On receiving the body-weight via hip-joint through the shaft of the femur, the condyles are now ready to handle the brunt.
Chapter – 3

STRUCTURE OF FEMORAL CONDYLES DISTRIBUTING WEIGHT TO LOWER PART OF THE LEG

Abstract

Bone lamellae of the femoral condyle distribute weight of the upper part of the body smoothly and rapidly. This has been analyzed mathematically.

The article includes:

a) Division of femur according to force / weight absorption.

b) Structure of parts of femur to absorb weight in short.

c) Parts of femoral condyle - its formation with geometrical correspondence and deductional correspondence.

d) The shortest way of distribution of weight due to its structure.
The femur is the longest and strongest bone of human body which facilitates the gait and transmits weight of the upper part of the body to the lower part. Femur may be effectively divided into (a) head; (b) neck; (c) intertrochanteric crest; (d) shaft and (e) condyle [Figure 1.3].

Head of the femur absorbs weight through its spherical structure due to its more than hemispherical shape and transforms it to the neck of truncated conical shape. Intertrochanteric crest resist breaking of head through neck and all the structures of the bone lamellae are in the form of circular helix. A curve traced on a surface of a circular cylinder cutting the generators at a constant angle distributes forces along the wall of the shaft to have greater efficiencies. We know from Galileo that strength of pipe is more than that of a rod with same diameter.

(a) *Lateral condylar surface transmit more force*:

At the condyle, circular helical distribution of weight turns into conical helices in lateral aspect which ultimately turns into equiangular spiral but remains in the form of circular helices in medial aspect as Archimedean Spiral.

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13 Helix was recognized by Archimedes, born at Sicily (modern Syracuse) in 287 B.C.; died at Syracuse 212 B.C., it was actually studied by his friend Conon (flourished in 250 B.C.). In the book, ON SPIRAL, he grieved for Conon, whose death, he said, was a great loss for Mathematics.

14 Galileo Galilei, born at Pisa, Italy, February 8, 1564; died at Florence, Italy, January 8, 1642.

15 Condyle: A rounded articular surface at the end of a bone. Femoral condyle is divided into two parts (a) lateral condyle (condylus lateris): The prominent expansion of bone at the lower end of the femur on the lateral aspect; (b) medial condyle (condylus medialis): The prominent expansion of bone at the lower end of the femur on the medial aspect.
The shaft of the femur is helical formation which absorbs weight with flexibility. Femoral condyles are prominences from shaft as convex appearance. It has intercondylar notch to accommodate patella which fix it from slipping. Both the medial and lateral condyles of femur are composed of spiralic grains for distribution of weight to the tibial condyles which are of concave shape.

The cylindrical helical structure of the shaft turns into conical helices at lateral condyle. Here it forms spirals. We know from the drawing of Albrecht Dürer\textsuperscript{16} that there is one-to-one correspondence between helix and spiral.

Each lamellae of the femoral lateral condyle appears as equiangular spiral or logarithmic spiral or logistique whose equation was given by René Descartes\textsuperscript{17}. A curve

\textsuperscript{16} Albrecht Dürer: born at Nürnberg, Germany, on May 21, 1471; died at Nürnberg on April 6, 1528; in his book: \textit{underweysung der messung mit dem zirckel und richtscheyi in Linien, ebnen, vnnd gantzem corpore}, Nürnberg, 1525 expressed geometrical representations.

\textsuperscript{17} In 1638 in his \textit{La Géométrie}. René Descartes: born at La Haye, Tourine, France on March 31, 1596; died: at Stockholm, Holland on February 11, 1650.
is defined as spiral that cuts radius vectors at a constant angle $\phi$ whose polar equation is $r = e^{a\theta}$ where $\alpha = \cot \phi$ as each line of grain turns into expanded conical prominence and projects as spiral. Thus a curve drawn on a right circular cone is an equiangular spiral.

