1.1 Some Problems in Process Control Methods:

Statistical Methods for controlling variations in the quality of an industrial product admit of improvements in various dimensions. The characteristic statistical tool for this purpose is a control chart. The statistical theory of control charts has mainly developed around the problem of stabilising the location (setting) and dispersion of a single measurable quality for a mass produced article in two separate representations. (The procedure is analogous to the use of two separate tests—one for central tendency and the other for dispersion of a normally distributed variable). The available theory makes use of certain assumptions not always warranted by evidences, takes recourse to certain approximations not always desirable and fails to recognise some economic aspects of the problem not always negligible.

Normality of the statistic plotted is one of the main assumptions in Control Chart methods. Although by virtue of central limit theorems one may argue that there is little difficulty in attributing normality to the distribution of sample mean, one must remember that the size of the sample frequently used in quality control work is only small and that such arguments do not apply to other measures of central tendency like the midrange and to measures of dispersion. For skewed distributions 3σ limits cannot tell us whether or not the process is in control. What is
needed is a set of exact probability limits derived from the use of a suitable curve for specification of different populations.

Another problem centres round the analysis of historical data to establish standard values. It does not follow that because a sample is unrepresentative with respect to its mean, it should be excluded for the purpose of estimating the process standard deviation. Nor does it follow necessarily that because a sample has too large a standard deviation, it should be excluded for the purpose of estimating the process mean, though perhaps its weight should be reduced. Usually if the dispersion of a particular sample is too large the sample is excluded for the recomputation of both mean standard deviation. It will be convenient for interpretation of such situations if we can develop joint control charts for the two statistics with a specified probability that either statistic will go beyond the limits assigned to it. Evidently, such joint charts are graphic representations of tests of the two-parameter simple hypothesis $H_0 (m = m_0, \sigma = \sigma_0)$ concerning a Normal population. Powers of the different usable tests in controlling various types of errors associated with various classes of alternatives have yet to be investigated.

Economic potentialities of control charts have sometimes been challenged on the grounds that the construction and use of process control charts involves additional costs in sampling and inspection of items, computation and plotting of points and investigation and correction of manufacturing troubles, apart from overhead costs for maintaining the required personnel. It seems
reasonable to incorporate such economic considerations in setting limits on a control chart, which, instead of being set rules applicable to all production processes, should depend on such factors as the rate of production or the discount on seconds apart from process parameters like the frequency and size of samples and the distribution of process level or dispersion. And this means the necessity for an economic design of control charts.

Yet another crucial problem is to decide upon the type of action which can be taken on the basis of control chart evidences. It has been empirically realised that if action has to be taken whenever a control chart goes out of limits process variability tends to be larger. Effects of such adjustments can be studied most conveniently in the control of dimensions, the action being an adjustment (often linear) in the machine for that dimension. A simple situation will arise when the adjustment equals the deviation between the process standard and the observed sample value. Consequences of such adjustments were not studied earlier.

There are various other problems including that of dependence between successive sample points. All these, however, relate to a single quality characteristic. But in modern days, characterisation of the quality of product-unit has become quite complex and has to be made in terms of several characteristics - some of which are mutually contradictory. The problem of controlling the quality of such a complex unit naturally urges the construction of a single chart for a composite characteristic which will suitably take into account the different elementary characteristics.
Thus the problems to be faced can be conceived of as problems relating to the construction and use of (i) separate control charts (for location and dispersion) for a single characteristic (ii) joint control chart for a single characteristic (iii) separate control charts for a composite characteristic and (iv) joint control chart for a composite characteristic. And solutions of these problems, in order to be usable, must have the property of simplicity.

1.2 Scope of the Present Work:

The object of this thesis (dissertation) has been to study some of these problems related to the construction, use and interpretation of variable control charts. Part I, which constitutes the bulk of the present work, is concerned with various statistical methods that can be applied for controlling severally the location and the dispersion of a single quality characteristic. It deals with the economic design of $\bar{x}$-charts, the development of asymmetrical control limits for various populations (sets of $\beta_1 - \beta_2$ values) derived from the use of the Edgeworth Series and the effects of process adjustments based on evidences from $\bar{x}$-charts. It also includes the development of modified control limits admitting the presence to tool-wear. An improvement over the existing control chart for coefficient of variation has been worked out, the current theory of group control charts has been corrected and an unconventional use of a control chart for checking the accuracy of coding in large scale data processing has been illustrated.

Part II is much smaller and deals only with the development of joint control charts for location and dispersion of a single
been given. Percentage points for the Pearsonian type III dis- 
tribution and for the Edgeworth series retaining terms up to order 
n^{-1/2} have also been obtained. Probabilities of type I error and 
differences in the areas of the two tails in using symmetrical 
\kappa\sigma-limits have been studied for different non-normal populations. 
Contours on the \((\beta_1, \beta_2)\) diagram for constant (including zero) errors 
and differences have been found. These results have been used to get 
probability limits on the \(\bar{x}\)-chart for samples from non-normal popu-
lations. Probability limits have been also derived for the sample 
S.D. in random samples from both Normal and Non-normal universes. 
In most of these situations exact probability limits are not known; 
where approximate limits exist they have been found to agree 
favourably with the corresponding values obtained in this chapter. 
Exact values of 3\sigma-limit factors on the chart for \(R\) and \(s\) and those 
for midrange (retaining terms up to \(B_4\) in the series given by Pillai, 
1950) have been obtained.

