Femoral condyles will now hand over the body-weight to the sheen through the knee-joint. Apart from weight bearing this joint is interposed to facilitate locomotion. It manifests flexion, extension for mobility and locking for stability.
Chapter – 4
GEOMETRY OF KNEE-JOINT IN FLEXION AND EXTENSION: LINES OF
FORCES ACTING THERE UPON

Abstract

Geometrical structure of the condyles at the knee-joint has been analyzed. The lines of forces active on femoral and tibial condyles at rest and during flexion or extension movements of the joint have been expressed by mathematical deductions.

The article includes:
(a) Composition of knee-joint along with geometrical appearance of patella, femoral and tibial condyles.
(b) Choice of axis for flexion and extension movements of the knee-joint.
(c) Geometry of the surface of articulation and relevant mathematical deductions during flexion and extension movements.
(d) The line connecting centers of the head of the femur, the knee-joint and the ankle-joint has been delineated and variations of angulations with vertical axis for males and females.
(e) Cycloidal movements of the femoral condyles due to flexion and extension of the knee.
(f) Consideration of evolute and involute of cycloid by the bone-structures on the surface of the femoral and tibial condyles for the smooth movements.
(a) Knee-joint is not a jam packed structure:

Knee is a tibiofemoral-joint. It is a large synovial joint with articular surfaces from tibia and femur. The covering capsule contains synovial fluid. Patella lies anteriorly within the quadriceps tendon. The distal aspect of femur is composed of two convex asymmetric condyles called lateral and medial condyles. The intercondylar fossa is U-shaped. The whole surface is covered with cartilages. The tibial surface has an inter-condylar prominence to fit into the groove of femur. The lateral and medial tibial condyles are concave to accommodate the femoral convex condyles. The articular concave surfaces are shallower than convex femoral condyles. The medial tibial articular surface is more concave than the lateral one. The internal structural pattern of the long bones has no immediate relevance, so that details are left out at present.

In Figure 4.1 vertical line over the center of ankle (C) enters the middle of the obturator foramen of the pelvis. Line of centers of the head of the femur (H) and (C) makes some angle with vertical line CD and the angle varies from $3^\circ$ to $6^\circ$ for male to female due to wider range of female pelvis. Axis of femoral shaft goes through OE, where O is the center of

Figure 4.1: Lines of centers joining head of femur, knee-joint, ankle.
Figure 4.2: A: The weight-bearing line indicates the line of force as it passes through knee-joint and requires radiographs extending from the hip to the ankle on a single film. Similarly, the mechanical axis must be drawn on long film. B: The anatomical axis, in contrast, can be measured on film including enough of the femoral shaft and tibial shaft to find the center of the modularly canals.

of the knee-joint and E is upper end point of the axis of the femoral shaft makes an angle $6^\circ$ to $8^\circ$ with HC as per changes according to sex mentioned above. For flexion and extension the transverse axis is XX'. This axis of rotation is perpendicular to the vertical axis CD.
Femoral neck protrudes beyond the shaft and the axis of the femoral shaft does not coincide with the axis of the leg HC. The angle between femoral shaft and CO varies from $166^0$ to $171^0$ i.e. $\angle EOC \equiv 166^0$ to $171^0$. This is known as angle of valgus. This determines the normal setting of knee-joint. Femoral condyles are at some angle to the horizontal line $XX'$. Ob divides the angle of valgus. It determines the inclination of the line of contact between femoral condyles and tibial condyles with the horizontal line $XX'$. The movement of knee due to flexion and extension occurs with respect to transverse axis $XX'$ and it acts as axis of hinge.

The angle of valgus varies according to the sex variation as well as physiological variation such as for female angle of valgus is less i.e. inclination of the line of contact between the condylar surfaces to the horizon is more.

Variations of angle of valgus:

(a) If the angle of valgus is reversed, it is called *genu varum* i.e. bandy legs or bow-leg; outward curving of the legs.

(b) If the angle of valgus is exaggerated then it is called *genu valgum* or knocked-knee i.e. deformity when knees come together and ankles are apart.

We can consider two types of joint surfaces ovoid\textsuperscript{28} or sellar\textsuperscript{29}. The curvatures of the opposing bones of the articulations are in turn either 'congruent'\textsuperscript{30} or 'incongruent' depending on its arc or curvature and the structural relationship of the two surfaces.

The congruous\textsuperscript{31} joint in bone articulation is not accepted by human body for want of

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\textsuperscript{28} Ovoid is a surface having uniformly convex or concave.

\textsuperscript{29} It is deep depression in the shape of Turkish saddle in the upper surface of the sphenoid bone.

