexity in load deviations. It is desirable that the response of the system should be much faster since the observer tends to act upon the error of the system. By thumb rule the desired observer location is made by having the following assumption:

Natural frequency of Oscillation (Observer Controller) $\approx 2$ to 5 times that of the natural frequency of oscillation of the system.

The sufficient condition for the design of full order state observer is that the system should be completely state observable. The observability of the systems is discussed now. The unforced system is considered which can be described by the following equations,

$$\begin{align*}
\dot{x} &= Ax \\
y &= Cx
\end{align*}$$  \hspace{1cm} (2.47)

The system is said to be completely observable if every state $x(t_0)$ can be determined from the observation of $y(t)$ over a finite time interval
\( t_0 \leq t \leq t_1 \). Within a shorter duration of time the observability helps to solve the problem of restoration of unmeasurable state variables from measurable variables. It is assumed that \( t_0 = 0 \) and the output vector is given by,

\[
y(t) = Ce^{At}x(0)
\]

(2.48)

The output equation can be obtained by using Sylverster’s Interpolation formula defined as,

\[
e^{At} = \sum_{i=0}^{n-1} \rho_i(t) A^i
\]

(2.49)

where \( \rho_i(t) \) is the Eigen values of \( A \) matrix. The output is now defined as,

\[
y(t) = \sum_{i=0}^{n-1} \rho_i(t) CA^i x(0)
\]

(2.50)

The above equation can be rewritten as,

\[
y(t) = \rho_0(t)Cx(0) + \rho_1(t)CAx(0) + \cdots + \rho_{n-1}(t)CA^{n-1}x(0)
\]

(2.51)

\( x(0) \) can be easily determined when the system is completely observable and the output equation \( y(t) \) is defined over a time interval \( 0 \leq t \leq t_f \). This condition requires the rank of this \( nm \times n \) matrix, 

\[
\begin{bmatrix}
C \\
\cdots \\
CA \\
\cdots \\
\cdots \\
CA_{n-1}
\end{bmatrix}
\]

to be \( n \).
This proves that the system is completely observable. In other words it can also be stated that the system is observable when no cancellation occurs in the system transfer function. The mathematical model of the observer is derived as follows for both the continuous and discrete time systems.

The Observer thus employed for the converters is called as Luenberger state observer. The difference between the measured and estimated output is called the error signal which is given as a feedback to the system and this error signal will correct the model continuously. Thus the estimation process can be speeded up and provides a constructive state estimation. The dynamic equation of the observer is defined as,

\[ \dot{x} = A\hat{x} + Bu + g(y - C\hat{x}) = (A - gC)\hat{x} + Bu + gy \] (2.52)

where \( \hat{x} \) is the estimated state and \( C\hat{x} \) is the estimated output. The measured output is \( y \) and the control input is \( u \). The observer gain matrix \( g \) acts as a rectifying term which includes the difference between the measured output \( y \) and the estimated output \( C\hat{x} \). This term improves the performance of the observer. The block diagram of the system with the full order state observer is shown in Figure 2.10.

![Figure 2.10 Block diagram of the Luenberger observer](image)
If the observer dynamic equation denoted by the Equation (2.52) is subtracted from system state equation which is represented by the Equation (2.47) we get,

\[ \dot{x} - \dot{\hat{x}} = Ax - A\hat{x} - g(Cx - C\hat{x}) = (A - gC)(x - \hat{x}) \] (2.53)

This difference is defined as the error vector and it is given by,

\[ e_{rr} = x - \hat{x} \] (2.54)

Thus Equation (2.53) becomes,

\[ e_{rr} = (A - gC)e_{rr} \] (2.55)

By the investigation of Equation (2.55), it is understood that the dynamic behaviour of the error vector is decided by the Eigen values of the matrix \( (A-gC) \). The error \( e_{rr} \) converges to zero for any initial value \( e_{rr}(0) \), when the matrix \( (A-gC) \) is stable. The error will lead to zero with an appropriate speed when the Eigen values of \( (A-gC) \) are asymptotically stable. This in turn makes the dynamic behaviour of the error faster. When the systems under consideration are completely observable, then it is possible to choose the values of \( g \) matrix so that the matrix \( (A-gC) \) has arbitrarily desired Eigen values. The value of observer gain matrix \( g \) can be determined as follows:

The observer gain matrix can also be obtained in a similar way as that of the state feedback matrix by direct substitution method. The observer gain matrix \( g \) for the third order system is given by,

\[ g = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} \] (2.56)
Substitution of this matrix into the desired polynomial equation and then equating the like power of \( \lambda \) as described below, the values of \( g_1, g_2, g_3 \) can be determined.

\[
|\lambda I - (A - gC)| = (\lambda - p_1)(\lambda - p_2)(\lambda - p_3)
\]  

(2.57)

where \( p_1, p_2 \) and \( p_3 \) are the desired pole locations for the systems under consideration. In order that the observation error converges to zero rapidly, the observer poles must be 2 to 5 times faster than the controller poles. This implies that the estimation error decomposes much faster (i.e.) two to five times faster than the state vector \( x \). This makes the controller poles to dominate the system response.

If the sensor noise is considered, the observer poles are to be selected in such a way that it should be two times slower than the controller poles so that the noise will be smoothened and at the same time bandwidth of the system will be lesser. Unlike control poles, different sets of observer poles have to be selected and these values are simulated in order to choose the best observer gain value to obtain speedy response and insensitivity to disturbances and noises. Thus in the above section, the general derivation of the full order state observer gain matrix for continuous time system has been discussed and a similar case can be derived for the discrete time system also which is discussed now.

In discrete system the observer is defined as a prediction observer since the estimate \( \hat{\mathbf{x}}(k+1) \) is one sampling period ahead of the measurement \( y(k) \) and the dynamic equation is defined as,

\[
\hat{\mathbf{x}} = G\hat{\mathbf{x}}(k) + Hu(k) + g_d(y(k) - C_d\hat{x}(k))
\]  

(2.58)
Let the state model of the discrete time system is defined as,

\[ x(k + 1) = Gx(k) + Hu(k) \]  \hspace{1cm} (2.59)

\[ y(k) = C_d x(k) \]  \hspace{1cm} (2.60)

Then by subtracting (2.59) from (2.58) gives the difference equation which describes the behaviour of the error and it is defined as,

\[ \ddot{x}(k + 1) = (G - g_d C_d) \ddot{x}(k) \]  \hspace{1cm} (2.61)

where, \( \ddot{x} = x - \hat{x} \). The observer gain matrix for the discrete time system can be obtained as follows:

The characteristic equation of the error is given by,

\[ |\lambda I - (G - g_d C_d)| = 0 \]  \hspace{1cm} (2.62)

The desired characteristic equation is assumed as,

\[ (\lambda - p_1)(\lambda - p_2) \cdots (\lambda - p_n) = 0 \]  \hspace{1cm} (2.63)

where \( p_1, p_2 \) and \( p_3 \) are the desired pole locations for the systems under consideration. Now \( g_d \) can be obtained by equating the like powers of \( \lambda \).

### 2.6 DERIVATION OF OBSERVER GAIN MATRIX FOR BUCK CONVERTER

In the above section the design of the full order state observer gain matrix for the DC-DC converters was explained. Now the design of full order state observer gain matrix particularly for the Buck converter will be discussed. The full order state observer gain matrix can be derived for the Buck converter under continuous time domain by the substitution method by
assuming appropriate natural frequency of oscillation and damping ratio as per the thumb rule. By assuming the damping ratio, \( \zeta = 0.5 \) and the natural frequency of oscillation, \( \omega_n = 211.2712 \times 10^3 \text{ rad/sec} \), the desired characteristic equation can be obtained as follows,

\[
\lambda^2 + 226.274 \times 10^3 \lambda + 442.879 \times 10^3 = 0 \tag{2.64}
\]

The polynomial equation with unknown values of observer poles is given by,

\[
\lambda^2 + (8.0032 \times 10^3 + g_2)\lambda + (159.999 \times 10^6 + 115.246 \times 10^3 g_1) = 0 \tag{2.65}
\]

Comparing the equations (2.64) and (2.65), the observer gain matrix is obtained. The values are \( g_1 = 442.879 \times 10^3 \) and \( g_2 = 218.2709 \times 10^3 \).

