CHAPTER FOUR

LEWIS' THEORY OF ENTAILMENT AS STRICT IMPLICATION

4.1 Foreword

It is clear from the discussions in the previous chapters that Russell's account of entailment in terms of material implication and later in terms of formal implication does not prove to be a satisfactory one. In identifying entailment with material implication Russell ignores an important element required for the nature of entailment, namely the element of necessity. Later, in defining entailment in terms of formal implication, even though Russell introduces the notion of necessity, he fails to give an adequate account of it.

Clarance Irving Lewis being dissatisfied with Russell’s analysis of entailment suggests a further analysis of the same by introducing the concept of modal logic.

He formulates the notion of strict implication and identifies it with entailment. He defines ‘strict implication’ thus: ‘p strictly implies q’ df ‘p ⥽ q’, [where ‘⥽’ (which is called ‘fish hook’) is the symbol for strict implication].

For a better understanding of Lewis’ definition of entailment in terms of strict implication let us first analyse the notions of necessity, possibility, impossibility etc.

4.2 Analysis of the Notions of Necessity, Possibility and Impossibility

Lewis began his system of modal logic by augmenting the propositional calculus with two unary operators, □(“box”) and ◊(“diamond”), the former denoting “necessity” and the latter “possibility”. These are sometimes written as N and M, meaning necessarily and possibly, respectively. Thus □P stands for “necessarily *p*”, likewise, ◊P signifies “possibly *p*”.

4.2.1 Interdefinibility of the Notions of Necessity And Possibility

Before we proceed further it would not be out of context to mention that each of these two operators is definable in terms of the other in the following manner.

□P (*p* is necessary) is equivalent to ~◊~P (“not possible that not-*p*”)

◊P (*p* is possible) is equivalent to ~□~P (“not necessary that not-*p*”) i.e.

$\Box P \leftrightarrow \sim\Diamond\sim P$

$\Diamond P \leftrightarrow \sim\Box\sim P$

To explain: If P = “it will snow in Darjeeling today” then the above formulae would mean that “it is *necessary* that it will snow in Darjeeling today if and only if it is *not possible* that it will *not* snow in Darjeeling today”; and “it is *possible* that it will snow in Darjeeling today if and only if it is *not necessary* that it will *not* snow in Darjeeling today”.

Thus $\Box$(“necessity”) and $\Diamond$(“possibility”) form a dual pair of operators which are interdefinable.

Let us now see what is meant by these terms.

4.2.2 Necessity

Necessary propositions or necessarily true propositions are propositions which are *true in all possible worlds*, for example: “2 + 2 = 4”; “all bachelors are unmarried” etc. Each of these is necessary for it holds in all the possible worlds. Whereas a proposition, like the grass is green, will be true if it holds in this actual (therefore, possible) world, but it will be necessary only when it is true in all the possible worlds. Further, to say that something is necessary means that it *must* be the case. Again, it also means that its negation leads to contradiction. Lewis in his

Symbolic Logic holds that to say that “p is necessary” is to say, “p is implied by its own denial”[1] or “The denial of p is not self-consistent”,[2] which according to Lewis also means that “the truth of p can be deduced from its own denial”.[3]

It will be worth explaining the notations used by Lewis first. Lewis states that ◊p stands for “p is self-consistent”[4] or “p does not imply its own negation”. According to him, when, ◊p is read as “p is possible”, then ~(◊p) can be read as “It is false that p is possible” or “p is impossible”; ◊(~p) can be read as “It is possible that p be false” or “p is not necessarily true”; whereas ~[◊(~p)] will be “It is impossible that p be false” or “p is necessarily true”. Again, (p o p) can be read as p is self-consistent. For convenience Lewis makes use of these expressions without parentheses thus:

~ ◊ p, ◊ ~ p and ~ ◊ ~ p.

Using the above notations, the notion of necessity can be shown in symbols thus:

18.14 ~◊~p · = · ~ p ⥽ p · = · ~ (~p o ~p) [5]

(p is necessary =p is implied by its own denial= The denial of p is not self-consistent)

Lewis also represents it as:

18.14 ~◊~p · = · ~ (~p o ~p) · = · ~ p ⥽ p [6]

[18.13] ~◊~p · = · ~ (~p o ~p) · = · ~[~(~ p ⥽ p)] [7]

(p is necessary= The denial of p is not self-consistent = the truth of p can be deduced from its own denial)

It has already been stated above that the two operators, viz, necessity and possibility are interdefinable; so necessity can also be defined in terms of possibility. Thus ‘p is necessarily true’ is equivalent to ‘what cannot possibly not be the case’.

This can be symbolically represented as follows.

□p is equivalent to ~◊~p

[*A Special Note on the Use of Dot Notation*

Lewis, in *Symbolic Logic*, uses the technique of dot notation, in place of brackets. Dot notation was first used by Bertrand Russell in *Principia Mathematica*. Let us explain dot notation, with the help of the following example.

(p v p') ⊃ (q v q'),

Here the use of brackets, or some scheme of punctuation equivalent to it is required.

The above expression can be written as follows by removing the two extreme brackets.

p v p') ⊃ (qv q'.

Now the remaining brackets can be replaced by dots:

p v p'. ⊃.q v q'.

One thing should be noted here that the brackets being asymmetrical, indicate the direction in which they operate, however, the dots are ambiguous in this context. So the convention must be introduced that a dot is always understood as operating away from the connective or other symbol next to which it occurs.

Let us now consider an expression which has two types of brackets, e.g., wider and narrower. E.g.,

$$[(r \vee r') \supset (s \vee s')] \vee [(t \vee t') \supset (u \vee u')],$$

This can be written as the following:

$$r \vee r') \supset (s \vee s')] \vee [(t \vee t') \supset (u \vee u';$$

Again, following the convention that a square bracket is to be considered to be stronger than a round bracket, in the sense that the former can have the latter within its scope, but not vice versa, then we have the following:

$$r \vee r') \supset (s \vee s']\vee [t \vee t') : (u \vee u'.$$

Then, square brackets can be replaced by double dots and round brackets by single dots thus:

$$r \vee r'. \supset. s \vee s': \vee: t \vee t'. \supset. u \vee u'.$$

The scope of a single dot can be considered to be closed by another single dot, or by two dots, but, the scope of two dots cannot be closed by a single one.

For example:

$$(p \vee q) \supset (q \vee p) \supset (p \vee s),$$

can be written as:

p v q . ⊃ . q vp. ⊃ . p v s,

or

p. v.q: ⊃: q. v. p : ⊃ :p. v. s.

These conventions can be applied to replace any number of brackets by dots.

***End of Special Note*]**

4.2.3 Impossibility

Impossible propositions are those propositions which are *true in no possible world. T*o say that a proposition is impossible is to say that it is necessarily false i.e. it *could not* have been the case**.** So an impossible proposition is that which can never be the case or that which is self-contradictory. For example: “Peter and Smith are older to each other”.

According to Lewis, to say that “p is impossible” is to say that “p implies its own negation” or “p is not self-consistent”.

In symbols:

18.12 ~◊p · = · ~(pop) · = · p ⥽ ~p [8]

[18.1] ~◊p · = · ~(pop) · =· ~[~(p ⥽ ~p)] [9]

(p is impossible=p is not self-consistent=from p its own negation can be deduced).

Lewis goes on to represent it also as:

$$8.12 \; \sim\Diamond\sim p \cdot = \cdot p \prec \sim p \cdot = \cdot \sim (p \circ p)$$ [10]

(p is impossible=p implies its own negation=p is not self-consistent).

