[INTRODUCTION]

Many years have passed since the integral transforms such as Laplace transform, Fourier transform etc have been introduced, and a good number of Mathematicians have worked on them till date. Even new transforms are being introduced. Professor E.C. Titchmarsh in the sixth chapter of his book [19] introduced an integral transform via a 2nd order boundary value problem viz.

\[-\frac{d^2y(x)}{dx^2} + \{y(x) - \lambda\} y(x) = 0 \quad 0 \leq x \leq b < \infty\]

\[\gamma(0) \cos \alpha + \gamma'(0) \sin \alpha = 0\]

\[\gamma(b) \cos \beta + \gamma'(b) \sin \beta = 0\]

\[0 \leq \alpha, \beta < \pi, \gamma(x) \text{ is real valued in } [0,b]\]

\[\lambda \text{ is a complex parameter.}\]

He established that a transform of any square integrable function \(f\) (\(f\) is said to be square-integrable or \(f \in L^2(0,\infty)\) if

\[\int_0^\infty |f(x)|^2 < \infty\]

may be defined by

\[F(\lambda) = \int_0^\infty \phi(x,\lambda)dx \quad (1)\]

where \(\phi(x,\lambda) = \cos \alpha, \phi'(x,\lambda) = \sin \alpha\). That is, he has established the existence of the integral on the right of (1) in some sense. In the light of this, one may view the classical Laplace transform \(\int_0^\infty e^{-\lambda x}f(x)dx\) as the one associated with the boundary value
problem
\[
\begin{align*}
\frac{d^2 y(x)}{dx^2} + \alpha^2 y(x) &= 0, \quad 0 \leq x \leq b < \infty \\
y(0) + y'(0) &= 0 \\
y(b) + y'(b) &= 0
\end{align*}
\]  \quad \cdots (2)

since, in this case we have \( \phi(x) = e^{-\alpha x} \), the Fourier transform \( \int_{-\infty}^{\infty} f(x) e^{-i\pi \alpha x} \) can be associated with the boundary value problem
\[
\begin{align*}
\frac{d^2 f(x)}{dx^2} + \alpha^2 f(x) &= 0, \quad 0 \leq x \leq b < \infty \\
f(0) + if'(0) &= 0 \\
f(b) + if'(b) &= 0
\end{align*}
\]  \quad \cdots (3)

where the \( \phi(x) = e^{i\pi x} \). In a similar manner one can associate other integral transforms with suitable boundary value problems. This association of an integral transform with a boundary value problem has not been exploited so far and to this investigation the present thesis has been devoted.

The first thing to be noticed is that, like Laplace transform, Fourier transform etc. the transform \( \mathcal{L} \) is not an operator in the sense that it may not carry a member of \( L^2([0,\infty)) \) to a member of itself; it depends on the way we attach a meaning to the integral in \( \mathcal{L} \). So, in the general case, one has to study the structure of the space of all transforms of members of \( L^2([0,\infty)) \).
referred to as the transformed space $\mathbb{T}_n$. Our first venture is to establish that $\mathbb{T}_n$ is a Hilbert space for which we need to define appropriate inner product in $\mathbb{T}_n$. Next we indicate that this idea of defining transforms can be extended to higher order ordinary differential equations. Finally we prove that for any $2m$th order ordinary differential equation, we can associate a suitable transform and the transformed space $\mathbb{T}_m$ is a Hilbert space provided an inner product is defined suitably in it.

In the first chapter we review the method adopted by Professor E.C. Titchmarsh for defining a transform pointing out the necessity of the Hilbert space structure of the transformed space. The utility of forming these transforms is well revealed when we succeed in linking up the convergence of the eigenfunction expansion of any $\xi \in \ell^1(0,\infty)$ (in relation to the boundary value problem under consideration) with the convergence of an associated sequence of members of the transformed space.

In the second chapter we first define the transform that is to be associated with a boundary value problem consisting of a fourth order ordinary differential equation coupled with boundary conditions. It has been shown that the transformed space $\mathbb{T}_2$ is a Hilbert space with, of course, suitable inner product defined in it.

The chapters three and four follow the same scheme as in
chapter two with the exception that they deal with sixth and
the 72th order differential equations respectively. Details
are being included in both chapter two and three so as to faci­
litate the development as done in chapter four.