Since the observer poles are mainly designed to check the robustness of the control law, it is essential that the estimated state variables and error variables should converge at zero from any non-zero initial value. This ensures the asymptotic stability of the system under consideration with the desired pole locations. It is demonstrated in the Figures 2.11 to 2.14 respectively. It is observed that the state variables \( x_1 \) and \( x_2 \) and the error variables \( e_1 \) and \( e_2 \) attain zero condition from any non-zero value. Thus it is understood that the system is dynamic with the desired observer pole locations.
Figure 2.11 Estimation of state variable 1

Figure 2.12 Estimation of state variable 2
Figure 2.13 Estimation of error variable 1

Figure 2.14 Estimation of error variable 2
Similar case can be derived for the discrete time system and it is discussed now. By assuming the damping ratio, \( \zeta = 0.5 \), sampling time \( T_S = 1 \, \mu\text{s} \) and the natural frequency of oscillation, \( \omega_n = 107.3264 \times 10^3 \, \text{rad/sec} \), the desired characteristic equation can be obtained as follows,

\[
Z = e^{-\zeta \omega_n T_S} e^{j \omega_n T_S \sqrt{1-\zeta^2}} = \\
e^{-(0.5 \times 1073264 \times 10^3 \times 1 \times 10^{-6})} \times \\
e^{(\pm j107.3264 \times 10^3 \times 1 \times 10^{-6} \times \sqrt{1-0.5^2})} \tag{2.66}
\]

The above equation can be simplified as,

\[
Z^2 - 1.5097Z + 0.65427 = 0 \tag{2.67}
\]

The polynomial equation with unknown values of observer poles is given by,

\[
z^2 + (g_{d2} - 1.9918)z + (0.1148g_{d1} - 0.9999g_{d2} + 0.991959) = 0 \tag{2.68}
\]

Comparing the Equations (2.67) and (2.68), the observer gain matrix for the Buck converter under discrete time domain is obtained. The values are \( g_{d1} = 1.2569 \) \( \text{and} \ g_{d2} = 0.482 \).

Thus the second stage for the controller design has been explained generally as well as for the Buck converter. The next stage is the final stage where the initial stage and the second stage are combined using Separation Principle which leads to the design of Observer controller and it will be discussed in the following section.
2.7 DESIGN OF OBSERVER CONTROLLER USING SEPARATION PRINCIPLE

2.7.1 Buck Converter under Continuous Time Domain

The separation principle makes the design procedure much simpler in which the state feedback gain matrix is designed by pole placement and then the full order state observer by the same technique, finally which can be combined together to provide a better dynamic compensation for the DC-DC converters under consideration. The main advantage of this principle is that the design of control law and the observer can be carried out independently and when both are used together the roots remain unchanged.

The configuration of the dynamic compensator (Observer Controller) based upon combining the state feedback control law and observer poles are shown in the Figure 2.15.

![Block diagram representation of dynamic compensator](image)

Figure 2.15 Block diagram representation of dynamic compensator

Now the dynamic equation of the continuous time system with control law is given by,

\[ \dot{x} = Ax - Bk\hat{x} = (A - Bk)x + Bk(x - \hat{x}) \]  \hspace{1cm} (2.69)
Substitution of the error equation, \( e_{rr} \) in the above equation yields the following,

\[
\dot{x} = (A - Bk)x + Bk\ddot{x} \tag{2.70}
\]

Combining the above equation and the error equation described in Equation (2.55), the dynamics of the observed state feedback control system is obtained as,

\[
\begin{bmatrix}
\dot{x} \\
\dot{\hat{x}}
\end{bmatrix} =
\begin{bmatrix}
A - Bk & Bk \\
0 & (A - gC)
\end{bmatrix}
\begin{bmatrix}
x \\
\hat{x}
\end{bmatrix} \tag{2.71}
\]

The characteristic equation of the system is given by,

\[
|sI - A + Bk||sI - A + gC| = 0 \tag{2.72}
\]

If the system is of \( n \)th order, the observer is also of \( n \)th order and the order of the characteristic equation for the entire closed loop system is \( 2n \). The observed state feedback control is given by,

\[
u = -k\ddot{x} \tag{2.73}
\]

Now by substituting the Equation (2.73) in the observer dynamic mathematical equation given by Equation (2.52), the equation for the observer is obtained as,

\[
\dot{\hat{x}} = (A - gC - Bk)\dot{x} + gy \tag{2.74}
\]

Assuming zero initial conditions and taking Laplace transform for Equation (2.74), we get the following equation,

\[
\hat{x}(s) = (sI - A + gC + Bk)^{-1}gy(s) \tag{2.75}
\]
Now the system can be represented by the block diagram as shown in the Figure 2.16.