Lewis upholds the view that necessary truths coincide with the set of tautologies, or truths that can be certified by only logic; impossible propositions, on the other hand, coincide with the set of those propositions which deny some tautology. Lewis says clearly:

> A necessarily true proposition – e.g.- "I am", as conceived by Descartes is one whose denial strictly implies it. …A true proposition is one which is materially implied by its own denial. This point of comparison throws some light upon the two relations. The negative of a necessary proposition is impossible or absurd. …A false proposition is one which materially implies its own negation.[11]

4.2.4 Possibility

Possible propositions are those propositions which are *true* in some possible world or *in at least one possible world* (for example: "The grass is green"). That is, when something is not necessarily false, when it *could* have been the case, it is said to be possible. It also means that it can be asserted without being involved in contradiction. Lewis holds that if the modal function ◊p (i.e. p is possible) had not been taken as a primitive idea, then it could be defined as p o p, i.e. p is consistent with itself or as ~(p⥽ ~p), i.e. p does not imply its own negation.[12]

Lewis goes on to prove it as follows:

18.1 ◊p · = · p o p · = · ~ (p ⥽ ~p)[13]

[12.3,12.7] ◊p · = · ~(~ ◊ p) · = · ~[~ ◊(p p)] · = · ~[~ ◊{ p ~(~p)}]

[11.02,17.01] ~[~ ◊{ p~(~ p)}] · = · ~ (p ⥽ ~p) · = · pop

(p is possible= p is self-consistent=p does not imply its own negation).

Lewis urges that the terms ‘possible’, ‘impossible’ and ‘necessary’ are ambiguously used in ordinary discourse. ◊p, for instance, has a *wide* meaning of ‘possibility’, i.e., that which is logically conceivable or that which has the absence of self-contradiction. And the corresponding meanings of ~◊p and ~◊~p are the *narrow* meaning of ‘impossibility’ and ‘necessity’, where the meaning of ~◊p can be “p is logically inconceivable”; and ~◊~p can mean “It is not logically conceivable that p should be false”.

Another meaning of ‘possible’, ‘impossible’ and ‘necessary’ that is more frequently used in common parlance, depends on the relation that the proposition or thing under consideration has to state of affairs, such as to any given data, or to our knowledge. ‘Possible’ in this sense, can mean that which is ‘consistent with the data’ or ‘with everything known’; again, ‘impossible’ stands for that which is ‘not consistent with the data, or with the known', whereas, 'necessary' can mean that which is 'implied by what is given or known'.

4.2.5 True, False and Contingent Propositions

It will not be out of context, however, to mention the meanings of true, false and contingent propositions here. Contingent propositions are those propositions which are true in some possible worlds and false in some. In other words, a proposition can be said to be contingent, if it is neither necessarily true, nor necessarily false. For example, "It is now raining in Kolkata" is *contingently true*, because as a matter of fact it is indeed raining in Kolkata now as I write this paragraph; and "It is now snowing in Kolkata" is *contingently false* because, as a matter of fact it is not snowing in Kolkata now, indeed Kolkata's climate being such that it never snows here. Again, true propositions are those propositions which are true in the actual world (for example: "the grass is green", Plato was a Greek Philosopher); while, false propositions are those propositions which are false in the actual world, (for example: Plato was a king).

Having analysed these concepts, let us now turn to Lewis' definition of entailment as strict implication.

4.3 Definition of Entailment as Strict Implication

4.3.1 The Nature of Strict Implication

Lewis defines entailment in terms of strict implication. He introduces a new symbol '⥽' to stand for "strict implication". This symbol is called a fish-hook. He goes on to symbolize 'p strictly implies q' as p ⥽ q. It means that it is not possible that p is true and q is false. This can be symbolically represented as:

$p \prec q \equiv \sim \Diamond (p \,\&\, \sim q)$

Lewis urges that strict implication is the necessity of material implication. To elaborate: One of the conditions for implication is that, in case p is true and q false, 'p implies q' must not hold. Thus in order to express "p implies q is a tautology", what is required is a relation R such that pRq means '"p true and q false", is a logically impossible combination,' or "It is necessarily true – true under all conceivable circumstances – that it is not the case that p is true and q false". That is, to assert 'p strictly implies q' is to assert the *impossibility* of the conjunction of 'p and ~q'. In other words, the statement "p is true and q is false" is inconsistent. This is exactly the meaning of strict implication. The relation p⥽ q holds whenever any truth – implication, 'p implies q' expresses a tautology, (i.e. is necessarily true). When p implies q is true but does not express a tautology (i.e. is not necessary), p⥽ q does not hold. According to Lewis, p⥽ q is equivalent to that relation which holds when q is deducible from p, and does *not* hold when q is *not* deducible from p. p⥽ q holds when it is impossible that p is true and q is false. It is worth mentioning here, that in *Symbolic Logic* Lewis symbolizes negation as ~, where ~p can be read as ‘not –p’, possibility as ◊, where ◊p can b read as ‘p is possible’, logical product as pq, or p·q, which can be read as ‘p and q’. Thus in Lewis’s own words: “The relation of strict implication can be defined in terms of negation, possibility and product: 11.02 p⥽ q · = · ~ ◊ (p~q). Thus “p implies q” or “p strictly implies q” is to mean, “It is false that it is possible that p should be true and q false” or “The statement ‘p is true and q false’ is not self-consistent.” When q is deducible from p, to say “p is true and q is false” is to assert, implicitly a contradiction”.[14]

4.3.2 Lewis’ Definition of Strict Implication in Terms of Necessity

As already mentioned in section 4.1.1, the two operators, viz, necessity and possibility are interdefinable; so necessity can also be defined in terms of possibility. ‘p is necessarily true’ is equivalent to ‘that which cannot possibly not be the case’. This can be symbolically represented as follows.

□p = ~◊~p

By substituting (p → q) for p we can obtain the following:

□ (p → q) = ~◊~ (p→q)

Again, by implication, double negation and De Morgan we can proceed to the following:

$\Box (p \rightarrow q) = \sim\Diamond (p \,\&\, \sim q)$.

Therefore, by definition of strict implication we can have:

$\Box (p \rightarrow q) = p \prec q$.

Thus, $p \prec q$ can be defined as the necessity of $p \rightarrow q$; i.e.:

$p \prec q =_{df} \Box (p \rightarrow q)$.

It can be seen that strict implication holds when material implication turns out to be necessary. As Lewis puts it:

> The strict implication, $p \prec q$ means, "It is impossible that *p* be true and *q* false"; or "p is inconsistent with the denial of q"; Similarly $\Phi x \prec \psi x$ means "It is impossible that Φx be true and ψx false", or "the assertion of Φx is inconsistent with the denial of ψx".[15]

Thus, in case of any two propositions p and q, where $p \supset q$ means that p materially implies q, there $\Box (p \supset q)$ means that p strictly implies q.

4.3.3 How Lewis' Definition of Entailment as Strict Implication Averts the Paradoxes of Material Implication

By defining entailment in terms of strict implication, Lewis was able to avoid the paradoxes of material implication. Let us consider the following e.g.:

(1) If the grass is blue, then India won the cricket World Cup.

Clearly (1) is false, most would say, because the colour of the grass has nothing to do with India's winning or, for that matter, losing the World Cup. (1) can be represented as

(2) The grass is blue ⊃ India won the cricket World Cup.

(2) Is true, simply because the antecedent is false. Hence, (2) is not an adequate translation of (1). Let us represent (1) as follows:

(3) □ (The grass is blue ⊃ India won the cricket World Cup.)

(3) Says (roughly) that in every possible world such that the grass is blue, India won the cricket World Cup. Since one can easily imagine a world with blue grass and India not winning the World Cup, so (3) is false. Hence, it is necessary that (The grass is blue ⊃ India won the cricket World Cup), does not hold. Thus, formulating (1) in terms of (3) seems to avoid the paradoxes of material implication.

