CHAPTER FIVE

EFFECTS OF PROCESS ADJUSTMENTS BASED ON CONTROL CHART EVIDENCES

5.1 Introduction:

It has been empirically realised (and illustrated by Cowden) that if action has to be taken whenever a point on a process control chart goes out of limits, process variability tends to be larger. In some cases, as in the control of dimensions, the action may simply consist of an adjustment in the machine for that dimension. Although such an adjustment is always expected to bring the process level to its initial correct setting it is not totally unlikely that the level may be further pulled away from the standard by such a process adjustment. Firstly because, on relatively infrequent occasions a control chart may give false alarms of assignable variations. Secondly, even when the alarm is a real indication of the situation, the circumstantial evidence provided by a point out of limits is subject to all the vagaries of a small sample. Lastly, the possibility of an automatic machine giving rise to a sporadic deviation and later restoring itself to the standard cannot entirely be ruled out.

The present author compares the proportion of effective articles from a manufacturing process under two situations: firstly, when the process is left to itself and secondly, when a process control chart is maintained and adjustments are based on indications given by it regarding the advent of assignable variations. Such a comparison has to take account of the limits used on the chart, the size of the sample observed, the nature of adjustment made on the basis of an out-of-limit point and the magnitude of deviation in the
process level. The case of a double limit $\bar{x}$-chart has been considered. For each point beyond (or below) a control limit an adjustment equal to the observed difference between the sample average and the central control line is made on the machine. A correct adjustment will be made only when the observed sample average and the true process level are identical — a situation having probability zero. Possible effects of sample size variation have also been investigated.

5.2 Process Fraction Effective with and without Adjustments:

Suppose quality is described in terms of a variable characteristic $x$ assumed to be distributed normally. Let $\mu_0$ be the standard value for $x$, and $\sigma^2$ be the process dispersion. It will be assumed that $\sigma$ remains sensibly constant throughout the operation. A unit of product will be said to be effective if its quality measure satisfies $\mu_0 - 3\sigma < x < \mu_0 + 3\sigma$, the unit will be defined as defective otherwise. Let $l_1$ and $l_2$ be the upper and the lower control limits on the $\bar{x}$-charts, placed at distances $n \sigma \bar{x}$ from $\mu_0$ ($\sigma \bar{x}$ being the standard error of $\bar{x}$ in random samples of size $m$).

Let us now suppose that an assignable source of variation takes place and shifts the process level to a value $\mu = \mu_0 + d \sigma \bar{x}$. Consider the process beginning from the instant the first sample of size $m$ is drawn from the deviated process. It is known that the first sample will give an average value ($\bar{x}$) lying within control limits with a probability

$$
\beta = \int_{l_1}^{l_2} P(\frac{\bar{x}}{\mu}) \, d\bar{x} = \int_{-(n+d)}^{n-d} \Phi(t) \, dt
$$
where \( P(\bar{x} | \mu') \) and \( \Phi(t) \) are probability densities for \( \bar{x} \) and for the standard normal deviate. With this probability the shift in process level remains undetected and the proportion of effective units becomes

\[
P(\mu') = \int P(x | \mu') \, dx = \int \Phi(t) \, dt
\]

\[
\mu' = \mu_0 + 3\sigma
\]

\[
\mu_0 - 3\sigma
\]

with the complementary probability \( 1 - \beta \) the first sample of \( m \) units will give an out-of-limit average and call for adjustment in the process level.

Let \( \bar{x} \) be at a distance of \( k \sigma \bar{x} \) from \( \mu_0 \), so that \( k \) is distributed normally with mean \( d \) and S.D. unity. For a given value of \( d \), the entire region on the control chart lying below \( l_1 \) and beyond \( l_2 \) can be divided into four mutually exclusive zones for \( \bar{x} \) (or \( k \)), and the application of the above adjustment will lead to four different situations with virtually different consequences. In each case the process level is brought down \( k \) S.E. units, so that the mean of the adjusted process becomes \( \mu' = \mu - k \sigma_{\bar{x}} = \mu_0 + (d - k) \sigma_{\bar{x}} \) and the proportion of effective units becomes

\[
P(\mu') = \int_{\mu_0 - 3\sigma}^{\mu_0 + 3\sigma} P(x | \mu') \, dx = \int_{-3 - \frac{d - k}{\sqrt{m}}}^{3 - \frac{d - k}{\sqrt{m}}} \Phi(t) \, dt
\]

