Chapter 2

Basic waveguides for surface plasmon waves using metal-dielectric interface: dielectric-metal-dielectric and metal-dielectric-metal structures
2.1 Introduction

Surface Plasmon Polaritons (SPP) are surface electromagnetic waves that propagate at the interface between a dielectric and a metal. The wave propagates along the interface between two media and remains evanescently confined in both the media in the direction perpendicular to the interface. These surface electromagnetic waves arise due to the coupling of the electromagnetic fields to oscillations of the metal's electron plasma. The single interface between a metal and dielectric (M-D) is the basic surface plasmon waveguide and the multilayer Dielectric-Metal-Dielectric (D-M-D) and its complementary structure Metal-Dielectric-Metal (M-D-M) are also considered to be the fundamental waveguides for SPP [1,2].

2.2 Metal Dielectric (M-D) single interface

The simplest geometry that sustains SPPs is a single, flat interface (figure 2.1) between a dielectric, non-absorbing half space (y>0) with positive real dielectric constant $\varepsilon_2$ and an adjacent metallic half space (y<0) described by a dispersive dielectric function $\varepsilon_1(\omega)$ introduced in chapter 1. The metallic character is manifested by the negative real part of $\varepsilon_1(\omega)$ and this condition is fulfilled at frequencies below the bulk plasma frequency $\omega_p$.

![Figure 2.1 Metal-Dielectric interface with coordinate system](image)

The propagating wave solution confined to the M-D interface can be of two types (a) TM modes and (b) TE mode (it will be shown subsequently that TE mode solution is not feasible). In case of TM type waves, because of the fact that (i) $\frac{\partial}{\partial x} = 0$ and (ii) $H_z = 0$ it can be shown that only three field components ($E_y, H_x, E_z$) will be non zero and the other
three field components \( (E_x, H_y, H_z) \) will be zero. There will be no \( x \)-dependence in the non zero fields and the field will be functions of \( y \) and \( z \) co-ordinates only. Starting from Helmholtz equation of the form \( \left[ \frac{d^2}{dy^2} + (k_0^2 \varepsilon_r - \beta^2) \right] \psi_r = 0 \), taking \( z \)-dependence as \( e^{-i\beta z} \) and assuming exponentially decaying fields on both sides of \( y=0 \) for \( H_x(y,z) \) and then deriving the expressions for \( E_y \) and \( E_z \) in both the media we can have a set of six equations for the three non zero fields in two media. The wave vector of the mode has two components in each media. \( \beta \) is the wave vector along \( Z \)-direction in both metal and dielectric for sustaining a propagating mode and \( h_{iy} \) (\( i=1, 2 \) for metal and dielectric respectively) represents the transverse wave vector in the direction perpendicular to the interface in the medium-\( i \). Imposing the continuity equations for fields we get a set of following three equations:

\[
\begin{aligned}
\frac{h_1}{h_2} &= -\frac{\varepsilon_2}{\varepsilon_1} \\
h_1^2 &= \beta^2 - k_0^2 \varepsilon_1 \\
h_2^2 &= \beta^2 - k_0^2 \varepsilon_2
\end{aligned}
\]

Combining the three equations we arrive at the central result of this section, the dispersion relation of SPPs propagating at the interface between the two half spaces:

\[
\beta = k_0 \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}}
\]

Let us now examine the possibility of TE type SPP mode excitation in the interface. For a TE mode to exist there will be three non zero field components \( (E_x, H_y, H_z) \).