Lewis' strict implication is a stronger relation than material implication. The latter only says that it *is not* the case that p is true and q is false, but strict implication states that it *cannot* be the case that p is true and q is false. Thus, it is clear that a false antecedent cannot by itself make a strict implication true. Because, for strict implication to hold, the material implication, ‘The grass is blue ⊃ India won the cricket World Cup’ has to be true in every possible world. While the falsity of the antecedent p, i.e. “it is not the case that ‘The grass is blue’”, makes p ⊃ q i.e. ‘The grass is blue ⊃ India won the cricket World Cup’ true in the actual world, but it doesn’t give us any information about the truth of the material implication in other worlds. It is always possible that in some world the grass being blue and India’s not winning the cricket World Cup holds. Thus the material implication is not necessarily true. So strict implication, i.e. □ (The grass is blue ⊃ India won the cricket World Cup.), or it is necessary that ‘The grass is blue ⊃ India won the cricket World Cup’, doesn’t hold. Thus, following the strict implication account, the conditional would be false. For the same reason, strict implication doesn’t turn out to be true by virtue of a true consequent either. Thus, Lewis’ notion of strict implication is able to avoid the paradoxes of material implication.

4.3.4 Lewis’ View that Strict Implication is an Appropriate Model for Entailment and Inference

Lewis’ concern lies not only about the paradoxes which material implication had brought along with it, he also seems dubious as to whether material implication at all represented the notion of entailment and thereby that of inference. Lewis in his *Survey of Symbolic Logic* points out:

> *Inference* depends upon meaning, logical import, *intension*, a⊂b is a relation purely of extension. Is this material implication, a⊂b, a relation which can validly represent the logical nexus of proof and demonstration?[16]

According to Lewis strict implication is a more appropriate model of actual inference. He claims that strict implication is the relation which justifies inference of a conclusion from the premises in case of a deductive argument. Indeed, by a deductively valid argument is meant that it is not possible that all the premises be true and the conclusion false; and this is exactly what strict implication states, i.e. it is impossible that p (the antecedent) be true and q(the consequent) be false.

Lewis regards strict implication to be an intensional notion unlike that of material implication. He considers material implication to be useless as a model of real inference, since it leads to the paradox, namely a false proposition implies every proposition. He urges that what we want to know is what would follow from a true proposition and, according to Lewis, this is tantamount to saying that it is strict implication which forms the real basis of inference, in which ‘p strictly implies q’ stands for ‘the truth of p is inconsistent with the falsity of q.’

Lewis points out one more important property of strict implication, which is different from that of material and formal implication. When "‘x is a man’ strictly implies ‘x is a mortal’", then even "‘Socrates is a man’ strictly implies ‘Socrates is a mortal’, and the converse also holds. He urges that propositions strictly imply one another when and only when propositional functions corresponding to them also imply each other. Lewis thinks, "'Socrates is a man' implies 'Socrates is a mortal'" is the kind of relation on which all inference depends.[17] He urges that strict implication is the symbolic representative of any inference which holds good, irrespective of whether its terms are propositions or propositional functions.

Though the notion of strict implication was introduced in order avoid the paradoxes of material implication, which it has been able to avert successfully, but this very notion itself is plagued by similar problems. We turn now to these in the following section.

4.4 Evaluation of Lewis' Theory of Strict Implication

4.4.1 Nature of the Paradoxes of Strict Implication

As already mentioned above, even though Lewis succeeded in providing an implication relation stronger than material implication, but, ironically, strict implication too has its own set of paradoxes, viz. any necessary proposition is strictly implied by any proposition and any impossible proposition strictly implies any proposition.

In symbols-

$$\sim\Diamond\sim p \rightarrow (q \prec p)$$

$$\sim\Diamond p \rightarrow (p \prec q)$$

The above paradoxes can be illustrated with the help of the following examples:

“2+2=4” being a necessary proposition, is strictly implied by any proposition whatsoever – be it “All men are mortal”, or “Socrates was a doctor”; an impossible proposition like “Today is Monday and today is not Monday”, on the other hand, strictly implies any other proposition whatsoever- from “The sky is blue” to “Plato was a Scientist”. However, this seems counterintuitive and even non-sensical and absurd that 2+2 being equal to 4 has anything at all to do with Socrates’ being a doctor, or for that matter, with all men being mortal; it is equally absurd that today’s being Monday and not being Monday has anything to do with the sky’s being blue or even with Plato’s being a scientist.

4.4.2 Lewis’ Defense of the Paradoxes of Strict Implication

Even though, these paradoxes seem counterintuitive and absurd, however slight amount of thinking makes it clear they are not paradoxical when taken as a truth about strict implication. The reason being that strict implication is what Lewis himself defines it to be. From Lewis’ definition of ‘strict implication’, it follows that the so called paradoxical statements i.e. ‘an impossible proposition strictly implies every proposition’ and ‘a necessary proposition is strictly implied by every proposition’ are necessary truths of logic. To explain: According to Lewis’s account of impossibility, to say that p is impossible is to say that it is a contradiction; then p being a contradiction, p.~q itself will contain the contradiction which p already is, irrespective of whatever q might be. Again, on similar lines, it can be said that if q is necessary then the denial of q, i.e., ~q will be impossible; ~q being impossible, will be a contradiction. From this it follows that, p.~q will contain the contradiction which ~q itself is, irrespective of whatever p might be. So, it can be seen that when either p is impossible or q is necessary, the conjunction p.~q contains a contradiction, and consequently, p strictly implies q holds. However, it is argued that even though they may be true of strict implication, they are not only paradoxical but also false of entailment.

Lewis himself was aware of the paradoxes which arose as a result of identifying strict implication with entailment; however, he did not regard them as paradoxes at all except in the sense of being unfamiliar and strange. Lewis comments:

> Unlike the corresponding paradoxes of material implication, these paradoxes of strict implication are inescapable consequences of logical principles which are in everyday use. They are paradoxical only in the sense of being commonly overlooked, because, we seldom draw inferences from a self-contradictory proposition.[18]

Lewis believes that these are not paradoxical in nature. These, according to Lewis, are consequences of the very definition of entailment itself. For, entailment is the converse of deducibility, i.e. to say that p entails q is to say that q is deducible from p. Thus, in case of the so called paradoxes of strict implication i.e. ‘an impossible proposition strictly implies any proposition’, and ‘a necessary proposition is strictly implied by any proposition', any proposition can be deduced from an impossible proposition, again a necessary proposition itself can be deduced from any proposition whatsoever: This can be demonstrated as follows:

1.	**p. ~ p**	**[impossible proposition]**
2.	**p**	**1. Simp.**
3.	**p ∨ q**	**2. add.**
4.	**~p**	**1. com, simp.**
5.	**q**	**3, 4, D.S.**

That is, an impossible proposition implies any proposition.

Again:

1.	p	[arbitrary proposition]
2.	p ∨ ~p	1, add

That is, a proposition, which is necessarily true, is implied by any proposition.

This simply shows, according to Lewis, that these paradoxes are truths about entailment (as there is a valid deduction of a necessary truth from any proposition and that of an arbitrary proposition from an impossible one).

Lewis equates 'p strictly implies q' with 'q logically follows from p', and tries to show that the paradoxes do not constitute an obstacle to this identification. In fact, Lewis has proved[19] these two theorems in his system in the following way:

19.74 ~◊p. ⥽ . p⥽ q [20]

[19.1: $^{\sim q}/_{q}$] ~ (p o p) ⥽ ~(p o~q) (1)

[18.12, 17.12] (1) = Q.E.D.

That is, a self-contradictory or impossible proposition implies any proposition.

19.75 $\sim\Diamond\sim p \cdot \prec \cdot q \prec p$[21]

[19.74: ${}^{\sim p}/_{p}$; $\sim{}^{q}/_{q}$] $\sim\Diamond\sim p \cdot \prec \cdot \sim p \prec \sim q$ (1)

[12.44] (1) = Q.E.D.

That is, a necessarily true proposition, is implied by any proposition whatsoever.