\textsuperscript{30} Ideal articulation: The surfaces of opposing bones comprising a joint are considered to be perfectly articulated if the curves on each surfaces will be in contact at each point along the articulating surface. Motion under this type of joint is called "true congruous joint motion". This type of joints is used in mechanical instruments and for lubrication spiral channels are scrapped on both surfaces of articulation.
lubrication of synovium\textsuperscript{32}. Congruous Joint will offer obstruction in free movements due to friction under greater area whereas in incongruous joint due to degree of incongruity\textsuperscript{33} the lubricant will move through the either sides of the joint for smooth movements.

In incongruous joint the articulating surfaces are varying and are in contact with small area. So, the remainder of the surfaces is more separated from each other.

The lubricant synovial\textsuperscript{34} fluid is both adhesive and viscous, being coated by hyaluronic\textsuperscript{35} acid. The hydrodynamic lubricant from edge shaped lubricating fluid and moves with the movements of bones having equal speed and even at rest it remains as layer between the two opposing joint surfaces.

Result in this section: Articular surface of the knee-joint is loosely packed to get proper lubrication which allows smooth movements.

(b) \textit{Curves on condyles}:

The section of ovoid or sellar is a cycloid and they are of different curvatures.

The articular surface of the femoral condyles is considered to be the surface formed by the movement of double pulley of aeroplane on the run-way with a point on the

\textsuperscript{31} True congruous joint implies direct contact of articular surfaces at every point around the curvature of the surfaces. The contact will create "close packed" relationship and would ‘blind’ the joint, i.e. no lubrication would evolve as there is no scarping of spiral channel on the articular surfaces.

\textsuperscript{32} Some glary fluid is secreted by synovial membranes. The fluid contains \textit{albumin, mucin, extractives and salts}. The word synovial, is taken from Greek word meaning egg-white, was invented by \textit{Paracelsus (Philippus Aureolus Paracelsus, 1493-1541)}, Swiss physician, alchemist and chemist, writer of medical books. His real name was \textit{Theophrastus Bombastus von Hohenheim}.

\textsuperscript{33} Incongruity of femoral condylar curve to the tibial plateau curve: convexity of the femoral condyles is greater than concavity of tibial plateau. This incongruity makes the inter-articular joint space to be wider laterally than centrally. Asymmetrical joint surfaces of incongruous joint make space to flow fluid towards the open articular area whereas the joint ligaments remain taut on the closing side and slack on the opening side.

\textsuperscript{34} A mucopolysaccharide found in all tissues and in joints to serve as a viscous agent.

\textsuperscript{35} A mucopolysaccharide is found in all tissues and serving as a viscous agent in the ground substance of tissues also as a lubricant in joints.
surface of the pulley i.e. a cycloidal surface. Here [In figure 4.2] $a = h$ as the point is on the circumference. So, equation of the cycloid is $x = at - a\sin t$; $y = a - a\cos t$ where $t$ is the parameter. Here $P(x, y)$ is the point on the circle MP, called the generating circle, which traces out the cycloid and let the line OMX on which origin is at O. $\angle PCN = t$ i.e., $t$ is the angle by which the circle turns and $P$ traces out the locus as cycloid.

As $P$ started from $O$ and circle rolls from $O$ to $M = \text{arc } PM = a t$ $x = OL = OM - LM = at - PN = at - a\sin t = a(t - \sin t)$; $y = PL = NM = CM - CN = a - a\cos t = a(1 - \cos t)$ where $PL$ and $CM$ are perpendicular from $P$ and $C$ on the $x$-axis respectively. So, parametric equation of the cycloid is $x = a(t - \sin t)$, $y = a(1 - \cos t)$. Configuration of lateral / medial condyles: Here we see that lines of curvature / radii of curvature i.e. normals are terminating with multi-centers are in such a way that surface generates

Figure 4.3: DC is femur & BA is tibia. E is evolute with 10 different contact point. K is the contour of femoral condyle. P is the Pole-curve or envelop i.e. locus of the point of intersection of the straight lines BC and AD.

Cycloid was first investigated by Marin Mersenne, 1588-1648, a French mathematician, in 1593. His life and work is in the book *Informer Mersenne d'une découverte, c'était la publier par l'Europe entière* by H. Bosmans. Cycloid is a locus of a point P attached to a circle rolling on a straight line. If 'a' is the diameter of the circle and 'h' is the distance $P$ from the center of the circle and the line of movement is $x$-axis i.e. $y = 0$ than the equation to the cycloid is $x = at - h\sin t$; $y = a - h\cos t$ when $\infty < t < \infty$.

Since $s = r\theta$; where $s$ = arc length, $r$ = radius and $\theta$ = angle subtended at the centre by the arc of the circle.
an evolute of the circular lines of curvature

Evolute of a curve $C: x = f(t), y = g(t)$ is the locus of the center of curvature.