Applying the similar techniques, the continuity of field components yields a condition \( h_1 + h_2 = 0 \), which is not possible because confinement to the surface requires \( \text{Re}[h_1] > 0 \) and \( \text{Re}[h_2] > 0 \). Thus we can conclude that SPP modes do not exist for TE polarization or in other words the SPP modes exist only for TM polarization [3,4]. Surface plasmon oscillations being longitudinal, never couples to the incident transverse electric (TE) fields.
A few important features of SPP excited in an isolated M-D interface can be summarized as follows:

a) The field decay exponentially on either side of the interface and the maximum value of the field occurs at the interface.

b) There is only one TM type SPP mode for which the propagation constant is obtained by equation (4).

c) Propagation constant $\beta$ is a complex number and therefore the propagation suffers loss. Extremely short propagation length due to absorption in metal is a typical characteristic of SPP mode.

d) Higher the values of $h_1$ and $h_2$, better would be the modal confinement in the transverse directions perpendicular to the interface.

e) The better the confinement, the lower is the propagation length. This characteristic trade-off between localization and loss is typical for plasmon waveguides.

f) The propagation length $L$ of travelling SPP is defined as $L = (2Im[\beta])^{-1}$, typically between 10 and 100 $\mu$m in the visible regime, depending upon the metal/dielectric configuration under consideration.

g) No propagating mode or free wave can excite SPP at the interface since $\beta > k_0$. A suitable evanescent field tail on one side of the interface can excite SPP at the interface. This aspect will be discussed in details in next chapter.

h) As an example, for the SPP mode supported at the interface of two different set of metal (dispersive permittivity $\epsilon_m$) and dielectric (permittivity $\epsilon_d$) at two different wavelengths the typical values of propagation lengths ($L$) are furnished in the following Table-1. It is noted that ‘$L$’ increases at higher value of $\lambda$. 

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Table-1

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\varepsilon_d$</th>
<th>$\varepsilon_m$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.55 $\mu$m.</td>
<td>12.25</td>
<td>-125.735 + j3.233</td>
<td>24.1853 $\mu$m.</td>
</tr>
<tr>
<td>0.6 $\mu$m.</td>
<td>2.25</td>
<td>-9.9 - j1.04</td>
<td>1.8435 $\mu$m.</td>
</tr>
</tbody>
</table>

2.3 Dielectric-Metal-Dielectric (D-M-D) double interface

SPPs in multilayer structures consisting of alternating metallic and dielectric thin films are also considered to be fundamental one dimensional wave guiding structures. In such a system, each M-D interface can sustain bound SPP modes. When the separation between adjacent interfaces is comparable to or smaller than the decay length of the interface mode, interaction between interface SPP modes results in formation of super-modes pertaining to the composite structure. Among the multilayer structures for supporting SPP modes, the two are fundamentally important (a) Dielectric-Metal-Dielectric (D-M-D) double interface and (b) Metal-Dielectric-Metal (M-D-M) double interface.

![Figure 2.2 Three layer dielectric - metal - dielectric (D-M-D) waveguide geometry](image)

Figure 2.2 Three layer dielectric - metal - dielectric (D-M-D) waveguide geometry
In this section we will consider only the three layer D-M-D structure. M-D-M structure will be considered in the next section. The modes in both the structures can be solved following the standard analytical techniques available for dielectric waveguide problems [5]. The geometry of the general asymmetric D-M-D structure is shown in figure 2.2 along with the coordinate system. The metal film of thickness \(2t\) and dispersive permittivity \(\varepsilon_m\) is sandwiched between dielectric half-spaces of permittivities \(\varepsilon_{d1}\) and \(\varepsilon_{d3}\). The structure is called symmetric if the upper and lower dielectrics are considered to be identical. The three regions in the structure are marked as regions 1, 2 and 3 as shown in figure 2.2. To find the prospective solution for TM SPP modes in such a structure, if we apply the conditions (i) \(\frac{\partial}{\partial x} = 0\) and (ii) \(H_z = 0\) it can be shown using Maxwell's equations that only three field components \((E_y, H_x, E_z)\) will be non zero and the other three field components will be zero. There will be no x-dependence in the non zero fields and the fields will be functions of \(y\) and \(z\) co-ordinates only. The general form of the field components of \(H_{xq}\) in three regions \((q = 1, 2, 3)\) can be written by inspection and the other two field components can be calculated using the following equations

\[
E_{zq} = -\frac{1}{\omega \varepsilon_0 \varepsilon_q} \frac{\partial H_{xq}}{\partial y} \quad (5)
\]

\[
E_{yq} = \frac{\beta}{\omega \varepsilon_0 \varepsilon_q} H_{xq} \quad (6)
\]