In all the four situations below, \( \bar{x} \) assumed out of control or that \( |n| < |k| \). We now enumerate the four situations as

**S_1 :** \( n < k < d \) (points in region A) – the adjusted process still
remains biased on the same side of $\mu_0$ as before, but the bias will now be smaller. Probability for this situations will be

$$P(S_1) = \gamma_1 = \Pr \left \{ l_2 < \bar{X} < \mu \text{ for } \mu > l_2 \right \}
\begin{cases}
0 & \text{otherwise} \\
= 0 & \\
\end{cases}
$$

Or

$$\gamma_1 = \int_{n-d}^{0} \Phi(t) \, dt \quad \text{for } d > n$$

$$= 0 \quad \text{for } d \leq n \quad \ldots (5.1)$$

$S_2: d < k < 2d$ (points in region B) - the adjusted process will be biased in the opposite direction, but with a smaller absolute amount of bias. The process level will be improved since $\mu_0 - \bar{X} < \mu < \mu_0$

$$P(S_2) = \gamma_2 = \Pr \left \{ \max (l_2, \mu) < \bar{X} < 2\mu \text{ for } \mu > \frac{l_2}{2} \right \}
\begin{cases}
0 & \text{otherwise} \\
= 0 & \\
\end{cases}
$$

Or

$$2 = \int_{\max(0,n-d)}^{\max(n-d,d)} \Phi(t) \, dt \quad \text{for } d > \frac{n}{2}$$

$$= 0 \quad \text{otherwise} \quad \ldots (5.2)$$

$S_3: k > d$ (points in region C) - the adjusted process will be more biased in the opposite direction than it was before.

$$P(S_3) = \gamma_3 = \Pr \left \{ \bar{X} > \max (l_2, 2\mu) \text{ for } \mu > \mu_0 \right \}
\begin{cases}
0 & \text{otherwise} \\
= 0 & \\
\end{cases}
$$

Or

$$\gamma_3 = \int_{\max(n-d,d)}^{\max(n-d,d)} \Phi(t) \, dt \quad \text{for } d > 0$$

$$= 0 \quad \text{otherwise} \quad \ldots (5.3)$$
$S_4: k < -n$ (points in region B) - the adjusted process will be more biased than the original and in the same direction.

\[
P(S_4) = \gamma_4 = \Pr \left\{ \bar{x} < l_1 \right\}
\]

\[
= \begin{cases} 0 & \text{for } \mu > \mu_0 \\ -(n+d) & \text{otherwise} \end{cases}
\]

Or
\[
\gamma_4 = \int \varphi(t) \, dt 
\]

\[
= \begin{cases} 0 & \text{for } d > 0 \\ \varphi & \text{otherwise.} \end{cases}
\]

Thus the total probability of detecting the shift in process level on the basis of evidence provided by the first sample is analysed as

\[
1 - \beta = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 
\]

... (5.5)

For a particular value of $d$ not all these four adjustments are admissible. Thus if the shift in process level be such that

(i) $0 < d < \frac{n}{2}$ only situations $S_3$ and $S_4$ are admissible

(ii) $d > \frac{n}{2}$ situations $S_2$, $S_3$ and $S_4$ are admissible

(iii) $d > n$ all the four situations are admissible.

Now for values of $k$ justifying situation $S_1$ the expected average fraction effective is given by

\[
P(m, n-d, 0) = \frac{d}{n} \int P(\mu') \Pr(k) \, dk = \frac{1}{n-d} \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \varphi(t)} \, dt \, dk
\]

\[
= \frac{1}{n} \int \Pr(k) \, dk
\]

\[
= \frac{F'(m, n-d, 0)}{1}, \text{ say}
\]