Parametric equation of the evolute is

$$x = f(t) - \frac{(f'^2 + g'^2)g'}{f'^2 \cdot g'' - f'' \cdot g'}$$
$$y = g(t) - \frac{(f'^2 + g'^2)f'}{f'^2 \cdot g'' - f'' \cdot g'}$$

where $f' = \frac{df}{dt}, g' = \frac{dg}{dt}, f'' = \frac{d^2f}{dt^2}, g'' = \frac{d^2g}{dt^2}$.

So, evolute of a cycloid is $x = a(t + \sin t), y = a(1 - \cos t)$.

Figure 4.4: Human knee with lateral femoral condyle having the cruciate linkage.

Figure 4.5: Left: CD marks the position of the femoral link in full extension. Here track of flexion axis relative to tibia. Right: Fixed femoral link CD. BA marks the position of tibial link at 140° of flexion. The track of flexion axis relative to femur.
Structure of tibial condyles is involute\(^{38}\) to the femoral condyles i.e. a cycloid.

General deduction of involute can be written as: In Figure 4.3 C is the cycloid. Let \(C_0\) is a point on C for which \(P\) lies on C: \(x = f(t), y = g(t)\). Then equation of the line\(^{39}\) is \(y - g(t) = \frac{dg}{dt}(x - f(t))\) and distance \(PQ = s\), the length of the arc \(C_0Q\) of C can be expressed as: \((x - f(t))^2 + (y - g(t))^2 = s^2\).

Solving the above equations we have equation of involute as:

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\begin{align*}
x &= f(t) - \frac{s \cdot df}{\sqrt{d^2 + dg^2}} \\
y &= g(t) - \frac{s \cdot dg}{\sqrt{d^2 + dg^2}}
\end{align*}
\]

So, involute of a cycloid is \(x = a(t - \sin t), y = a(1 - \cos t)\).

For cycloid, evolute and involute are equal curves and we know that: If the evolute itself be regarded as the original curve, a curve of which it is evolute is called an involute.

Result in this section: Curves on tibial condyles are inverse curves to the cycloid on the femoral condyles but of different curvatures.

(c) Geometrical analysis of flexion and extension movements:

In this special anatomical situation the involute of a cycloid is geometrically inverse curve of its evolute but diameter of the circle forming it is greater and varies for medial and lateral aspect of the condyles.

\(^{38}\) Involute was investigated on cycloid by Christian Huygens (Born: 14th April, 1629; died: 8th June, 1695). In the second chapter of *Horologium Oscillatorium* published in Paris 1673, he showed that cycloid is *tautochronus* (Greek word means 'exactly same'). It is a curve in which if a line \(L\) rolls as tangent, without slipping, along a fixed curve, \(C\), any fixed point \(P\) on \(L\) is an involute of \(C\). All involutes of \(C\) are parallel.

\(^{39}\) Equation of the line is of the form \(y - y_1 = m(x - x_1)\).
Surfaces on femoral and tibial condyles are surfaces formed by cycloids and there are two different surfaces on each of the femoral and tibial condyles respectively whereas tibial condyles are composed of blunt eminence in the middle to resist it from slipping or dislocation as it enters into the femoral condylar notch.

Structurally knee-joint is bicondylar joint of hinge variety buffered by cartilage.

As we know that inverse of a curve is opposite to it keeping lines of curvature inverted. As the structure of femoral medial and lateral condyles are of helicoidally structured having end surface in the form of cycloidal surface then involutes of it constitute a family of parallel inverse cycloidal surfaces.
In equations of evolute and involute we see that difference of the angles between the axes of the two cycloids = 180°.

The cycloidal\textsuperscript{40} shaped femoral condyles roll over\textsuperscript{41} tibial condyles.

Figure 4.7: Rolling of femoral condyle in flexion. Figure 4.8: Formation of cycloid on rolling in flexion.

In Figure 4.8 it has been shown to demonstrate the movement of femoral condyles is on cycloid.

Considering the point of full extension as starting point of the movements of femoral condyles, the condyles start rolling without sliding. Then on further progression of movement at the end of flexion, femoral condyles slide without rolling. The initial rolling movement is more in lateral condyle than the medial condyle.

This happens because the movements of femoral condyles over the lateral condyles cover space medially than laterally.

Figure 4.9: Sliding movement without rolling.

\textsuperscript{40} It is formed by rolling of a circle with a point on the circumference.
\textsuperscript{41} Without sliding.
Result in this section:

Geometrically it is obvious that medial condyle gets more space to play whereas lateral condyle acts as a guard and check post for any chance of slip.

Applied considerations: The geometrical ideation of flexion and extension must be properly respected in planning prosthetic correction.