Now we can write the prospective SPP modal solutions for \(H_{xq}\) fields in the following form

\[
H_{x1} = A_1 \exp\{h_1(y + t)\} \quad (7)
\]

\[
H_{x2} = A_{21} \exp\{h_2(y - t)\} + A_{22} \exp\{-h_2(y + t)\} \quad (8)
\]

\[
H_{x3} = A_3 \exp\{-h_3(y - t)\} \quad (9)
\]

Where

\[
h_{1(3)}^2 = \beta^2 - k_0^2 \varepsilon_{d1(3)} \quad (10)
\]

\[
h_2^2 = \beta^2 - k_0^2 \varepsilon_m \quad (11)
\]

Now using equation (5) and (6) we can derive the expressions for \(E_{zq}\) and \(E_{yq}\) for \(q = 1, 2\) and \(3\). The derived expressions for \(E_{zq}\) at different regions are as follows:
\[ E_{z1} = \frac{j}{\omega_0} h_1 A_1 \exp\{h_1 (y + t)\} \]  
\[ E_{z2} = \frac{j}{\omega_0} h_2 \left[ A_{z1} \exp\{h_2 (y - t)\} - A_{z2} \exp\{-h_2 (y + t)\} \right] \]  
\[ E_{z3} = \frac{j}{\omega_0} h_3 A_3 \exp\{-h_3 (y - t)\} \]

Matching of \( E_z \) and \( H_z \) fields at \( y = \pm t \) yields the following two equations
\[ A_{z1} - A_{z2} \exp\{-2h_z t\} = -\frac{h_2}{h_2} \frac{\epsilon_m}{\epsilon_d} A_3 \]  
\[ A_{z1} + A_{z2} \exp\{-2h_z t\} = A_3 \]

Similarly, matching of \( E_z \) and \( H_z \) fields at \( y = -t \) yields the following two equations
\[ A_{z1} \exp\{-2h_z t\} - A_{z2} = \frac{h_1}{h_2} \frac{\epsilon_m}{\epsilon_d} A_1 \]  
\[ A_{z1} \exp\{-2h_z t\} + A_{z2} = A_1 \]

Using the last four equations we may derive the following dispersion equation
\[ e^{-4h_z^2 t} = \frac{\epsilon_{d1} h_2 + \epsilon_m h_1}{\epsilon_{d1} h_2 - \epsilon_m h_1} \frac{\epsilon_d h_3 + \epsilon_m h_3}{\epsilon_d h_3 - \epsilon_m h_3} \]

In case of a symmetric structure \( \epsilon_{d1} = \epsilon_{d3} = \epsilon_d \), and in that case the generalized dispersion equation can be reduced to the following two different forms
\[ \tanh(h_z t) = -\alpha \]  
\[ \coth(h_z t) = -\alpha \]

Where \( \alpha = \frac{h_3}{h_1} \frac{\epsilon_d}{\epsilon_m} \)

Dispersion equations (20) and (21) correspond to asymmetric and symmetric type TM modes respectively supported in the three layer D-M-D structure. For infinite thickness of the metal layer the dispersion relations reduce to that of a single M-D interface. A few important features of SPP excited in an isolated D-M-D interface can be summarized as follows:

(a) TM SPP mode is excited as a super-mode with field coupling between two interfaces. The predominant field is \( E_y \) and the other two fields are \( H_x \) and \( E_z \).
b) Two classes of bound modes (symmetric: $s_b$ and asymmetric: $a_b$) may exist depending upon field pattern of $E_y$. In case of a symmetric 3 layer structure ($\varepsilon_{d1} = \varepsilon_{d2}$) the modal symmetry (asymmetry) is determined by presence of electric (magnetic) wall at $Y=0$ plane.