Let us consider right circular cone with semi-vertical angle $\alpha$, so as to cut all the generators at the same angle $\beta$. Consider the vertex of the cone as origin and the axis as z-axis. Let $C$, be the projection of $P$, the point considered, on the axis, where $CP = r$, $OP = R$. Considering direction $OX$ as initial line we can write $(r, \theta)$ is the polar co-ordinate of $P$ on any plane perpendicular to the axis of the cone when $\angle PCX = \theta$. Let $Q$ be any point on the curve of the cone making an angle $\delta \theta$ with radius vector at $P$. Then

\[ \text{from figure 3.4} \] and \[ dr = dR \sin \alpha = dz \tan \alpha \] where \[ dR = ds \cos \beta = r \, d\theta \, \cot \beta \]

\[ ds = r \, d\theta = \text{small length of the curve. Then} \]

\[ \frac{dr}{r} = \frac{dR \sin \alpha}{r} = \frac{r \, d\theta \, \cot \beta \, \sin \alpha}{r} = \cot \beta \sin \alpha \, d\theta \]

This differential equation reduces to \[ \log r = k\theta + \log A \] by integration, where \[ k = \]
\[\cot \beta \sin \alpha\] and \(A\) is the arbitrary constant of integration. So, it is a logarithmic spiral / equiangular spiral whose equation may be expressed as \(r = Ae^{k\theta}\).

Now,
\[
\frac{dz}{ds} = \frac{dR \sin \alpha \cot \alpha}{dR \sec \beta} = \cos \alpha \cos \beta = \text{constant}
\]
shows that tangent to the curve makes a constant angle \(\gamma\) with the z-axis such that \(\cos \alpha \cos \beta = \cos \gamma\). When the radius of curvature\(^{18}\) of the curve is \(\rho\) and radius of torsion\(^{19}\) is \(\sigma\) then \(\sigma = \rho \tan \gamma\) and
\[
\frac{d^2 z}{ds^2} = \frac{d}{dR} \left( \frac{dz}{ds} \right) = \frac{d}{ds} \left( c \right) = 0 \quad \text{where} \quad c = \text{constant}.
\]

For polar form of Cartesian co-ordinates we consider \(x = r \cos \theta, y = r \sin \theta\), then
\[
\frac{dr}{ds} = \frac{dR \sin \alpha}{dR \cos \beta \sin \alpha} \quad \text{or} \quad r' = \cos \beta \sin \alpha = \text{constant}.
\]

Again
\[
r.\theta' = \frac{d\theta}{ds} = \frac{dR \tan \beta}{dR \sec \beta} = \sin \beta. \quad \text{So, from} \quad x = r \cos \theta, \quad x' = r' \cos \theta - r \sin \theta \cdot \theta' = r' \cos \theta - \sin \theta \cdot \sin \beta \quad \text{and} \quad y = r \sin \theta, \quad y' = r' \sin \theta + r \cos \theta \cdot \theta' = r' \sin \theta + \sin \beta \cos \theta;
\]
\[
x'' = -r'. \sin \theta \cdot \theta' + r' \cos \theta - \cos \theta \cdot \theta' \sin \beta = -\left( r' \sin \theta + \sin \beta \cos \theta \right) \cdot \theta'; \quad y'' = r' \cos \theta \cdot \theta' + r'. \sin \theta - \sin \beta \sin \theta \cdot \theta' = \left( r' \cos \theta - \sin \beta \sin \theta \right) \cdot \theta' \quad \text{in both cases} \quad r'' = 0 \quad \text{since} \quad r' = \text{constant. But} \quad \frac{1}{\rho^2} = \left( \frac{d^2 x}{ds^2} \right)^2 + \left( \frac{d^2 y}{ds^2} \right)^2 + \left( \frac{d^2 z}{ds^2} \right)^2 \quad \text{since} \quad \frac{d^2 z}{ds^2} = 0 \quad \text{as} \quad \frac{dz}{ds} = \text{constant.}
\]
Therefore,
\[
\frac{1}{\rho^2} = \left( x'' \right)^2 + \left( y'' \right)^2 = \left( r'' + \sin^2 \beta \right) \theta'^2 = \frac{\sin^2 \beta \left( 1 - \cos^2 \alpha \cos^2 \beta \right)}{r^2} = \frac{\sin^2 \beta \left( 1 - \cos^2 \gamma \right)}{r^2} = \frac{\sin^2 \beta \sin^2 \gamma}{r^2}
\]
So,
\[
\rho = \frac{r}{\sin \beta \sin \gamma}; \quad \sigma = \rho \tan \gamma = \frac{r}{\sin \beta \cos \gamma}. \quad \text{Then} \quad \frac{\rho}{\sigma} = \frac{\cos \gamma}{\sin \gamma} = \text{constant.}
\]