Lewis argues that the so called paradoxical theorems do not create any problem with equating strict implication $p \prec q$ with “q is deducible from p”. He remarks:

> It is not, therefore, a legitimate objection to strict implication, or to any other implication –relation, that if fails to coincide with logistic deducibility in Logic itself. The only legitimate demand which can be made is that a relation shall hold between a premise and a conclusion whenever, *all valid modes of deduction being allowed*, that conclusion can be derived from that premise; and that it shall fail to hold when such deductive derivation is not possible. The relation of strict implication satisfies this requirement: no relation of truth-implication does. In this sense, the system of Strict Implication constitutes an

adequate canon (for unanalyzed propositions) of deductive inference".[22]

According to Lewis, "q is deducible from p" can be said to hold only when the conjunction 'p true and q false' is impossible. This, he argues, is precisely what is meant by p ⥽ q. He therefore, thinks that strict implication can serve as the basis of inference.

4.4.3 Consideration of Some Objections to Lewis' Theory

The 'paradoxes' of strict implication are considered to be as shocking as Lewis considered the paradoxes of material implication to be. Moreover, it can be said that strict implication does not capture the sense of deducibility required in valid deduction and hence cannot be identified with entailment.

4.4.3.1 Pranab Kumar Sen's Objections to Lewis' Theory

Pranab Kumar Sen, in his book, *Logic, Induction and Ontology,* attacks Lewis' proof of the paradoxes. He reconstructs the proof which Lewis gives for the paradoxes and shows them to be invalid. Sen's presentation of Lewis' proof for the paradox, 'an impossible proposition entails any proposition whatsoever', is the following.

1. $p \cdot \sim p$
2. p 1, Simp.
3. $\sim p \cdot p$ 1, Com.
4. $\sim p$ 3, Simp.
5. $p \vee q$ 2, Add.
6. q 5, 4, D.S.

Sen shows the above proof to be invalid. An examination of the proof reveals that q is deduced from the combination of the two premises 5 and 4 i.e. from, p v q and ~p. According to Lewis, the derivation of q from p v q holds, because ~p is the contradictory of p. Thus, when ~p is true, p cannot be true also; again, if p is not true then the other disjunct of p v q, i.e., q, must be true. It is clear that in deducing the conclusion q, Lewis presumes that both the propositions p and ~p cannot be true at the same time; in other words, the conjunction p·~p can by no means turn out to be true. However, Lewis considers this very conjunction as the main premise of the deduction. Sen argues that, though, an inference, 'which is confessedly *per impossible'*,[23] can begin with an impossible proposition, however, this premise itself 'cannot be contradicted either explicitly or implicitly, within the course of the same inference'. Sen argues that consistency is an important condition of any inference and Lewis's proof fails to satisfy this condition.

What Sen here objects to is not D.S. in general but the use of D.S. in this particular deduction. In the deduction of q from p·~p, p·~p is taken as the premise. So, p·~p is taken to be true. However, in the course of the very same argument it is assumed that both p and ~p cannot be true at the same time, (i.e. p·~p is not true), so from pvq, and ~p, q is inferred. What Sen points out is that D.S. is invalid in this context, as the falsity of p·~p was suspended at first (as it was taken to be true) and later during the course of the same deduction, its falsity is counted upon. But it is clear that this proof fails to satisfy a very important criterion which every proof demands, namely, that of consistency.

One might argue that Sen's objection can be countered by a slight modification of Lewis' proof. One might demonstrate Lewis' proof in the following manner:

1.	**p . ~p**	
2.	**p**	**1, Simp**
3.	**~p**	**1, Simp**
4.	**~pvq**	**2, Add**
5.	**p⊃q**	**4, df. Impl.**
6.	**q**	**5, 2, M.P.**

What Sen objects to is the use of D.S. in particular. However, if the fundamental basis of this objection could be shown to be defective then the objection no longer holds. The argument, as it can be seen, has been reconstructed by using M.P., rather than D.S.. Hence Sen's objection against D.S. no longer holds.

However, the above proof to encounter Sen's objection does not seem to be tenable. One might object here that p⊃q i.e. material implication is nothing but disjunction in disguise, and modus ponens is nothing but disjunctive syllogism in disguise.[24]

Sen's objection however, is not without flaws. It may be pointed out that Sen's conclusion that the deduction of q from p·~p is an invalid one, is itself dependent on p.~p; its invalidity can be proved by taking recourse to p·~p itself. Sen holds that p·~p is contradictory and nothing can be deduced from it; but in order to prove the invalidity of Lewis' proof, he himself has to derive the argument from p·~p itself- thereby committing the same mistake. So the charge which he brings against Lewis can also be brought against him as well.

Pranab Kumar Sen raises another objection against Lewis. He points out that, following Lewis, "p entails q" can be defined as, "it is necessary that p⊃q". This, in turn, is the same as saying that the contradictory of the above material implication, i.e., p·~q, is again, impossible. According to Lewis, to say that a proposition is impossible, is to say that it contains a contradiction. Though, Sen accepts that, to say that p entails q is to say that 'p⊃q' is necessary and 'p·~q' is impossible, but what he doubts is that when p·~q is impossible then it contains a contradiction by necessity. As Sen puts it:

> But even if it be true that from any proposition which is impossible some contradiction or other can be derived, it is not true that it will *contain* some such contradiction.[25]

4.4.3.2 Bronstein and Tarter's Criticisms of Lewis' theory

Bronstein and Tarter counter Lewis's view that 'p $\prec$ q' is equivalent to 'q is deducible from p'; i.e., when p strictly implies q, then q is deducible from p. They point out that Lewis goes on to show this by demonstrating that the paradoxes of strict implication state facts about deducibility; in fact, he gives proofs for the paradoxes. They argue that in the proof of the paradox, i.e., 'any proposition, say, q is deducible from an impossible one, say, p•~p, Lewis takes it for granted that p v q can be deduced from p. However, to say that any proposition, for instance, p v q, is deducible from another proposition, say p, means that "p implies pvq". Since implication, according to Lewis, is strict implication, so it must be said that 'p strictly implies p v q'. It follows from this, that Lewis supposes strict implication to be a sufficient condition for deducibility, but his purpose is to establish that strict implication is the converse of deducibility. Bronstein and Tarter urge that, 'any proposition can be deduced from an impossible one', holds in Lewis' system of strict implication. According to them, laws of implication are presupposed by the laws of deducibility; laws of implication in turn are determined by the definition of implication as given in a particular system; thus “paradoxes” of implication can in no way be justified on the basis of their deducibility. Bronstein and Tarter contend that “p strictly implies q” cannot be identified with “q is deducible from p”, because, “p v q” cannot be deduced validly from p. They argue that “if p is a proposition, and q is a proposition, then it does not follow that ‘p v q’ is a proposition”.[26]

According to them, this proposition can rightly be said to belong to an extensional logic, such as that of material implication. They urge that strict implication does not stand for that particular relation that holds between the postulates and theorems of a deductive system, because the sense in which the conclusion of a valid syllogism is implied by its premises is not the same as that in which that conclusion is implied by an impossible proposition. They maintain that ‘strict implication is not strict enough’.

4.4.3.3 Nelson’s Objections to Lewis’ Theory of Strict Implication

Nelson[27] rejects strict implication as implication. He rejects Lewis’ proof of the so called paradox $(A\cdot \sim A)\rightarrow B$, on the grounds that there is no inner connection between the impossible premise and the conclusion.[28]

4.4.3.4 Duncan Jones on the Paradoxes of Strict Implication

According to Duncan Jones, the paradoxes of Strict implication are ‘outrageous’ He proposes “to use the word entailment for a relation which only holds between two propositions, p and q, when q arises out of the meaning of p”.[29] He maintains that strict implication cannot be equated with entailment.