c) There is no existence of higher order modes.

d) The symmetric bound mode has much smaller attenuation compared to that of asymmetric bound mode and symmetric bound mode is often termed a Long Range Surface Plasmon Polariton (LRSPP). The asymmetric mode is called Short Range Surface Plasmon Polariton (SRSPP).

e) In case of symmetric structure having identical dielectric constant of both top and bottom cladding dielectrics, neither the LRSPP mode nor the SRSPP mode has any cut-off thickness of metal, however if the structure is not symmetric, the LRSPP mode (and not the SRSPP mode) has a cut-off thickness of metal film below which the mode does not propagate.

f) With decreasing metal film thickness the SRSPP mode becomes slower and more lossy whereas LRSPP mode becomes faster and less lossy. However both the LRSPP and SRSPP are slow-wave modes.

g) For a symmetric three layer structure, the LRSPP and SRSPP modes become degenerate at larger thickness and $\beta$ becomes that of a single M-D interface.

As an illustrative example we have first considered a three layer symmetric D-M-D structure (Figure 2.2) at an operating wavelength $\lambda = 1.55 \mu m$, the dielectric constant of the homogeneous upper and lower claddings are identical $\varepsilon_{d1} = \varepsilon_{d3} = 12.25$ and the metal is silver having dielectric constant $\varepsilon_m = -125.735 + j3.233$ at 1.55 $\mu m$. The parameters have been chosen as in [6]. The dispersion plots of real and imaginary parts of the mode effective index with the metal layer thickness are shown in figures 2.3 and 2.4 respectively. The modal effective index is defined as $n_{eff} = \frac{k}{k_o}$ and the same is computed for asymmetric and symmetric modes supported in the D-M-D structure after solving equations (20) and (21) respectively. The typical filed distribution patterns for symmetric
and asymmetric modes in a symmetric D-M-D structure look like the plots presented in figure 2.5. A discussion on the modes in a symmetric D-M-D structure can be found in [4].

Figure 2.3 Dispersion characteristics of real part of fundamental mode effective index with metal layer thickness in a three layer symmetric D-M-D structure.
Figure 2.4 Dispersion characteristics of imaginary part of fundamental mode effective index with metal layer thickness in a three layer symmetric D-M-D structure.

Figure 2.5 Typical symmetric and asymmetric field ($H_y$) patterns in Dielectric-Metal-Dielectric SPP waveguide structure.

The dispersion characteristics of a general asymmetric D-M-D structure (Figure 2.2) is studied using equation (19) at an operating wavelength $\lambda = 0.633 \, \mu m$ having the dielectric constants of upper and lower dielectrics $\varepsilon_{d3} = 4.00$ and $\varepsilon_{d1} = 3.61$ and the metal slab is of silver having permittivity $-19 - j0.53$ at the operating wavelength. The parameters are chosen from [7]. The modes here are not truly symmetric or asymmetric in...
respect of $y=0$ plane, however the field distributions are symmetric-like or asymmetric-like and the modes are still called symmetric and asymmetric modes. The dispersion plots of real and imaginary parts of the mode effective index for an asymmetric D-M-D structure with the metal layer thickness are shown in figures 2.6 and 2.7 respectively. It may be seen that unlike symmetric D-M-D structure the symmetric and asymmetric modes are not degenerate at higher metal thickness, rather they exist as isolated SPP modes supported at upper and lower D-M interfaces.

![Dispersion characteristics of real part of fundamental mode effective index with metal layer thickness in a three layer asymmetric D-M-D structure.](image)

**Figure 2.6** Dispersion characteristics of real part of fundamental mode effective index with metal layer thickness in a three layer asymmetric D-M-D structure.
Figure 2.7 Dispersion characteristics of imaginary part of fundamental mode effective index with metal layer thickness in a three layer asymmetric D-M-D structure.