![Block Diagram of DC-DC Converter with Controller](image)

**Figure 2.16 DC-DC Converter with the controller**

The transfer function \( k|sI - A + gC + Bk|^{-1}g \) acts as a controller for the systems under consideration. This is defined as the observer controller transfer function defined as,

\[
T(s) = -\frac{U(s)}{Y(s)} = k(sI - A + gC + Bk)^{-1}g
\]  

(2.76)

By substituting all the necessary values in the above equation the transfer function of the Observer Controller for Buck converter under continuous time domain is obtained as follows,

\[
T(s) = \frac{2.486 \times 10^7 s + 3.358 \times 10^{16}}{s^2 + 2.463 \times 10^5 s + 5.561 \times 10^{10}}
\]  

(2.77)

The closed loop control system for the DC-DC converter with Observer controller feedback is shown in the Figure 2.17.

![Functional Block Diagram of Closed Loop Control for DC-DC Converter](image)

**Figure 2.17 Functional block diagram of closed loop control for DC-DC converter**
The ultimate aim in designing the Observer controller is to minimize the error between $V_o$ and $V_{ref}$. As seen from Figure 2.17, the important functional blocks that are evident are: Observer Controller, Pulse Width Modulation (PWM) and DC-DC converter. The Observer controller acts as a compensator and generates the control signal by compensating the error signal ($V_e$). PWM block is for the generation of driver signal obtained from the compensator. The error ($V_e$), which is the difference between the output voltage ($V_o$) and reference voltage ($V_{ref}$), is processed by the compensator block with Observer Controller algorithm and the processed control signal in turn acts as a reference for the inductor current of the DC-DC converter. The control signal significantly affects the converter characteristics and therefore effective tuning of the controller is one of the desired aspects of the control system. The fine tuned Observer controller generates the duty cycle command corresponding to the error signal which is then converted as switching pulses using the PWM functional block. These pulses are then fed to the semiconductor switch employed in the DC-DC converter for obtaining the preferred output voltage.

### 2.7.2 Buck Converter under Discrete Time Domain

The previous section clearly explains the derivation of the transfer function of Observer controller for the Buck converter under continuous time domain. In this section, a similar case can be derived for the discrete time system also, which is discussed now. The dynamic equation of the discrete time system with control law is given by,

$$\ddot{x} = Gx - Hk\hat{x} = (G - Hk)x + Hk(x - \hat{x})$$  \hspace{1cm} (2.78)

Substitution of the error equation, $e_{rr}$ in the above equation yields the following,
Combining this equation and the error equation the dynamics of the observed state feedback control system is obtained as,

\[
\begin{bmatrix}
\dot{x}
\end{bmatrix} = \begin{bmatrix}
G - Hk & Hk \\
0 & (G - g_d C_d)
\end{bmatrix} \begin{bmatrix}
x \\
\hat{x}
\end{bmatrix} \tag{2.80}
\]

The characteristic equation of the system is given by,

\[
|zI - G + Hk||zI - G + g_d C_d| = 0
\tag{2.81}
\]

Now by substituting the Equation (2.73) in the observer dynamic mathematical equation given by (2.58), the equation for the observer is obtained as,

\[
\dot{x} = (G - g_d C_d - Hk)\hat{x} + g_d y \tag{2.82}
\]

Assuming zero initial conditions and taking Z-transform for Equation (2.82), we get

\[
\hat{x}(z) = (zI - G + g_d C_d + Hk)^{-1}g_d y(z) \tag{2.83}
\]

Now the system can be represented by the block diagram as shown in the Figure 2.18.