... (5.6)
Similarly when situations $S_2$, $S_3$ and $S_4$ arise the expected average fraction effective from the adjusted process are given by respectively

$$F(m, 0, d) = \frac{F'(m, 0, d)}{Y_2}, \quad F(m, d, \infty) = \frac{F'(m, d, \infty)}{Y_3}$$

and $F(m, \infty, -n + d) = \frac{F'(m, -\infty, -n + d)}{Y_4}$

Thus when the process level has shifted $d$ S.E. units from the standard, the expected average fraction effective between the first and the second samples will be $F(\mu)$ with probability $F(m, \mu - d, 0)$ with probability $y_1$, $F(m, 0, d)$ with probability $y_2$, $F(m, d, \infty)$ with probability $y_3$ and $F(m, -\infty, -n + d)$ with probability $y_4$. Thus the expected over-all average fraction effective from the process (when a control chart is used) between the first and second sample is

$$F' = P(\mu)\beta + F'(m, n, d) + F'(m, d, 2d) + F'(m, 2d, \infty) + F'(m, -\infty, -n)$$

$$= P(\mu)\beta + F''$$

say

$$\ldots (5.7)$$

If however no control chart has been used to detect shifts in the process level the fraction effective from the process during the interval between the first and second samples would be $F = P(\mu)$. Thus the gain in fraction effective from a process with level $\mu$ by using a control chart is

$$G = F' - F = F'(m, n, d) + F'(m, d, 2d) + F'(m, 2d, \infty)$$

$$+ F'(m, -\infty, -n) - (1 - \beta) P(\mu)$$

$$= F'' - (1 - \beta) P(\mu).$$

$$\ldots (5.8)$$

Yet another harmful adjustment may be called for with probability

$$\alpha = \int_{-\infty}^{n} \rho(t) \, dt + \int_{n}^{\infty} q(t) \, dt$$

$$\ldots$$

$$\ldots$$
when the process level has actually remained at the standard but
$|k| > |n|$, so that the process level is drawn from the standard to
the value $\mu' = \mu - k \sigma_x$ producing an expected average fraction
effective

$$F_1 = \frac{\int P(\mu') \Pr(k) \, dk + \int P(\mu) \Pr(k) \, dk}{\Pr\{|k| > |n|\}} = \frac{F'_1}{\alpha} \text{ say} ... (5.9)$$

and this will occur with probability $\alpha$, so that the gain will be
$G = F'_1 - F$ in this case. However this case is covered by the
earlier situation when $d = 0$. Thus we shall start with a process
which has been set at $/\mu_0$ and shall assume that at the time of
drawing the first sample the process level has shifted to

$$\mu = /\mu_0 + d \sigma_x (\infty < d < \infty).$$

5.3 The Integral $F(m, n, d) = \int \frac{1}{2\pi} e^{-k^2/2} \int \frac{1}{\sqrt{m}} e^{-y^2/2} \, dy \, dk$:

The function $F(m, n, d)$ can be written as

$$F = \iint_R \frac{1}{2\pi} e^{-\frac{1}{2} (x^2 + y^2)} \, dx \, dy \text{ where } R \text{ is the}$$

parallelogram defined by

$$R: \begin{cases} n - d < x < 0 \\ -3 - \frac{d-x}{\sqrt{m}} < y < 3 - \frac{d-x}{\sqrt{m}} \end{cases}$$

Introducing the linear transformation $z = y - \frac{x}{\sqrt{m}}$ and $x = x$
the parallelogram $R$ becomes the rectangle

$$R': n - d < x < 0 \text{ and } -3 - \frac{d}{\sqrt{m}} < z < 3 - \frac{d}{\sqrt{m}}$$

and we can write
Thus \( F \) is the volume \( \Delta_1 \) under the bivariate Normal surface with correlation \( \rho = \frac{-1}{\sqrt{m+1}} \) over the region
\[ n - d < x < 0 \]
\[ -\frac{3\sqrt{m} - d}{m+1} < y < \frac{3\sqrt{m} - d}{\sqrt{m+1}} \]
Values of \( F \) can be obtained by interpolation from tables of bivariate Normal probability

\[
L(h, k, \rho) = \frac{1}{2\pi(1-\rho^2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{1}{2(1-\rho^2)}[x^2 - 2\rho xy + y^2]} dx dy
\]

... (5.11)