\(^{18}\)Curvature: It is the rate of change of direction of the curve with respect to the arc, or roughly speaking the curvature is the rate at which the curve curves. Radius of curvature is the inverse of curvature.

\(^{19}\)The rate of turning of the binormal is called the torsion of the curve (Binormal is the normal to the osculating plane to the curvature in the plane containing two consecutive tangents and therefore three consecutive points on the curve). Radius of torsion is the inverse of torsion.
From the above deductions we see that radius of curvature and radius of torsion are proportional i.e., curvature and torsion are proportional.

Then we can conclude that force is distributed evenly on the conical helix and it can be projected to a plane curve as spiral.

But we know that on an equiangular spiral\(^2\), force is proportional \(\frac{1}{r^3}\) and it can be proved as:

Equation of equiangular spiral \(r = a.e^{k\theta}\)

Let us take \(\frac{1}{r} = u\) then \(1 = a.u.e^{k\theta}\)

or, \(\log_e 1 = \log_e a + \log_e u + k\theta\)

Differentiating with respect to \(\theta\) we get

\[
0 = \frac{1}{u} \frac{du}{d\theta} \text{; or, } \frac{du}{d\theta} = -uk; \text{ or, } \frac{d^2u}{d\theta^2} = -k \cdot \frac{du}{d\theta} = uk^2; \text{ or, } u + \frac{d^2u}{d\theta^2} = u(1 + k^2)
\]

So, force \(P = h^2u^2\left(1 + \frac{d^2u}{d\theta^2}\right) = \frac{h^2(1+k^2)}{r^3}\); i.e., \(P \propto \frac{1}{r^3}\). Therefore the smaller radius exerts more force.

From the Figure 3.5 we see that less force is exerted on the condyle of tibia and more forces / weights are distributed laterally and that forces / weights transmitted to lower portion by the muscles and ligaments connected from the upper portions of the femur to the lower portions of the tibia. It also helps us to resist application of more force on the articular surface i.e., to resist porosis of articular surface both convex and concave appearance at the femoral and tibial condyles.

\(^2\)Equation of equiangular spiral is considered to be \(r = a.e^{k\theta}\).
Grains of medial femoral condyle appears like Archimedean Spiral \((r = A\theta)\). It possesses one-to-one correspondence with circular helix\(^{21}\). Let 'a' be the radius of the cylinder and its axis be taken as the z-axis. The plane through the axis and the point \((x, y, z)\) on the helix is inclined to zx-Plane at an angle \(\theta\) such that \(x = a\cos \theta, y = a\sin \theta, z = a\theta\tan \alpha\).

Then position vector \(r\) at the point \((x, y, z)\) can be expressed as \(r = (a\cos \theta, a\sin \theta, a\theta\tan \alpha)\) where \(a, \tan \alpha\) are constant.

From the above equation we see that projection of helix on the plane transforms to \(r = a\theta\tan \alpha = A\theta\) which is Archimedean spiral.

As it corresponds to a curve on a circular helix whose position vector \(r\) relative to a fixed origin is \(r = (a\cos \theta, a\sin \theta, a\theta\tan \alpha)\) and it is known as skew or tortuous or twisted curve.