Chapter 4: In the first section difficulties involved in securing 
extact probability integrals of the sample coefficient of variation 
(\(v\)) and approximate methods available for the purpose have been 
noted. Mokey's \(\chi^2\)-approximation to the distribution of 
\[ \frac{n \frac{v^2}{1 + v^2}}{1 + v^2} \left( \frac{1}{\sqrt{v^2 + 1}} \right), \] 
\(v\) being the population coefficient of variation, 
has been found suitable for calculating upper (probability) limits 
on a \(v\)-chart as 
\[ v_1 = \sqrt{\frac{\chi_1^2}{C - \chi_1^2}} \] 
where 
\[ C = n \left( \frac{1}{\sqrt{v^2 + 1}} \right) \] 
\[ \chi_1^2 = \chi_1^2 - P \] 
Values of \(v_1\) have been obtained for \(n = 5\) 
(1) 30, \(v = .01 (.01) .30 \) and \(P = .975 \) and \( .995\), and have been 
compared with results obtainable from Balf's approximation.
characteristic again. Power properties of different sets of charts have been studied in some details. Part III is only an indication of a simple approach to the problem of analysing composite characteristics.

1.3 A Summary of the Work:

What follows is a brief summary of the work reported in various chapters of this dissertation.

Part I:

Methods for Controlling Severally the Mean and the Dispersion of a Single Quality Characteristic.

Chapter 2: Assuming a production model, an expression for the gain (in terms of fraction defective, rate of production and discount on seconds) resulting from the use of an $\bar{x}$-chart has been derived. This has been used in section I to determine the economically optimum control limits for given size ($n$) and frequency ($k$) of samples. For different ranges of the quantity $r \times k$ optimum control limits have been worked out assuming three normal distributions (with different S.D.'s) and a rectangular distribution for deviations in process mean. For $2\sigma$- or $3\sigma$- control limits optimum sample size has been found in section II to increase with $z$. A justification for this uncommon behaviour of $n$ has been provided by bringing in the concept of loss associated with each of a sequence of graded alternatives and equalising the expected loss for each.

Chapter 3: Methods of getting probability integrals and percentage points for the Edgeworth series retaining terms upto order $n^{-1}$ have
The second section deals with the problem of getting sloping control limits on the modified control chart for sample mean assuming product quality to decrease (increase) proportionately with time. The range estimate of the regression coefficient (given by Bose, 1938) has been used to define a sloping band of constant width connecting the pairs of points \( \left\{ Y_i (x) \pm F \sigma, \gamma \right\} \), where \( F \sigma = 3 \sqrt{\frac{1}{n} + \frac{2(x - \bar{x})^2}{(n-1)^2}} \) and \( F_1 = F_n = F \). Values of \( F \) for \( x, n = 1 \) (1) 10 have been tabulated.

Derivation of 3σ-limits and probability limits on the group chart for average and range using moments of order statistics given by Godwin (1949) and Hastings et al (1947) is the subject of section III. Values of appropriate control limits in situations with and without standard values given have been obtained for \( n \) and \( k \) (number of sources) varying between 2 and 10. Expressions for operating characteristics of \( \bar{X} \) and \( R \) group charts have also been derived.

Chapter 5: Effects of process adjustments by way of changing the process mean through the amount \( \{ \text{standard value} (\mu_0) - \text{sample average} (\bar{x}) \} \), when \( \bar{x} \) is out of control limits, on the proportion of effective articles produced have been studied. Assuming a shift \( \delta \sigma_{\bar{x}} \) on the \((n + \sigma)\) \( \bar{x} \)-chart and observing a sample average (\( \bar{x} \)) at a distance of \( k \sigma_{\bar{x}} \) from \( \mu_0 \), the sample space outside the control limits has been divided in four regions and the process fraction defective on making a correction of \( k \sigma_{\bar{x}} \) in \( \bar{x} \) has been obtained in terms of integrals of the Bivariate Normal density. The average fraction defective considering the various possibilities has been obtained for each amount of deviation and compared with the fraction defective if no chart were maintained.
Chapter 6: The use of a group control chart for fraction defective (limits whereon have been derived in the chapter) in controlling non-sampling errors associated with the transferring of information from schedules to worksheets using codes in large scale surveys has been illustrated. The method can be used to study intrateat variation in scholastic tests due to differences between examiners as shown by Bose (1954).

Part II:
Methods for Simultaneous Control of Location and Dispersion of a Single Quality Characteristic

Chapter 7: The problem of controlling the location and dispersion (of a single product quality) simultaneously by using joint control charts has been discussed. The problem is analogous to the test of the hypothesis $H_0 (m = m_0, \sigma = \sigma_0)$ for a Normal Population. The Joint test based on $(\bar{x}, R)$ proposed in this chapter along with the two based on $(\bar{x}, s)$ and $(t, s)$ studied by Walsch, Randi and Bhattacharyya, and the single chart for extreme values given by Howell have been studied for their operating characteristic surfaces. Detailed properties of the $(\bar{x}, R)$ and the $(\bar{x}, s)$ tests in leading to correct decisions and in controlling various types of errors associated with the classes of alternatives $H (m_0, \sigma)$, $H (m, \sigma_0)$ and $H (m, \sigma)$ have been investigated. The concept of loss has been introduced to get the average one-way and two-way errors. It has been found suitable to recommend $(\bar{x}, R)$ test generally for the class of alternatives $\sigma < \sigma_0$. 
Part III:
Methods for Controlling Composite Characteristics

Chapter 7: The problems of combining the magnitudes of different characteristics of a manufactured product into a single composite characteristic and of setting limits for sampling variations in it have been considered. Limits for the desirability function $D$ of Harrington (1965) have been given first by considering the probit transformation and then by modifying the definition of component desirability. A model for desirability has been developed from the Pearsonian type III distribution. An alternative approach of using the weighted total of normal equivalent deviates has also been indicated.

References:


