CONCLUDING REMARKS

A survey of the different theories of entailment has been made in the previous chapters. In the course of this undertaking, it has been seen that attempts to ascertain the meaning of entailment as understood by G.E. Moore have failed. Because every attempt to give the definition and meaning of entailment leads to consequences which are more enigmatic and more problematic.

In estimating the theories of entailment, Russell's early account of entailment as identical with material implication has been our starting point. However, it is seen that his theory fails to capture the meaning of entailment in Moorean sense, i.e., as a relation which is the converse of 'follows from'. Such a definition of entailment leads to paradoxical consequences like the following:

(i) '~p ⊃ (p⊃q)' i.e., any false proposition (no matter how irrelevant it is to its implicate) implies any proposition and

(ii) 'q ⊃ (p⊃q)' i.e., any true proposition is entailed by any proposition, (no matter how irrelevant it is to its implicans).

Russell, however, was himself aware of the paradoxical consequences of material implication. In order to overcome them he formulated the notion of formal implication. But it has been seen that this theory, too, is not free from paradoxes and so is unacceptable.

Thus, Russell's attempts to give a definition of entailment in terms of material implication and later in terms of formal implication fall short. In identifying entailment with material implication Russell ignores an important element in understanding the nature of entailment, namely, the element of necessity. When he defines entailment in terms of formal implication, Russell introduces the notion of necessity but fails to give an adequate account of it.

Lewis, in order to overcome these paradoxes, introduces the notion of strict implication and identifies entailment with it. However, Lewis's theory, too, is plagued by similar paradoxes.

For, from such a definition, it follows at once that:

(i)   a necessary proposition is entailed by any proposition and

(ii)   a self-contradictory proposition (i.e. a proposition that contains a contradiction or a proposition that cannot possibly be true) entails any proposition.

Strawson's definitions of entailment- both earlier and revised formulations- have been found to be unsatisfactory.

Some logicians, such as the Relevance logicians, notably Anderson and Belnap uphold the view that for a proper explanation of the nature of entailment what is required is not only the element of necessity but also the condition of relevance. They argue that in any logical implication worth having the name, the antecedent

should be relevant to the consequent. There should be some meaning connection between the antecedent and the consequent. The question, however, arises as to whether relevance itself is relevant to and a necessary condition for entailment and validity. Many logicians think that this condition is not necessary for capturing the real nature of entailment. Relevance as a condition for the analysis of entailment has been severely criticized and it has been seen that relevance theory is not tenable. Some of the objections against this theory are as follows:

To begin with, entailment is an objective concept, whereas, the notion of relevance is laden with subjective elements which are relative to one's psychological conditions. Hence relevance cannot be a necessary condition for entailment. Moreover, the word 'relevance' itself is extremely ambiguous and vague, it is in need of a clear explanation. Furthermore, relevance is also subject to and dependent upon the subject's knowledge, skill and experience.

The more serious problem with Anderson and Belnap's theory of relevance is that it does away with the transitivity of entailment, which the classical account preserves very well. In case of entailment it can be held that if P entails Q and Q entails R, then P entails R. The schematical representation of the same is as follows:

$\sim \Diamond (P \wedge \sim Q) \wedge \sim \Diamond (Q \wedge \sim R)$ entails $\sim \Diamond (P \wedge \sim R)$

However, in case of relevance transitivity fails. Let us consider the following argument:

P∧Q is relevant to Q∧R

Q∧R is relevant to R∧S

∴ P∧Q is relevant to R∧S

The aforementioned argument is not a valid one. Here, though Q is common to both P∧Q and Q∧R, and R is common to both Q∧R and R∧S, thereby making the premises true, but nothing is common between P∧Q and R∧S, therefore the conclusion is false. It is common place that the relation of entailment is transitive. But it seems to us that when the condition of relevance is considered, entailment loses its transitivity. Thus, it is clear that relevance fails to preserve the transitivity of entailment, which is a time-honoured principle of classical logic. So, relevance is not indispensable for entailment.