![Figure 2.18 DC-DC converter with the controller](image-url)
The transfer function \( k[zI - G + g_dC_d + Hk^{-1}g_d] \) acts as a controller for the DC-DC converter under consideration. This is defined as the prediction observer controller transfer function which is defined as follows,

\[
T(z) = \frac{-b(z)}{y(z)} = k(zI - G + g_dC_d + Hk)^{-1}g_d
\]  
(2.84)

On substituting the required values in the above equation the transfer function can be obtained as follows,

\[
T(z) = \frac{1772z - 1637}{z^2 - 1.182 + 0.4533}
\]  
(2.85)

The digital control objective is to drive the discrete state feedback control and the observer based controller similar to the Kalman-like filter using pole placement technique. The main intention of the state feedback control is to make the system to track the reference signal which is considered as a step input. Prediction Observer acts as a load estimator and measures the unmeasurable variables. It effectively ensures the robustness of the control law. By Separation Principle, digital state feedback control and Observer poles are combined to provide a dynamic compensation. The closed loop control is illustrated in Figure 2.19. The output voltage is regulated by using the feedback. The feedback ensures that the output must be insensitive to load disturbances, stable and provides good transient response thereby improving the dynamic performances. The error voltage is fed to A/D Converter which samples them at a sampling rate equal to at least twice the switching frequency. The function of the discrete time compensator is to process the error signals. The output samples control the switch by generating gating pulses after being processed through Digital Pulse Width Modulation (DPWM) block.
Figure 2.19 Closed loop control of DC-DC converter (discrete time domain)

The DPWM block includes a sample and hold and it acts as a demodulator. The delay time ($t_d$) is included in the feedback loop. It includes the A/D conversion time, computational delay, modulator delay and the switch transition time which is taken equal to the switching period. The additional advantages of digital state feedback control include: (i) Implementation of discrete time systems is very easy including Prediction (ii) Effective utilization of observed state variables instead of measured ones and (iii) synchronization of controller operation and pulse width modulation can be achieved effectively. Figure 2.20 shows the Analog to digital converter block. It is an effective device that converts a continuous time signal to a discrete time signal by using sampling. The converter block includes a delay, zero order hold and a quantizer.

Figure 2.20 Analog to digital converter
The total time between sampling the error signal and updating the duty cycle command at the beginning of the next switching period is carried out by the delay block. The zero order hold is added mainly for modelling the sampling effect. It reconstructs the continuous time signal by holding the sample one by one for each sampling interval. Quantizer is mainly used to map a larger set of input values to a smaller set such as rounding values to some unit of precision. The discrete time Compensation block is shown in the Figure 2.21.

![Figure 2.21 Discrete time compensation](image)

**Figure 2.21** Discrete time compensation

The output of the A/D converter called quantized error is fed to the discrete zero-pole block which in turn is converted into Pulse using DPWM block shown in Figure 2.22. The compensator thus designed minimizes the error and send the command signal to the switch in form of pulses in order that the output tracks the reference signal.

![Figure 2.22 Digital pulse width modulation](image)

**Figure 2.22** Digital pulse width modulation
The output of the compensator is compared against the ramp signal in order that the duty cycle command for the semiconductor switch is obtained. Thus the design of the Observer controller for the Buck converter is finally obtained and the simulation results for the Buck converter with Observer controller in continuous time domain and discrete time domain are discussed in the following sections.

2.8 RESULTS AND DISCUSSIONS

2.8.1 Buck Converter (Continuous Time Domain)

In all the above sections, the design of the Buck converter, its modelling and derivation of the Observer controller transfer have been dealt and in this section the simulation results are being discussed in detail. The design and the performance of Buck converter is accomplished in continuous conduction mode and simulated using MATLAB/ Simulink. The ultimate aim is to achieve a robust controller inspite of uncertainty and large load disturbances. The performance parameters of the converter under consideration are rise time, settling time, maximum peak overshoot and steady state error, which are shown in the Table 2.2. It is evident that the converter settles down at 0.015 s and the rise time of the converter is 0.0125 s. No overshoots or undershoots are evident and the steady state error observed is of the order of 0.02 V.

Table 2.2 Performance parameters of Buck converter (Continuous time domain)

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Settling Time (s)</td>
<td>0.015</td>
</tr>
<tr>
<td>2</td>
<td>Peak Overshoot (%)</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Steady State Error (V)</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>Rise Time (s)</td>
<td>0.0125</td>
</tr>
<tr>
<td>5</td>
<td>Output Ripple Voltage (V)</td>
<td>0</td>
</tr>
</tbody>
</table>
The simulation of the Buck converter is also carried out by varying the load, not limiting it to $R$ load and it is illustrated in the Table 2.3.