For the purpose of the present study, we shall require values of \( F(m, n, d) \) for \( 1 \leq n \leq 4, \ m = 3 \) and 15 and \( 0 \leq d \leq 5.0 \) with the restriction \( n \geq d \). For these values of the parameters, \( R'' \) lies partly in the first and partly in the fourth quadrant. In this case \( F(m, n, d) \) can be determined as \( \Delta = \Delta_1 - \Delta_2 - \Delta_3 + \Delta_4 \) where \( \Delta_1 = L(h_1, k_1, \rho), \Delta_2 = L(h_2, k_1, \rho), \Delta_3 = L(h_1, k_2, \rho) \) and \( \Delta_4 = L(h_2, k_2, \rho) \) where \( h_1 = n-d, \ h_2 = 0, \ k_1 = \frac{-3\sqrt{m} - d}{\sqrt{m+1}} \) and \( k_2 = \frac{3\sqrt{m} - d}{\sqrt{m+1}} \). For the given values of \( n, m \) and \( d \), \( h_1 \) and \( h_2 \) are always positive but \( k_1 \) is always negative. \( \rho \) is negative in all the integrals \( F \). For negative \( k \), the recursion relation

\[
L(h, -k, r) = -L(h, k, -r) + \frac{1}{2} \left[ 1 - \alpha(h) \right]
\]
may be used. ... (5.12)
The other properties to be used are:

\[ L(-h, k, r) = -L(h, k, -r) + \frac{1}{2} \left[ 1 - \alpha(k) \right] \]

\[ L(-h, -k, r) = L(h, k, r) + \frac{1}{2} \left[ \alpha(k) + \alpha(h) \right] \quad \ldots (5.13) \]

and \( Pr(X < h, Y > k, r) = L(-h, k, -r) \)

where \( \alpha(x) = \int_{-x}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \)

Thus \( F(m, n, d) = L(n, k_1, r) - L(n, k_2, r) - L(d, k_1, r) + L(d, k_2, r) \)

\[ = 1 - P(n) - L(n, -k_1, r') - L(n, k_2, r) - 1 + P(d) \]

\[ + L(d, -k_1, r') + L(d, k_2, r) \text{ where } r < 0 \text{ and } r' = -r > 0 \]

\[ = [P(d) - P(n)] + [L(d, -k_1, r') + L(d, k_2, r)] \]

\[ - [L(n, -k_1, r') + L(n, k_2, r)] \quad \ldots (5.14) \]

\( F(x) \) being \( \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \Phi(x) \)

The function \( F(m, d, 2d) \) will define a region \( R \) which lies partly in the first and partly in the fourth quadrants for \( d > 0 \). For negative values of \( d \) this region will lie partly in the second and partly in the third quadrants. Hence for positive values of \( d : F(m, d, 2d) = [P(2d) - P(d)] + [L(2d, -k_1, r') + L(2d, k_2, r)] \)

\[ - [L(d, -k_1, r') + L(d, k_2, r)] \quad \ldots (5.15) \]

The integral \( F(m, 2d, \infty) \) is easier to be obtained as

\[ F(m, 2d, \infty) = L(2d, k_1, r) - L(2d, k_2, r) \]

\[ = 1 - P(2d) - [L(2d, -k_1, r') + L(2d, k_2, r)] \quad \ldots (5.16) \]

For \( F(-\infty, -m) \) the region lies partly in the second and partly in the third quadrants and use has to be made of the recursion relation
Values of $F$ can now be obtained from the following simplified expressions:

For $0 < d < \frac{n}{2}$,

$$F'' = \left[1 - P(d)\right] + \left[1 - P(d+n)\right] - \left[L(d, k_2, r') + L(n+d, k_2, r')\right]$$

For $\frac{n}{2} < d \leq n$,

$$F'' = \left[1 - P(0)\right] + \left[1 - P(d+n)\right] - \left[L(0, k_2, r') + L(n+d, k_2, r')\right]$$

For $d > n$,

$$F'' = \left[P(-k_1)+P(k_2)-1\right] + \left[1 - P(n+d)\right] - \left[L(n+d, k_2, r')\right]$$

$$- \left[L(n+d, -k_1, r) + L(n+d, k_2, r')\right]$$

...(5.18)

5.4 Preparation of Tables:

For simplicity a small sample of size 3 has been considered initially giving a correlation $\rho = -0.5$. To determine the effect of sample sizes a second sample of size 15 has also been taken. Table I gives approximate values of $k_1$ and $k_2$ for different values of $d$ corresponding to $n = 3$ and 15 along with values of $F(\mu)$. In table II are given values of $1 - \beta$ while table III presents values of $F$ and $G$. Values of $L(h, k, r)$ and $L(h, k, r')$ are available from tables prepared by the Dept. of Commerce, National Bureau of Standards, U.S.A.