Moreover, entailment is a property of propositions alone, relevance, on the other hand, may be a property of propositions, things, and events as well. Thus, it can only be said in case of a proposition that one entails the other, but this does not hold in case of anything other than propositions. On the contrary, relevance, in addition to propositions, being also the property of things and events, can be said to hold

between the latter too. Thus, the following statements of relevance are intelligible in a like manner.

(i)   Studying hard is relevant to one's good results.

(ii)   Treatment is relevant to heal a wound.

(iii)   Good health is relevant to one's well being.

Furthermore, relevance is matter of degree, whereas, the question of degree doesn't arise in case of entailment. Either P entails Q or it does not. It cannot be said that P partially entails Q. Thus, it would be nonsensical to say that a propositions P entails Q more than R. However, if one says that the number of years of experience in a particular field is more relevant to a person's getting a job in that field in any company than that of his educational qualification, then his statement is intelligible.

If relevance is considered to be a necessary condition of entailment then it has to be agreed that the degree of entailment varies with the variation of the degree of relevance. But it seems absurd to speak of degree of entailment. So it appears that this intensional condition of relevance need not be brought in the consideration of entailment.

Neil Tennant, a contemporary logician, offers intuitionistic relevant logic by overcoming some of the defects of Anderson and Benlap's theory of entailment. Even though Tennant's view seems to be better that the other views, however, accepting his theory leads to the sacrifice of a fundamental principle of classical logic, namely, the principle of unrestricted transitivity. He makes room for only restricted transitivity.

The question naturally arises at this juncture: Is it not possible to give any definitional meaning of entailment? Is it impossible to explicate the meaning of entailment in anyway? It may be said here that the search for an adequate definition of entailment is on; attempts are still being made to make the notion of entailment clear. Whatever account about the nature of entailment be given, none can do away with the classical account. By classical account in this context is meant Lewis' theory of strict implication. The classical account remains as the foundational principle for all such accounts, although admittedly it is not without flaws.

So all that can be said about the meaning of entailment is that whenever a proposition p entails another proposition q, it means that it is impossible that p is true and q is false. Again, to say that 'it is impossible that p is true and q is false' is to say that 'p→q' is a tautology. And this is exactly what the classical account states. The classical account has stood the test of time and successfully serves in logic, mathematics etc. It is good enough and so there is no reason to do away with it.

The classical account in spite of its shortcomings is tolerably satisfactory. There is scope for working upon the classical account, without however sacrificing its fundamental principles.

Proposal for a New Definition of Entailment

After reflecting on Lewis' definition of entailment in terms of strict implication, it appears to me that a slight modification of the classical notion will help us avert one of the paradoxes of implication, namely, 'an impossible proposition

implies any proposition'. Adding the clause 'either p is possible or q is necessary' to Lewis' definition of entailment helps us solve one of the paradoxes of strict implication. So, we arrive at a new definition of entailment. 'p entails q' means that "'it is impossible that p is true and q is false' and 'either p is possible or q is necessary'". This can be symbolized as follows:

p ⩾ q = ~ ◊(p & ~q) & (◊p v □q)

(Where ⩾ stands for entails; ~ for negation; ◊ for possibility; & for conjunction and □ for necessity)

Defining entailment in this way avoids the paradoxes. Let us consider the following example: 'If the earth is round and it is not the case that the earth is round then it will rain today'. Under Lewis' definition of implication, it turns out to be true, for the antecedent 'the earth is round and it is not the case that the earth is round' is impossible.

However, according to my solution this paradoxical implication turns out to be false. It is false simply on the ground that it fails to fulfill the second conjunct of the above definition of entailment – namely (◊p v □q), i.e. either p is possible or q is necessary. For in the aforementioned example, p, (i.e. the earth is round and it is not the case that the earth is round) is impossible, so it cannot be possible, again, q (i.e. it will rain today), is only possible and not necessary, thus the condition that (◊pv□q), i.e. either p is possible or q is necessary, is not being satisfied. Thus, the above paradox of strict implication is averted with the help of the proposed solution.

However, this definition cannot solve the second paradox, namely that 'a necessary proposition is implied by any proposition. Even though this solution is not full proof, it, nevertheless, opens the scope for further research to tackle the paradoxes of strict implication.