4.4.3.5 P.F. Strawson’s Criticism of Lewis’ theory

P.F. Strawson, another critic of Lewis, upholds the view that necessary and impossible propositions cannot form a part of entailment relation. In other words, entailment can hold only between contingent statements and not between necessary or impossible statements. Strawson[30] accordingly writes:

> I propose to use the word “entails” [in such a way] that no necessary statement and no negation of a necessary statement can significantly be said to entail or be entailed by any statement. That is, the function “‘p’ entails ‘q’ ” cannot take necessary or self-contradictory statements as arguments.[31]

It can be pointed out that Lewis himself questions in his *Survey of Symbolic logic* that: “Inference depends upon meaning, logical import, intension, a ⊂ b is a relation purely of extension. Is this material implication, a ⊂ b, a relation which can validly represent the logical nexus of proof and demonstration?"[32] He raises this question against material implication, it seems that this very objection can be raised against strict implication too. In fact many of his critics have raised such questions.

4.4.3.6 Anderson and Belnap's Criticisms of Lewis' Proof of the Paradoxes of Strict Implication

Anderson and Belnap[33] vehemently criticize Lewis' definition of entailment in terms of strict implication. They think that a necessary condition for the validity of any inference is the relation between premises and conclusion, which logicians including Lewis have ignored. Anderson and Belnap's objection is considered in details in chapter five.[34]

Thus, it can be seen that several logicians have rejected the view that strict implication is equivalent to entailment.

4.4.4 A Defense of Lewis' Position

4.4.4.1 Consideration of Strawson's Criticisms of Lewis' View

Strawson's objection can be blocked by saying that in mathematics, geometry etc, entailment relation is regarded to hold between necessary propositions. So limiting the concept of entailment, following Strawson's view, will limit the scope of entailment, whereby it will not be applicable to mathematics etc. which make much use of the concept.[35]

4.4.4.2 Consideration of Anderson and Belnap's Criticisms of Lewis' Theory

Anderson and Belnap's attack on Lewis' proof for the paradox $[(A \cdot \sim A) \rightarrow B]$ on the grounds that it uses the principle of disjunctive syllogism, doesn't seem to be tenable.[36]

We have already seen above that Strawson is one of the critics of Lewis' theory of strict implication. Let us now turn towards Strawson's notion of entailment.

4.5 Strawson's Notion of Entailment

P. F. Strawson in his *Introduction to Logical Theory* offers a definition of the notion of entailment, first in terms of the notion of inconsistency and then in terms of necessity. Let us first see how he defines entailment in terms of the notion of inconsistency.

4.5.1 Strawson's Definition of Entailment in Terms of Inconsistency

P. F. Strawson in his *Introduction to Logical Theory* offers a definition of the notion of entailment in terms of the notion of inconsistency. Initially, Strawson explains the meaning of "entails" and subsequently by a process of transformation of this explanation, he arrives at the intended definition. At first he defines "entails" as follows: "To say that one statement entails another is to say that it would be inconsistent to make the first and deny the second". [37] The above explanation of "entails" is based on the idea of what passes as entailment in ordinary logical discussion. But this explanation does not refer to any particular statement, but to statements in general. Strawson goes on to express the above statement in terms of S_1 and S_2 in the following way, where the subscripts, according to him, are to be taken to mean "'one', 'another', 'first' and 'second'". "To say that S_1 entails S_2 is to say that it would be inconsistent to make S_1 and deny S_2". [38] The above explanation introduces the notion of denial or negation which is very much related to the notion of contradictoriness, for to deny any statement is to assert its contradictory. On the basis of such additional explanation the above can be re-stated as follows: "To say that S_1 entails S_2 is to say that it would be inconsistent both to assert S_1 and to assert the contradictory of S_2". [39] Generally 'contradiction' is expressed by the phrase 'not', so in the above explanation, the expression "the contradictory of S_2" can be represented as not-S_2: Thus the above can be restated as: "To say that S_1 entails S_2 is to say that it would be inconsistent both to assert S_1 and to assert not-S_2". [40] If it is inconsistent to assert S_1 and not-S_2 together, then the above explanation can be re-paraphrased in the following manner: "To say that S_1 entails S_2 is to say that S_1 and not-S_2 is inconsistent". [41] On the basis of the final explanation of 'entails', as offered above, its definition can be stated as follows: "'S_1 entails S_2' $=_{Df}$ 'S_1 and not-S_2 is inconsistent.'" [42]

4.5. 2 Strawson's Definition of Entailment in Terms of Logical Necessity

Strawson, in his *Introduction to Logical Theory,* argues about the inter definition of logical words. He suggests not only that two logical words are inter-definable but also that the same logical word may be defined in terms of more than one logical word. He proceeds to show how 'entails' can be defined by both the terms, 'inconsistent' and 'logically necessary'. After having defined 'entailment' in terms of the term 'inconsistent', Strawson attempts to define the same word by the phrase 'logically necessary'. He does this by establishing the logical connection between inconsistent and the logically necessary statement.

Strawson states that a logically necessary statement is made by negating an inconsistent statement. A logically necessary statement is identical with an 'analytic statement' or a 'logically true statement' or a 'necessary truth'. Establishing the connection between inconsistency and logical necessity, Strawson provides an explanation of entailment in terms of logical necessity and then he provides the definition of the word 'entails' in terms of the phrase 'is logically necessary'. He formulates the first explanation as follows:

1. "To say that S_1 entails S_2 is to say that the contradictory of S_1 and not-S_2 is logically necessary".[43]

In the above explanation the phrase 'the contradictory of S_1' and 'not-S_2'may be represented as 'not-(S_1 and not-S_2)'. Then we shall have the following explanation:

2. "To say that S_1 entails S_2 is to say that 'not-(S_1 and not-S_2)' is logically necessary".[44]

As per the practice of truth-functional logic, the explanation 'not-(S_1 and not-S_2)' may be replaced by $S_1 \supset S_2$ which, in turn, leads to the following abbreviated definition or explanation:

3. To say that S_1 entails S_2 is to say that S_1 implies S_2 is logically necessary.

This final explanation leads to another definition of 'entails' in terms of the expression 'is logically necessary'-

"'S_1 entails S_2'$=_{Df}$ '$S_1 \supset S_2$ is logically necessary'".[45]

The Strawsonian route from explanation to definition brings to light the hidden connection between a logical statement with its linguistic counterpart. As in the case of the definition of 'entails' in terms of the expression 'is inconsistent', here also Strawson shows how 'entails' can be defined in terms of the expression 'is logically necessary'. In the process, he brings to light the missing link between logical and linguistic statements.

4.5.3 Paradoxical Consequences of the Above two Definitions of Entailment and Strawson's Solution

According to Strawson, the above two definitions of entailment give rise to certain paradoxes which are as follows: -

1. An inconsistent statement entails any statement whatsoever.

2. A logically necessary statement is entailed by any statement whatsoever.

Let us suppose following Strawson, that we have two separate statements one- an inconsistent statement, and another- a synthetic or informative statement.

1. Mr. X is both over 6 ft. tall and under 6 ft. tall (S_t)
2. X mas day, 1900 was a fine day (S_f)

It can be seen that 'S_t is inconsistent and S_f' is informative. As S_t is inconsistent, $S_t \& \sim(S_f)$ is also inconsistent.

3. $S_t \& \sim S_f$ is inconsistent.

According to first definition of entailment, S_1 entails S_2 if S_1 and not- S_2 is inconsistent. Here, S_t entails $S_{f,}$ if and only if S_t and not-S_f is inconsistent. It may be pointed out that any conjunctive statement can be said to be inconsistent, based on the inconsistency of one or both of its conjuncts. Here, since, S_t is inconsistent, so the entire conjunctive statement turns out to be inconsistent. If this be the case, then it has to be said that S_t entails S_f, but this is something which conflicts with our common sense. One statement is said to entail another when one logically follows from another. But, here S_f does not logically follow from S_t. Thus, it can be seen that the first definition of entailment gives rise to the above problem.