For values of $k$ intermediate between tabled values linear interpolation was employed. The character of interpolation depends
on $h + k$ rather than on $h$ and $k$ separately since the dominant part of the integrand comes from the term of the exponent involving $(h + k)^2$. From schedule 3 given in the NBS tables it is found that for $P < .6$ the number of decimal places obtainable is at the least 6 for $h + k > 3.9$. In our computations the least value of $h + k$ is 1.9 and most values are larger than 3.9. Thus linear interpolation can be safely used to give five places of decimal as we have done in the tables.

5.5 Discussion:

For any deviation wider control limits ensure larger gain, but even with very wide limits more defective articles are produced for deviations beyond a certain small magnitude. It is found that the gain in fraction effective increases with $n$ upto $n = 3$ quite rapidly and rather slowly beyond $n = 3$. Practically for lower values of $d$ the gain is almost invariant especially beyond $n = 3$. However, even with very wide limits there occurs a loss in proportion effective for deviations which do not exceed $\sigma$ in magnitude, and for such small deviations $\sigma$-limits ensure gains which are definitely greater than those envisaged by tighter limits and which differ only slightly from those resulting from the use of wider limits.

Apart from the production of more defective articles such interruptions impede the rate of production and thus indirectly lead to other losses. Thus process adjustments should be made only when the evidence of assignable causes provided by control charts is confirmed by an engineering investigation. Dimensional adjustments based on control chart evidences can be recommended for deviations of magnitude $\sigma$ at the meet. $\sigma$.

As the size of the sample increases use of control charts involves greater losses. $\sigma$-limits for deviations of magnitude $\sigma$ or less.
References:


For any set of control limits gain in proportion effective is found to decrease with the magnitude of deviation $d$ over each of the intervals $0 \leq d \leq \frac{n}{2}$, $\frac{n}{2} < d \leq n$, $n < d \leq 2n$, $2n < d$, taking a jump at the beginning of each subsequent interval.
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</table>

\( \gamma \) and \( \phi \) values for \( K_1 \) and \( K_2 \).
<table>
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<th>0.4</th>
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<tbody>
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<td>12.0</td>
<td>16.0</td>
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Table 5.2: Values of $1 - P$
<table>
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<th>( \theta )</th>
<th>( \phi )</th>
<th>( p )</th>
<th>( q )</th>
<th>( \rho )</th>
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<tr>
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<td>1.0</td>
<td>1.0</td>
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Table 5.3 Values of \( F^* \) and \( G \)
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<th>$n = 2.5$</th>
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<tr>
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Table 5.3 (contd.)
<table>
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</table>

Table 5.3 (Contd.)
\[
\begin{array}{c|cccc}
 n & 0 & 1 & 2 & 3 \\
 \hline
 0 & 0.0 & 0.5 & 1.0 & 1.5 \\
 1 & 0.2 & 0.4 & 0.6 & 0.7 \\
 2 & 0.3 & 0.5 & 0.7 & 0.8 \\
 3 & 0.4 & 0.6 & 0.8 & 0.9 \\
 4 & 0.5 & 0.7 & 0.9 & 1.0 \\
 5 & 0.6 & 0.8 & 1.0 & 1.2 \\
 6 & 0.7 & 0.9 & 1.2 & 1.5 \\
 7 & 0.8 & 1.0 & 1.5 & 2.0 \\
 8 & 0.9 & 1.2 & 2.0 & 2.5 \\
 9 & 1.0 & 1.5 & 2.5 & 3.0 \\
 10 & 1.2 & 2.0 & 3.0 & 3.5 \\
 11 & 1.5 & 2.5 & 3.5 & 4.0 \\
 12 & 2.0 & 3.0 & 4.0 & 4.5 \\
 13 & 2.5 & 3.5 & 4.5 & 5.0 \\
 14 & 3.0 & 4.0 & 5.0 & 5.5 \\
 15 & 3.5 & 4.5 & 5.5 & 6.0 \\
\end{array}
\]

Table 5.3 (Contd.)