### Table 2.3 Output response of Buck converter for load variations (Continuous time domain)

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Load</th>
<th>Reference Voltage(V)</th>
<th>Output Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.44</td>
<td>12</td>
<td>11.995</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>12</td>
<td>11.99</td>
</tr>
<tr>
<td>4</td>
<td>14.44</td>
<td>10 x 10^6</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>100 x 10^6</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>100 x 10^6</td>
<td>5</td>
</tr>
</tbody>
</table>

It is obvious that the Buck converter with observer controller is efficient enough to track the output voltage irrespective of the load variations. When the load resistance is varied as 14.44 $\Omega$, 20 $\Omega$ and 10 $\Omega$, the converter is able to track the output voltages as 11.995 V, 12 V and 11.99 V respectively for the reference voltage of about 12 V. Again when the inductance of 10 $\mu$H and 100 $\mu$H are added to the resistances of 14.44 $\Omega$ and 20 $\Omega$, the output thus obtained is of the order of 12.01 V and 12 V respectively. The steady state error observed is of very considerable order of about 0.01 V only. The simulation is also carried out again using RLE load with a resistance of 15 $\Omega$, inductance of 100 $\mu$H and an ideal voltage source of about 5 V. The response of the converter is such that the controller is capable to work under all the load transients thereby tracking the voltage as 12.008 V.

The simulation is also carried out by varying the input voltage and load resistance and the corresponding, input voltage, load resistance, output voltage, inductor current and load current are shown in the Figure 2.23. The input voltage is first set as 44 V until 0.125 s and again varied from 44 V to
46 V upto 0.25 s. Again at 0.25 s it is varied to 48 V and at 0.35 s it has been varied to 50 V respectively.

Simultaneously the load resistance is also varied from 20 Ω to 15 Ω, 10 Ω and again to 15 Ω respectively at 0.125 s, 0.25 s, and 0.35 s. The corresponding output response of the Buck converter shows fixed output voltage regulation. Undershoots and Overshoots are not observed and the steady state error is also not apparent. The inductor current and load current are also shown in the Figure 2.23, which shows no evidence of current ripples. In order to check the dynamic performance of the system, the $L$ and $C$ parameters of the Buck converter are varied and the output response of the system is shown in the Table 2.4.

![Figure 2.23](image)

**Figure 2.23** Output response of Buck converter with observer controller

($V_s$ – Input Voltage, $R_o$ – Load resistance, $V_o$ – Output Voltage, $I_L$ – Inductor current, $I_o$ – Load Current)
It is obvious from the Table 2.4 that the system is very much dynamic in tracking the reference voltages inspite of the variations in the inductance and capacitance values. The inductance values are varied as 720.28 µH, 600 µH, 500 µH, 100 mH and 10 mH and the corresponding capacitance values are 0.8677 µF, 1 µF, 5 µF and 1 µF respectively. For all the changes the Buck converter along with the Observer controller is efficient enough to track the references. The system does not show any overshoots or undershoots and it settles down fast with a settling time of about 0.015 s for all the values. The steady state error thus noticeable is within the tolerable limits. The load current, output power, losses and efficiency of the Buck converter is determined and is illustrated in Table 2.5.

Table 2.5 Efficiency of the Buck converter with analog observer controller

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>I₀(A)</th>
<th>P₀(W)</th>
<th>Losses(W)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6048</td>
<td>7.2576</td>
<td>0.4158</td>
<td>96.590</td>
</tr>
<tr>
<td>2</td>
<td>0.6275</td>
<td>7.5300</td>
<td>0.2603</td>
<td>96.659</td>
</tr>
<tr>
<td>3</td>
<td>0.8350</td>
<td>10.0158</td>
<td>0.3535</td>
<td>96.178</td>
</tr>
<tr>
<td>4</td>
<td>0.8586</td>
<td>10.3007</td>
<td>0.3654</td>
<td>96.570</td>
</tr>
<tr>
<td>5</td>
<td>0.8682</td>
<td>10.4271</td>
<td>0.3703</td>
<td>94.580</td>
</tr>
<tr>
<td>6</td>
<td>1.2000</td>
<td>14.3880</td>
<td>0.5715</td>
<td>96.577</td>
</tr>
</tbody>
</table>
The efficiency of the Buck converter remains more or less same with the increase in the load current. It is very well understood that the Buck converter with observer controller is highly efficient and the highest efficiency is obtained as 96.659% at a load current of about 0.6275 A and the corresponding output power is 7.53 W. The output voltage for the references of 5 V and 8 V are shown with respective input voltage variations in Figure 2.24. In this Figure, for the output voltage graph, blue line represents the set value and the green line represents the actual output voltage. It is evident that the Observer controller is sturdy and efficient enough in tracking the reference voltages irrespective of the input voltage variations and disturbances.