Now tangent vector 't' by Serret-Frenet\(^{22}\) formulae \(r' = t' = \kappa n\) where \(\kappa\) = curvature and \(n = \) principal normal to unit vectors and \([r' r'' r'''] = \kappa^2\tau\) where \(\tau\) = torsion and \([r' r'' r''']\) denotes scalar triple product of vectors \(r', r'', r'''.\) Therefore, \(\tau = r' = a(-\sin \theta, \cos \theta, \tan \alpha)\). As this is a unit vector, we have \((a^2\sin^2\theta + a^2\cos^2\theta + a^2\tan^2\alpha)\theta' = 1;\) Or, \(a^2\theta' = \cos^2\alpha\) i.e., \(\theta'\) is constant as \(a\) and \(\alpha\) are constant. Again \(r'' = \kappa n = a(\cos \theta, \sin \theta, 0)\theta'\) but

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\(^{21}\) The circular helix: This curve is on the circular cylinder, cutting the perpendicular to the generators at a constant angle \(\alpha\).

\(^{22}\) Joseph Alfred Serret: born August 30, 1819; died: March 2, 1885 best known for his Cours d'algébre supérieure (1849) on the Differential Geometry. Jean Frederic Frenet: born: February 17, 1816; died June 12, 1900, both were of France.
principal normal being unit vector \( n = -(\cos \theta, \sin \theta, 0) \) and \( \kappa = a \cdot \theta^2 = \frac{\cos^2 \omega}{a} = \) constant. For torsion we consider \( r'' = a(\sin \theta, -\cos \theta, 0) \theta^2 \). Therefore, \( r'' \times r''' = \) vector cross product = \( a^2 (0, 0, 1) \theta^2 \). Hence
\[
\kappa^2 \tau = \left[ r'' r''' \right] = a \cdot \tan \alpha \cdot \theta^2; \text{or, } \tau = \frac{a^2 \cdot \tan \alpha \cdot \theta^2}{a^2 \theta^4} = \frac{\sin \alpha \cdot \cos \alpha}{a} = \text{const.}
\]

Therefore, curvature and torsion are both constant and eventually their ratio is constant. The principal normal intersects the axis of the cylinder orthogonally.

By the above mentioned helices, surfaces of lateral femoral condyle and medial femoral condyle generated in the form of helicoids.

Forces / weights along the femur to the condyle are along the geodesics. We know that geodesics are the path of shortest distance on the surface between two given points. It remains on a path of distribution of weight even after deformation. Geodesics remain geodesics on the deformed surface.

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23 Helicoids: It is a surface generated by a curve which is simultaneously rotated about a fixed point and translated in the direction of the axis with a velocity proportional to the angular velocity of rotation. The plane sections through the axis are called meridians. Considering axis of rotation as z-axis let us take \( u \) to be the perpendicular distance of a point from the axis, and \( v \) the inclination of the meridian plane through the point \( \alpha \)-plane the current co-ordinates of a point on the surface are \( x = u \cdot \cos v, y = u \cdot \sin v, z = f(u) + cv \). The parametric curves \( u = \text{constant} \) are obviously helices.

24 Geodesics on a surface may be defined as a curve whose osculating plane at each point contains the normal to the surface at the point. Osculating Plane: the plane containing two consecutive tangents and therefore three consecutive points at a point P is called osculating plane or plane of curvature.
Geodesics on such a surface\textsuperscript{25} of revolution are helices. Let us consider the surface of revolution $x = u \cos v$, $y = u \sin v$, $z = f(u)$ where $u$ and $v$ are parameters.

Thus $p = \frac{1}{u} \frac{df}{ds}$, $q = \frac{y}{u} \frac{df}{ds}$ where $s$ = arc length.

But for geodesics $p \frac{d^2 y}{ds^2} = q \frac{d^2 x}{ds^2}$. Therefore, $y \frac{d^2 x}{ds^2} - x \frac{d^2 y}{ds^2} = 0$ or, $\frac{d}{ds} \left( y \frac{dx}{ds} - \frac{dy}{ds} \right) = 0$

Hence $y \frac{dx}{ds} - x \frac{dy}{ds} = -C$ where $C$ is an arbitrary constant, but

\[
\frac{dx}{ds} = \frac{du}{ds} \cos v - u \frac{dv}{ds} \sin v, \quad \frac{dy}{ds} = \frac{du}{ds} \sin v + u \frac{dv}{ds} \cos v;
\]