Strawson has given another definition of entailment in terms of logical necessity. It is known that an implication turns out to be logically necessary when its consequent is logically necessary. To show that $S_t \supset S_f$ is logically necessary, S_f must stand for some logically necessary statement. Let us take the logically necessary statement, 'the wall is blue or not blue'. Let it be referred to as S_f. Let S_t stand for any statement whatsoever, say the chair is brown. Now, the entire implication turns out to be logically necessary, irrespective of the value of S_t, simply on the grounds that $S_{f,}$ which is the consequent, is logically necessary. If this be the case, then it has to be said that 'S_t entails S_f'. But this usually strikes us as nonsensical because here, S_f does not logically follow from S_t. But accepting this definition of entailment leads to the paradoxical consequence that a logically necessary statement is entailed by any statement whatsoever. Thus these two definitions of 'entailment', as given by Strawson, gives rise to the following two paradoxes:

1. An inconsistent statement entails any statement whatever.
2. A logically necessary statement is entailed by any statement whatever.

Strawson seeks to resolve these difficulties in two ways. According to him, one way by which the difficulty can be avoided is by making it a rule that "no conjunctive statement is to be called inconsistent simply on the ground that one or both of its conjuncts are inconsistent".[46] Once such a rule is made, then $S_{t,}$ i.e., 'Mr.X is both over 6 ft. tall and under 6 ft. tall' and the negation of $S_{f,}$ i.e., 'X mas day, 1900 was a fine day' can no longer be considered to be inconsistent. Thus, the question of S_t entailing S_f does not arise. Therefore, the paradox does not arise.

To explain further, the paradoxes can be avoided by adding to the definition of entailment "the proviso that the inconsistency of conjunctive statement does not result simply from the inconsistency of one or both of its conjuncts".[47] By adding such further clause, it can be said that in the example in question, S_t does not entail S_f.

In other words, according to Strawson we can avoid the paradoxes by imposing certain restrictions as explained above. The first paradox arose because it had been assumed that the entire conjunction turns out to be inconsistent, on the ground that one of the conjuncts is inconsistent. Now, if it is possible to rule out this assumption, i.e. if it can be shown that a conjunction does not turn out to be inconsistent simply on the basis of one of its conjuncts being inconsistent, then the problem can be avoided. Another way out is to revise the definition of entailment by adding a further condition in it. These are the two solutions which are offered by Strawson to solve the paradoxes.

4.5.4 Another Solution Proposed by Strawson: Entailment-Statements are not Necessary Statements

Strawson in his 'Necessary Propositions and Entailment-Statements'[48] states that an entailment statement cannot have for its arguments, necessary and impossible statements. i.e. necessary and impossible propositions cannot be a part of entailment statements.

As he puts it:

> I propose so to use the word "entails" that no necessary statement and no negation of a necessary statement can significantly be said to entail or be entailed by any statement. That

is, the function " ' p ' entails ' q ' " cannot take necessary or self-contradictory statements as arguments.[49]

Thus, Strawson proposes to provide a solution to the paradoxes of implication by suggesting that necessary and impossible propositions cannot form a part of entailment statements. According to Strawson, what is meant by the expression "'p' entails 'q'" is that 'p⊃q' is necessary, but 'p' or 'q' is not necessary or self-contradictory; it may also mean that 'p·not-q' is impossible, but at the same time 'p'or 'q', and even their contradictories are not necessary. To explain: In case 'p' is an impossible statement or 'q' a necessary one, then "p entails q" cannot be said to hold on the grounds that 'p' is an impossible statement or 'q' a necessary statement. Strawson goes on to explain how the addition of the provision that terms like 'necessary', 'impossible' etc, are not appropriate for 'entailment' helps to avoid the paradoxes of strict implication. Suppose "p_1" stands for a contingent proposition, while "q_1" expresses a necessary proposition. It is clear then, that, "p_1.not -q_1" is impossible on the ground that "not -q_1" is impossible. But "q_1" is necessary. Thus, by the provision above, "p_1" cannot be said to entail "q_1". What Strawson wants to put through is that entailment statements are not necessary statements. As he puts it: "No entailment-statements are necessary statements".[50]

Strawson goes on to bring to light the distinction between an expression and the statement that the expression is necessary or impossible; what he states through this distinction is that " 'p' entails 'q'" is a "meta-statement" about "p" and "q", and is in turn of a higher order than “p” and “q”. He equates a true entailment statement with a contingent intensional statement that mentions a necessary or impossible statement. He continues:

> Thus every contingent intensional statement of the form " ' p ' entails ' q ' " is logically equivalent to another contingent intensional statement of the form " ' p ⊃ q ' is necessary "; or to a contingent intensional statement of the form " ' p· not-q' is impossible".[51]

Strawson calls attention to the fact that on the basis of his suggestions it can be asserted that entailment-statements are contingent statements; and that neither necessary statements nor their contradictories can entail nor be entailed by any statement. This, Strawson thinks, offers a solution to the paradoxes of implication without leading to any unwanted result. Strawson remarks:

> Thus, by adopting the views (i) that entailment-statements (and other intensional statements) are non-necessary, and (ii) that no necessary statement or its contradictory can entail or be entailed by any statement, we avoid the paradox that a necessary proposition is entailed by any proposition; and, as will be evident, all the other associated paradoxes.[52]

However, Strawson himself anticipates an objection to his view and says that ‘the cure is worse than the disease’.[53] That is to say, Strawson anticipates that someone may object to his view by saying that the solution offered by him is even worse than the problem of paradox. For accepting that necessary and impossible propositions cannot be a part of entailment statements; i.e. rejecting that necessary propositions entail or are entailed by any proposition “is too high a price to pay for the solution of the paradoxes.”[54] The reason being that, necessary propositions do form a part of entailment statements in any deductive system. In other words, in any deductive system there are entailment statements which have necessary propositions as arguments. Therefore, rejecting the view that the relation of entailment can hold between necessary propositions will be unfavorable towards the possibility of any deductive system.

Strawson, nevertheless, answers this objection by calling ‘this alarm’ ‘baseless’[55] He continues to stick to his earlier claim that necessary propositions cannot form a part of entailment statements. According to him, no deductive system consists of necessary propositions that are connected by entailment relation. Rather, they consist of contingent intensional statements which, in turn, state that certain expressions express necessary propositions. These contingent intensional statements are the ones which have the relation of entailment between them.

According to him: “One expression expressing a necessary proposition does not “necessitate” another such expression; but one expression's membership of a system of expressions expressing necessary propositions may necessitate another expression's membership of that system”.[56]

Thus, Strawson goes on to call the intensional logical systems like those of C. I. Lewis’, “a mistaken attempt”[57] in so far as it has expressions with strict implication, which is equated with 'entailment', as rules (i.e. necessary statements) of the system.

4.5.4.1 Examination of Strawson's View

Strawson maintains that in any deductive system no necessary propositions are related by the relation of entailment. However, it is common place that in any deductive system there are necessary propositions which are related by entailment. These are found for instance, in mathematics, geometry etc. and even in case of inconsistent statements, as seen in the case of *reductio ad absurdum*. Strawson's above theory will lead to serious difficulties, as limiting the concept of entailment will limit its scope. So it will no longer be applicable to mathematics etc. which make an extensive use of the concept.