![Figure 2.24 Output response of Buck converter for variation in the reference voltages](image)

### 2.8.2 Buck Converter (Discrete Time Domain)

In this section simulation results of Buck converter with prediction observer controller is discussed. Typical simulation has been carried out for the Buck converter under discrete time domain and the performance parameters are tabulated in Table 2.6. It is understood that the system settles down much faster at 0.01 s and the rise time is only 0.005 s as shown. The overshoots and undershoots are not seen and there is no peak overshoot. The steady state error thus observed for load variations is much lesser than 2%.
The performance parameters for the Buck converter with digital state feedback matrix shows improved performance when compared with the analog state feedback matrix.

The output response with load variations is shown in the Table 2.7. When the load resistance is varied as 14.44 Ω, 20 Ω and 10 Ω, the Buck converter with discrete controller is able to track the output voltages as 12.01 V, 12 V and 11.95 V respectively for the reference voltage of about 12 V. Again when the inductance of 10 µH and 100 µH are added to the resistances of 14.44 Ω and 20 Ω, the output thus obtained is of the order of 12.04 V and 12.05 V respectively. The steady state error observed is of very minimal order of about 0.04 V and 0.05 V respectively. The simulation is also carried out again using RLE load with a resistance of 15 Ω, inductance of 100 µH and an ideal voltage source of about 5 V. The response of the converter is such that the controller is capable to work under all the load transients thereby tracking the voltage as 12.02 V.

Table 2.6 Performance parameters of Buck converter (Discrete time domain)

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Settling Time (s)</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>Peak Overshoot (%)</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Steady State Error (V)</td>
<td>0.005</td>
</tr>
<tr>
<td>4</td>
<td>Rise Time (s)</td>
<td>0.005</td>
</tr>
<tr>
<td>5</td>
<td>Output Ripple Voltage (V)</td>
<td>0</td>
</tr>
</tbody>
</table>

The simulation is also carried out by varying the input voltage and load resistance and the corresponding, input voltage, load resistance, output voltage, inductor current and load current are shown in the Figure 2.25. The input voltage is first set as 48 V until 0.01 s and again varied to 46 V upto
0.02 s. Again at 0.02 s it is varied to 48 V and at 0.03 s it has been varied to 50 V up to 0.04 s and finally it is varied to 48 V at 0.05 s respectively. Simultaneously the load resistance is also varied as 2.5 \( \Omega \), 2 \( \Omega \) and 1.5 \( \Omega \) respectively and the corresponding output response of the Buck converter shows fixed output voltage regulation. Undershoots and Overshoots are not observed and the steady state error is also not apparent. The inductor current and load current are also shown in the Figure 2.25, which shows no evidence of current ripples. In order to check the dynamic performance of the system, the \( L \) and \( C \) parameters of the Buck converter are varied and the output response of the system is shown in the Table 2.8.

Table 2.7 Output response of Buck converter for load variations (Discrete time domain)

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Load R(( \Omega ))</th>
<th>Load L(H)</th>
<th>Reference Voltage(V)</th>
<th>Output Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.44</td>
<td>--</td>
<td>12</td>
<td>12.01</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>--</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>--</td>
<td>12</td>
<td>11.95</td>
</tr>
<tr>
<td>4</td>
<td>14.44 10 x 10(^{-6})</td>
<td>--</td>
<td>12</td>
<td>12.04</td>
</tr>
<tr>
<td>5</td>
<td>20 100 x 10(^{-6})</td>
<td>--</td>
<td>12</td>
<td>12.05</td>
</tr>
<tr>
<td>6</td>
<td>15 100 x 10(^{-6})</td>
<td>5</td>
<td>12</td>
<td>12.02</td>
</tr>
</tbody>
</table>

Table 2.8 Output response of Buck converter with variable parameters (Discrete time domain)