With increasing metal thickness, the asymmetric mode evolves into the single interface SPP mode supported at the metal and dielectric of higher permittivity whereas the symmetric mode evolves as that supported in the metal and dielectric of lower permittivity. A discussion on the modes in an asymmetric D-M-D structure can be found in [7].

2.4 Metal-Dielectric-Metal (M-D-M) double interface

The schematic diagram of a three layer metal-dielectric-metal waveguide is shown in figure 2.8 where the dielectric layer is sandwiched between two metallic half spaces. The analysis presented in section 2.3 can be readily applied to the M-D-M structure also taking care of the dielectric constants of the different regions.
Such a structure is often called 'Gap Plasmon Waveguide' and supports classical TE/TM metallic waveguide modes and the TM type SPP modes. The equations for transverse wave vectors in M-D-M structure take the following form

\[ h_{1(3)}^2 = \beta^2 - k_0^2 \varepsilon_{m1(3)} \]  
\[ h_2^2 = \beta^2 - k_0^2 \varepsilon_d \]  

The dispersion equation is:

\[ e^{-4h_2t} = \frac{\varepsilon_m h_2 + \varepsilon_d h_3}{\varepsilon_m h_2 - \varepsilon_d h_3} \cdot \frac{\varepsilon_m h_2 + \varepsilon_d h_3}{\varepsilon_m h_2 - \varepsilon_d h_3} \]  

In case of a symmetric structure \( \varepsilon_{m1} = \varepsilon_{m3} = \varepsilon_m \), and in that case the generalized dispersion equation can be reduced to the following two different forms

\[ \tanh(h_2t) = -\alpha \]  
\[ \coth(h_2t) = -\alpha \]  

Where \( \alpha = \frac{h_2 \varepsilon_m}{h_2 \varepsilon_d} \)

Dispersion equations (26) and (27) correspond to asymmetric and symmetric type TM modes respectively supported in the three layer M-D-M structure.
Dielectric layer thickness (in micrometers)

Figure 2.9 Dispersion characteristics of real part of fundamental mode effective index with dielectric layer thickness in a three layer M-D-M structure.

For the symmetric three layer M-D-M structure ($\epsilon_{m1} = \epsilon_{m2}$) the dispersion relations are solved for a different set of parameter values $\lambda = 0.6\mu m$, the dielectric constant of the homogeneous core dielectric $\epsilon_d = 2.25$ and the metal is having dielectric constant $\epsilon_m = -9.9 - j1.04$ at $\lambda = 0.6\mu m$. The dispersion plots of real and imaginary parts of the mode effective index with the dielectric layer thickness are shown in figures 2.9 and 2.10 respectively. The structure in 2.8 behaves as a three layer dielectric waveguide as long as $\beta < k_0\epsilon_d^{0.5}$. This condition will sustain oscillating wave in the core with exponentially decaying tails in the metallic cladding. For values $\beta > k_0\epsilon_d^{0.5}$, there will be exponential decaying/growing fields inside the core dielectric and this situation corresponds to generation two types of SPP modes depending upon the symmetry of the dominant field component. The dispersion relations for both symmetric and asymmetric SPP modes in M-D-M are solved and like D-M-D structure, here also we observe that if the thickness of
the guide is large enough, the two interfaces can support two individual SPP modes of
same $\beta$ (for a symmetric structure) and with reduced thickness the individual SPP modes
at interface interact with each other forming a super-mode – symmetric or asymmetric
type. From the dispersion plot in figure 2.10 it is seen that asymmetric wave exhibits
more losses and therefore propagates less compared to the symmetric mode. M-D-M
structure offers better confinement in the core due the metal cladding, therefore it is
especially suitable for wave guidance purpose, however for sensing applications M-D-M
structure is not suitable.

Figure 2.10 Dispersion characteristics of imaginary part of fundamental mode effective
index with dielectric layer thickness in a three layer M-D-M structure.

References


[2] Berini P 2009 Long-range Surface Plasmon Polaritons Advances in Optics and
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