So, $y \frac{dx}{ds} - x \frac{dy}{ds} = -u^2 \frac{dv}{ds} = -C; i.e. u^2 \frac{dv}{ds} = C$

Now if we consider $\psi$ as the angle at which the geodesics cuts the meridian then

$u \frac{dv}{ds} = \sin \psi$ So, $u \sin \psi = C$ \hspace{1cm} (a)

Thus, for a circular cylinder $u$ = constant. From the equation (a) we see that $\psi$ is also constant as their product is constant. Thus the geodesics which cut the generators at a constant angle are, therefore, helices and we can have numbers of geodesics / helices within the circular cylinder which turns into \textit{Archimedean spiral}. It can be shown that there are number of geodesics on a cone.

Let us suppose the points on the cone are A and B and consider $xz$-plane is passing through A. Let the semi-vertical angle of the cone be $\alpha$ and the plane $BOZ$ make an angle $\beta$ with $xz$-plane and also consider that A and B are at a distance 'a' and 'b' from the vertex.

\textsuperscript{25} Surface may be regarded as the locus of a point whose position vector $r$ is a function of two independent parameters $u, v$ and a relation $f(u, v) = 0$ represents a curve on the surface. (a) If $r$ becomes a function of only one independent parameter then locus of the point is a curve. (b) Curves on the surface will be parametric curves in which one parameter remains constant. The parameters $u, v$ constitute a system of curvilinear co-ordinates for points on the surface and the position of a point is determined by the values of $u, v$. 

43
The circular sections of the cone through A and B becomes arcs of concentric circles of radii 'a' and 'b'. Now \( \angle A_1OD_1 = \text{arc } A_1D_1/O_1A_1 = \text{arc } AD/OA = \beta \sin \alpha = \gamma \) (say).

The geodesic develops into \( A_1B_1 \) and if \( P \), any point on \( A_1B_1 \), has polar co-ordinates \( r, \psi \) referred to \( O_1A_1 \) as initial line, since \( \Delta O_1A_1P_1 + \Delta O_1P_1B_1 = \Delta O_1A_1B_1 \); or, \( ar \sin \psi + br \sin (\gamma - \psi) = ab \sin \psi. \)

But the relations between the cylindrical co-ordinates \( u, \theta \) in space and the polar co-ordinates \( r, \psi \) in the planes are \( u = r \sin \alpha, \psi = \theta \sin \alpha \), so the co-ordinates of any point of the geodesic satisfy the equation \( u \{a \sin(\theta \sin \alpha) + b \sin(\gamma - \theta \sin \alpha)\} = ab \sin \alpha \sin \gamma \). This equation represents a cylinder which intersects the cone in the geodesic.

An infinite number of geodesics can be drawn through two points A and B of a cylinder and, therefore, we have infinite number of geodesics on a cone.

So, we can say a bone lamellae consisting of infinite numbers of grains like helices on cylindrical form and transforming into conical in some aspects are geodesics and it is of least curvature and shortest distance on the surface.

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26 If any number of sheets is unwrapped from the cylinder and \( A', A'', A''' \ldots, B', B'', B''' \ldots \), are the positions of A, B on the plane so formed, the line joining any one of the points \( A', A'', A''' \ldots \) to any one of the points \( B', B'', B''' \ldots \) becomes a geodesic when sheets are wound again on the cylinder.

27 It was detected by John Baptiste Marco-Charles Meusnier De La Place, born: Tours, France on June 19, 1754; died: Mainz, Germany on June 17, 1793.
Result in this section: Spiralic structure of condyles transmit more force laterally, having smaller radius, through the muscles and ligaments connecting the femur with tibia whereas it exert less force directly on tibia through the medial condyle having greater radius. It helps better lubrication as well as stability of the articular surface between the bones.

In conclusion we can derive that weight from upper part of body transmits to the lower part is evenly distributed to maintain the stability and flexibility for a long period and it is arranged geometrically so as to be deducted mathematically.