<table>
<thead>
<tr>
<th>Sl.No</th>
<th>Inductance, ( L )</th>
<th>Capacitance, ( C )</th>
<th>Reference Voltage(V)</th>
<th>Output Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>720.28 ( \mu )H</td>
<td>0.8677 ( \mu )F</td>
<td>12</td>
<td>12.00</td>
</tr>
<tr>
<td>2</td>
<td>600 ( \mu )H</td>
<td>1 ( \mu )F</td>
<td>12</td>
<td>12.01</td>
</tr>
<tr>
<td>3</td>
<td>500 ( \mu )H</td>
<td>5 ( \mu )F</td>
<td>12</td>
<td>11.992</td>
</tr>
<tr>
<td>4</td>
<td>100 mH</td>
<td>1 ( \mu )F</td>
<td>12</td>
<td>12.981</td>
</tr>
<tr>
<td>5</td>
<td>10 mH</td>
<td>1 ( \mu )F</td>
<td>12</td>
<td>12.981</td>
</tr>
</tbody>
</table>
It is clear that the system is very much dynamic in tracking the reference voltages inspite of the variations in the inductance and capacitance values. The $L$ and $C$ values are varied in the same manner as that of the analog converter and the results thus obtained act as an evidence for the efficient discrete controller design. The Buck converter with prediction observer controller very well tracks the reference value of 12 V against the parameter variations. The system does not show any overshoots or undershoots and it settles down fast with a settling time of about 0.01 s for all the values. The steady state error thus noticeable is within the tolerable limits. The load current, output power, losses and efficiency of the Buck converter with prediction observer controller is determined and is illustrated in Table 2.9.

![Output response of Buck converter with prediction observer controller](image)

Figure 2.25  Output response of Buck converter with prediction observer controller

$(V_s$ – Input Voltage, $R_o$ – Load resistance, $V_o$ – Output Voltage, $I_L$ – Inductor current, $I_o$ – Load Current)
The efficiency remains more or less same with the increase in the load current. It is very well understood that the system is highly efficient and the highest efficiency is obtained as 96.687% at a load current of about 0.6283 A and the corresponding output power is 7.57 W. The efficiency against the load current for the Buck converter under both continuous and discrete time domain is shown compared in the Figure 2.26. The efficiency is high for all the load variations and the Observer acts as a dynamic and robust controller under both the continuous and discrete time domain.

Table 2.9 Efficiency of the Buck converter with prediction observer controller

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>I₀(A)</th>
<th>P₀(W)</th>
<th>Losses(W)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5213</td>
<td>6.266</td>
<td>0.2213</td>
<td>96.588</td>
</tr>
<tr>
<td>2</td>
<td>0.6093</td>
<td>7.3177</td>
<td>0.35382</td>
<td>95.333</td>
</tr>
<tr>
<td>3</td>
<td>0.6283</td>
<td>7.5710</td>
<td>0.2595</td>
<td>96.687</td>
</tr>
<tr>
<td>4</td>
<td>0.8325</td>
<td>9.9900</td>
<td>0.35209</td>
<td>96.596</td>
</tr>
<tr>
<td>5</td>
<td>0.8360</td>
<td>10.065</td>
<td>0.3519</td>
<td>96.622</td>
</tr>
<tr>
<td>6</td>
<td>1.195</td>
<td>14.283</td>
<td>0.5642</td>
<td>96.199</td>
</tr>
</tbody>
</table>

Figure 2.26 Comparison of efficiencies for Buck converter
In order to confirm the dominance of the digital controller over its analog counter parts, the output response of the Buck converter with prediction Observer controller is compared against the response produced by the converter with analog observer controller and it is shown in the Figure 2.27. It is noticeable that the digital controller demonstrates much better performance than the analog controller in terms of performance specifications. The discrete system settles down much faster when compared with the analog system. The steady state error observed in the case of discrete system is zero where as in the case of analog controller, steady state error of 0.02 V is obvious.

![Figure 2.27 Comparison between digital and analog controller responses for Buck converter](image)

2.9 CONCLUSION

A state feedback control approach has been designed for the Buck converter under both continuous time domain and discrete time domain using pole placement technique and separation principle. The load estimator has been designed by deriving full order state observer to ensure robust control for the converter. The separation principle allows designing a dynamic
compensator which very much looks like a classical compensator since the design is carried out using simple root locus technique. The mathematical analysis and simulation study show that the Buck converter with both analog and digital Observer controller thus designed achieves tight output voltage regulation, shows good dynamic performances and highly efficient.

In the next chapter the design and modeling of the Boost converter with Observer controller is carried out.