It is worth mentioning here that Strawson himself was aware of such difficulties and he at a later stage modified his earlier view. He admitted that, the view that entailment can hold between contingent statements can lead to grave difficulties in mathematics etc. So he made an attempt to give a new definition for entailment in his article, 'Entailment and its Paradoxes'.[58]

4.5.5 Strawson's Revised Definition of Entailment

In the above section we have seen that Strawson tries to avoid the paradoxes of strict implication by saying that entailment can hold only between contingent propositions. Necessary and impossible propositions cannot be part of entailment relations. However as we have already discussed, such a view seems to be untenable. Apart from Hampshire[59] and Geach,[60] Pranab Kumar Sen also objects to this view of Strawson. According to Sen, the "solution to the paradoxes is too heavy a price".[61]

Strawson, in his article 'Entailment and its Paradoxes', acknowledged that Sen regarded his previous effort of defining entailment as one that contained 'the germ of a promising approach'.[62]

Strawson himself admitted that his previous effort contained 'serious errors', and that his 'claim' to have solved the problem of entailment 'was unjustified'[63,] Thus Strawson goes on to give a new definition of entailment. He defines the notion of entailment with the help of the notion of 'primary entailment' and 'primary entailment schema'. 'Primary entailment' according to him is one in which both the entailing and the entailed statements are contingent, and which satisfies the condition that 'it is impossible that the antecedent is true and the consequent is false'. In other words, primary entailment between two propositions p and q holds when the following two conditions are satisfied.

1. It is impossible that the antecedent (p) is true and the consequent (q) is false.
2. p and q stand for contingent propositions.

The concept of primary entailment can be exemplified with the help of the following examples.

I. The conjunction of ‘it is raining $\supset$ the ground is wet’ and ‘it is raining’ $\rightarrow$‘the ground is wet’.

II. ‘All members of girls club are spinsters’ $\rightarrow$‘All members of girls club are unmarried’

The above two entailment statements are true, though in the second example the entailing and the entailed statements are false.

After having explained the notion of primary entailment, Strawson proceeds to define ‘primary entailment schemata’. He defines it as an entailment schema that is exemplified by only true primary entailment statement and not by any false primary entailment-statement. In other words, primary entailment schemata is an entailment schema which has only true primary entailment-statement as its substitution instance and no false primary entailment-statement as its substitution instance.

Corresponding to the above two primary entailment statements are the following two primary entailment schemata.

Ia $[(P \supset Q) \& P] \rightarrow Q$

IIa ‘All G’s are spinsters’ $\rightarrow$ ‘all G’s are unmarried’

Strawson now proceeds to define ‘entailment’ with the help of the notion of primary entailment statement and primary entailment schemata. His definition of entailment goes as follows; ‘one proposition entails another if and only if the statement that it does so exemplifies a primary entailment-schema, whether the propositions in question are contingent, necessary or self-contradictory’. To explain.

One proposition –say, p, can be said to entail another proposition –say, q, when ‘p→q’ is a substitution instance of a primary entailment schema, irrespective of p, q being either contingent, necessary or self-contradictory propositions.

Strawson’s new definition of entailment can be explained with the help of the following examples:

Ib The conjunction of ‘(x) (x is smaller than 5 ⊃ x-1 is smaller than 5)’ and ‘2 is smaller than 5’ entails ‘2-1 is smaller than 5’;

IIb ‘All unmarried adult females are spinsters’ entails ‘all unmarried adult females are unmarried’.

It can be seen that in the above examples both the entailing and the entailed propositions are non-contingent, but they are substitution instances of contingent entailment schema, (a contingent entailment schema is one which can have, necessary or impossible or contingent propositions as substitution instance).

The point that Strawson emphasizes upon in this article is that when a proposition p entails another proposition q, then it must not be the case that it entails only due to the fact that p is impossible or q is necessary. To elucidate: Even if the entailing proposition is necessary or the entailed proposition is impossible, the entailment should still hold if the propositions were contingent instead. In other words, even if the entailing and entailed proposition is necessary or impossible, the relation of entailment between them still continues to hold even if both were replaced by contingent statements instead. This condition can be fulfilled by laying down the condition that the entailing and the entailed propositions be substitution instances of a contingent schema, i.e. of a schema whose substitution instances are neither necessarily true nor necessarily false; i.e. one which has necessary, impossible, contingently true, contingently false propositions as its substitution instance.

The definition of entailment as given by Strawson succeeds in solving the paradoxes of implication. Let us explain this with the help of the following example. ‘The grass is green & ~(The grass is green)’. The above proposition entails the propositions ‘The grass is green’ and ‘~(The grass is green)’ separately, but it does not entail each and every other proposition. For example, it does not entail the proposition ‘The sea is blue’. Following the law of simplification it can be said that if a proposition is a substitution instance of the contingent schema ‘p&q’, then it entails the substitution instance of ‘p’ or ‘q’ that corresponds to it, but doesn’t entail any and every proposition. On this account it can be said that ‘The grass is green & ~ (The grass is green)’, i.e. (p & q) would entail ‘The grass is green’, i.e. (p) or ‘~(The grass is green)’, i.e. (q), but not ‘The sea is blue’, since, ‘The sea is blue’ is neither the substitution instance of ‘p’ nor of ‘q’. So it is not entailed by ‘p & q’. Thus it seems that Strawson’s new definition of entailment succeeds in blocking the paradoxes of entailment.

4.5.5.1 Merits of Strawson’s Revised Definition of Entailment

Pranab Kumar Sen, in his article ‘P.F. Strawson on Entailment and its Paradoxes’[64] goes on to commend Strawson by saying that his new definition of ‘entailment’ not only succeeds in providing a solution to the paradoxes of implication, but also captures the basic intuition about entailment that no irrelevant propositions entail each other. For e.g. 'The grass is green & ~(The grass is green) doesn't entail the irrelevant proposition 'The sea is blue', for the latter is not the substitution instance of the contingent entailment schema which the former exemplifies.[65]

4.5.5.2 An appraisal of Strawson's Revised Definition of Entailment

However, the revised definition of entailment is not free from defects. Strawson himself is quick to point out that 'there are sacrifices to be made' and that accepting his new definition of entailment leads to the rejection of 'certain generally accepted principles of entailment',[66] namely the principle of transitivity of entailment.

Strawson points out that the derivation of Q from P&~P cannot be said to be invalid, as claimed by professor Sen. For, the steps involved in the derivation, according to Strawson, exemplifies a primary entailment schema. For e.g. 'the step from *P&~P to P',* 'exemplifies the primary entailment schema (P&Q) →P'. So, it seems that the paradox has to be accepted that any proposition is entailed by a contradiction or, as Strawson puts it, that any proposition is 'validly deductively derivable from a formal contradiction'. However, Strawson maintains that such paradoxes need not be accepted. For this is not the relation of entailment; it is something for which he thinks another name must be accepted, i.e. that of being 'the converse of validly deductively derivable from'. And (according to him) being 'the converse of validly deductively derivable from' is not what he calls entailment. He thinks that the relation of being the converse of validly deductively derivable from is nothing other than the classical notion of implication, which he prefers to call the 'logical implication'. Strawson maintains that it is this 'logical implication' or 'the converse of validly deductively derivable' from, for which the paradoxes are acceptable, and which is transitive. For entailment both of the above do not hold, i.e. in case of entailment, neither do the paradoxes hold nor is it transitive. To explain Strawson suggests that where p, q, r, stand for propositions, if r is validly deductively derivable from q and q is validly deductively derivable from p, then r is validly deductively derivable from p. This shows that transitivity holds for the relation of being validly deductively derivable from. Had entailment been the 'converse of validly deductively derivable from' then transitivity would hold good for entailment too. However, Strawson thinks that 'converse of validly deductively derivable from' is not entailment but strict implication as defined by Lewis.

Entailment according to Strawson's revised definition is not transitive. But, we have seen above that the relation of deducibility is transitive. Now, if 'entailment' is 'the converse of deducibility', then deducibility being transitive, so must entailment be. But according to Strawson, entailment is not transitive. And Strawson doesn't consider this to be a problem for entailment at all. According to him, entailment is not the converse of validly deductively derivable from. He considers the classical notion of strict implication as validly deductively derivable from, and 'reserves' the name 'logical implication' for it.

Following Pranab Kumar Sen,[67] it may be objected here, that the notion of 'entailment' has been coined by Moore as the converse of deducibility. It is worth mentioning that Strawson's 'converse of validly deductively derivable from' is nothing but 'converse of deducibility'. But Strawson doesn't regard entailment to be the 'converse of validly deductively derivable from'. That means Strawson doesn't regard entailment to be the converse of deducibility, (as 'converse of validly deductively derivable from' is nothing but 'the converse of deducibility'). The question that Sen raises here is how can we divert from Moore's sense of the term 'entailment'?; as the term 'entailment' was coined by Moore, to mean the converse of deducibility.[68] Thus, it may be objected here that what else is 'converse of validly deductively derivable from' than 'converse of deducibility', which is exactly what Moore meant by entailment.

It seems to me that Strawson's revised definition of entailment is far from satisfactory. My objection against Strawson is twofold. The first objection which I would like to raise against Strawson's new definition of entailment is that it involves circularity. On the one hand, Strawson states that primary entailment schema is an entailment schema which is exemplified by only true primary entailment statements and again, he goes on to define entailment as that which exemplifies a primary entailment schema.

Secondly, we would like to point out that Strawson's new definition of entailment boils down to 'true' primary entailment statement. For if 'primary entailment schema' is that which is exemplified by only true primary entailment statement, and if entailment is that which exemplifies a primary entailment schemata, then it is clear that, according to Strawson entailment is nothing but true primary entailment statement.

NOTES AND REFERENCES

1. Clarence Irving Lewis and Cooper Harold Langford, *Symbolic Logic*, Dover Publications, 1951, p. 248.
2. *Ibid.*, p.248, and p.160.
3. *Ibid.*, p. 160.
4. *Ibidem*.
5. *Ibid.*, p. 248.
6. *Ibid.*, p. 160.
7. *Ibidem*.
8. *Ibid.*, p. 160.
9. *Ibidem*.
10. *Ibid.*, p. 248.
11. Clarence Irving Lewis, *A Survey of Symbolic Logic*, Berkeley, University of California Press, 1918, p. 304.
12. Clarence Irving Lewis and Cooper Harold Langford, *Symbolic Logic*, *op. cit.*, p.160.
13. *Ibid.*, p. 159.
14. *Ibid.*, p. 124.
15. Clarence Irving Lewis, *A Survey of Symbolic Logic*, *op. cit.*, p. 332 ff.
16. *Ibid.*, p. 328.

17. *Ibid.*, p. 335.

18. Clarence Irving Lewis and Cooper Harold Langford, *Symbolic Logic, op. cit.*, p.174.

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20. *Ibidem*.

21. *Ibidem*.

22. *Ibid.*, p. 255.

23. Pranab Kumar Sen, *Logic Induction and Ontology*, Macmillan India Press, Madras, Calcutta 1980, p. 24.

24. For a survey on attacks on the various steps in Lewis' proof of the paradoxes, see Jonathan Bennett, 'Entailment', *The Philosophical Review*, lxviii, I969, pp.197-236.

25. Pranab Kumar Sen, *Logic Induction and Ontology*, *op. cit.,* p .43.

26. Daniel J. Bronstein, and Harry Tarter, 'Reviews on C.I. Lewis, Symbolic Logic', in The Philosophical Review, vol-XLIII No 3, New York, 1934, p.308, (of pp. 305-309).

27. Everett J. Nelson, 'Intensional Relations', in *Mind*, Vol. 39, No. 156, Oct., 1930, pp.440-453.

28. For a detailed discussion of Nelson's view see below 5.4.

29. Austin E. Duncan-Jones, 'Is Strict Implication the same as Entailment'? *Analysis*, Vol. 2, No. 5, Apr., 1935, p. 71, (of pp. 70-78).

30. For a detailed discussion of Strawson's view, see 4.5.

31. P. F. Strawson, Necessary Propositions and Entailment-Statements, Mind, New Series, Vol. 57, No. 226, Apr., 1948, Oxford University Press, p. 186, (of pp. 184-200).

32. Clarence Irving Lewis, *A Survey of Symbolic Logic*, *op. cit.*, p. 328.

33. Alan R. Anderson, Nuel D. Belnap, Jr., *Entailment: The Logic of Relevance and Necessity*, vol. I. Princeton U.P., 1975.

34. For a detailed discussion of Anderson And Belnap's attack on Lewis' Proof of the Paradoxes of Strict Implication see chapter five.

35. See this chapter, sec 4.5 for a detailed discussion of Strawson's view.

36. For a detailed discussion of objections against Anderson and Belnap's see chapter five, sec- 5.8.1 and 5.8.2.

37. P.F. Strawson, *Introduction to Logical Theory*, Methuen & Co Ltd, London, p.19.

38. *Ibid.*, pp. 19-20.

39. *Ibid.*, p. 20.

40. *Ibidem*.

41. *Ibidem*.

42. *Ibidem*.

43. *Ibid.*, p. 23.

44. *Ibidem*.

45. *Ibidem.*

46. *Ibid.*, p. 24.

47. *Ibidem*.

48. P. F. Strawson, "Necessary Propositions and Entailment-Statements", *Mind*, Vol. 57, No. 226, Oxford University Press, pp. 184-200.

49. *Ibid*, p. p.186.

50. *Ibid.*, p. 195.

51. *Ibid.*, p. 185.

52. *Ibid.*, p. 187.

53. *Ibid*, p. 187.

54. *Ibid.*, p. 187.

55. *Ibid.*, p. 187.

56. *Ibid*, p. 188.

57. *Ibid.*, p. 195.

58. P.F. Strawson, 'Entailment and its Paradoxes', D. P. Chattopadhyaya, et al eds., *Realism: Responses and Reactions, Essays in Honour of Pranab Kumar Sen*, New Delhi, India, Indian Council of Philosophical Research, 2001, pp.22-31.

59. For objections against Strawson's view, see also S. N. Hampshire, 'Mr. Strawson on Necessary Propositions and Entailment Statements', in *Mind*, Vol. 57, No. 227, Jul., 1948, pp. 354-357. S. N. Hampshire, in his article 'Mr. Strawson on necessary propositions and entailment-statements', alleges that

Strawson is confronted with paradox and contradiction in so far as he ignores his distinction between statements used within a deductive system and meta-statements i.e. statements about statements or about the system.

60. Also see for objections against Strawson's view, P. T. Geach, 'Necessary Propositions and Entailment-Statements', in *Mind*, New Series, Vol. 57, No. 228, Oct., 1948, pp. 491-493. P. T. Geach in his article, 'Necessary propositions and entailment-statements', raises a twofold objection to Strawson's view. First and foremost, he points out that Strawson's use of quotation marks with variables leads to the production of expressions that are nonsense. Geach rather offers a simpler way of differentiating an expression from its name. The Second objection to which he subjects Strawson is that, Strawson's view regarding the deduction of a necessary proposition from another one is demonstrably wrong for simple cases also.

61. Pranab Kumar Sen, 'P.F.Strawson on Entailment and its Paradoxes', D. P. Chattopadhyaya, et al eds., *Realism: Responses and Reactions, Essays in Honour of Pranab Kumar Sen, op. cit.*, pp. 543, (of 539-544).

62. P.F. Strawson, 'Entailment and its Paradoxes', *op. cit*, p. 22, (of 22-31).

63. *Ibidem*.

64. Pranab Kumar Sen, 'P.F. Strawson on Entailment and its Paradoxes', *op. cit.*, pp.543-544, (of 539-544).

65. For detailed discussion on the merits of Strawson's revised definition of entailment see section 4.5.5.

66. Strawson, P.F., ‘Entailment and its Paradoxes’, *op. cit.,* pp. 28, (of 22-31).

67. Pranab Kumar Sen, ‘P.F. Strawson on Entailment and its Paradoxes’, *op. cit.*, p.543 (of 539-544).