CHAPTER ONE

INTRODUCTION

1.1 The Need for a Proper Understanding and Analysis of the Notion of Entailment

Entailment being the central notion of logic has gained immense attention and interest of logicians since time immemorial. George Edward Moore in his ‘External and Internal Relations’,[1] introduced the term “entailment” as the relation which is the converse of the relation of deducibility. In Moore’s words:

> …“p entails q” is a natural expression for “q follows from p,” i.e., “entails” can naturally be used as the converse of “follows from.[2]

Moore upholds the view that “p entails q” and “q follows from p” are related to each other in the same manner in which “A is greater than B” is to “B is less than A”. According to him, the proposition “This is coloured” is deducible from the proposition “This is red, and all red things are coloured things”. Hence, the proposition “This is red, and all red things are coloured things” entails the proposition “This is coloured”. As Moore puts it:

> We require, first of all, some term to express the converse of that relation which we assert to hold between a particular proposition q and a particular proposition p, when we assert that q follows from or is deducible from p. Let us use the term “entails” to express the converse of this relation. We shall then be able to say truly that “p entails q,” when and only when we are able to say truly that “q follows from p” or “is deducible from p,” in the sense in which the conclusion of a syllogism in Barbara follows from the two premises, taken as one conjunctive proposition; or in which the proposition “This is coloured” follows from “This is red.” “p entails q” will be related to “q follows from p” in the same way in which “A is greater than B” is related to “B is less than A.[3]

Unarguably the relation of entailment is one of the necessary conditions for the validity of a deductive inference. Going a step ahead, many consider it to be the sufficient condition for validity as well. If logic is an investigation and study of the principles of valid reasoning; and if the relation of entailment, as holding between premise and conclusion, is a necessary condition for the validity of inference, then the concept of entailment turns out to be a key notion of logic. Hence, logicians have evinced a keen interest in this concept. It is the fundamental task of the pioneers of logic to give an adequate analysis of the concept, to give its "proper" meaning.

However, before entering into the arduous task of analyzing the notion of entailment, it would be proper to consider the distinction between deductive and inductive arguments, and also show that the question of entailment arises in case of the former alone.

1.2 Arguments

The act of passing from one or more judgements to another, the truth of which is believed to follow from that of the former is known as inference or reasoning and an argument is nothing but an inference expressed in language. An argument is a group of statements, some of which are intended to provide evidence, justification or support for the truth of another. It will not be out of context to mention that a statement is a declarative sentence that is either true or false (according to classical logic); e.g. "The earth revolves around the sun". An argument consist of one or more *premise* and a *conclusion*; those statements that provide support, evidence or reason for the argument's conclusion are known as premise; while the conclusion is that statement which the premises support. Let us explain the structure of an argument. A standard form categorical argument consists of three propositions; a major premise, a minor premise and a conclusion. It contains three terms, the major term, the minor term and the middle term. The major premise forms the first premise and it contains both the middle and major terms. The minor premise is the second premise and it has both the middle and minor terms. And the conclusion is that which contains the minor and the major terms. The *major term* is that which is present in the major premise and it also forms the predicate of the conclusion. While, the *minor term* is the one that is found in the minor premise and is also the subject term of the conclusion. Again, the *middle term* is that which is found in both the premises but not in the conclusion.

Let us understand this with the help of an example:

All planets revolve around the sun

The Earth is a planet

∴ The Earth revolves around the sun

The statement followed by "∴" **(**therefore) i.e., 'The Earth revolves around the sun', is the conclusion of this argument. The other two statements are the premises, which are offered as reasons or justification for the truth of this claim. In the above example of a deductive argument, 'All planets revolve around the sun' is the major premise; 'The Earth is a planet' is the minor premise, while, 'The Earth revolves around the sun' is the conclusion. Again, 'Planets' is the middle term (M), 'revolve around the sun' is the major term, while, 'The Earth' is the minor term. In the conclusion, 'The Earth' is the subject term (S), while 'revolves around the sun' is the predicate term (P). It can be seen here that the conclusion logically and necessarily follows from the premises. That is, the conclusion, ‘The Earth revolves around the sun’, necessarily follows from the premises ‘All planets revolve around the sun’ and ‘The Earth is a planet’.

It is with arguments that logic is primarily concerned. Because it is with the help of arguments that evidence or support can be provided for a certain claim. Words such as, “therefore”, “hence”, “thus”, “so” or “consequently”, are often present before the conclusions of arguments. While, the premises are often associated with words or expressions like, “because”, “since”, “for the reason that”, “inasmuch as”, etc. These “indicators” help in identifying the conclusion of the argument, which often comes last in the series of statements that form the argument.

1.2.1 Deductive and Inductive Arguments

Arguments can further be divided into two types, namely deductive argument and inductive argument. Let us see what is meant by each of them.

1.2.1.1 Deductive Argument

A deductive argument is an argument in which the premises provide an absolute guarantee for the truth of the conclusion. In a deductive argument, it is impossible for the premises to be true and the conclusion to be false. In other words, in a deductive argument, if the premises are true then the conclusion must also be true; i.e. the conclusion follows necessarily from the premises. This can be explained with the help of the following example:

Whatever is destructible is non-eternal (Premise)

Pot is destructible (Premise)

∴ Pot is non-eternal (Conclusion)

The truth of the conclusion, 'Pot is non-eternal' follows necessarily from the truth of the premises, 'Whatever is destructible is non-eternal' and 'Pot is destructible'. Thus, it can be seen that if the premises are true, then it impossible for the conclusion to be false. In case of any deductive argument, the premises provide conclusive grounds for the truth of the conclusion. To state it differently, deductive arguments claim certainty, those deductive arguments which meet this claim are called "valid", while those which do not meet this claim are called "invalid." Therefore, every deductive argument is either valid or invalid. A deductive argument is valid if the conclusion necessarily follows from the premises and it is *invalid* otherwise. It is worth noting that, a valid deductive argument may be sound or may not be so. A deductive argument is sound, only when, along with being valid, its premises are true also. It is unsound otherwise.

In a deductive argument, the conclusion is already contained in the premises. The conclusion only restates the premises more clearly. Thus, accepting the premises means accepting the conclusion.

1.2.1.2 Inductive Argument

Let's us now see what is meant by an inductive argument. An inductive argument is one in which the premises support the conclusion but do not guarantee it. In case of inductive argument one moves from the particular to the general. Particular observations are gathered in the form of premises, then reasoning is made from these particular premises to obtain a general conclusion. In the most common form of inductive reasoning, evidence is collected of some observed phenomena (e.g. observing 2000 crows for black colour), then a general conclusion is drawn about this phenomena based on the collected evidence (whether all crows are black). In other words, inductive reasoning involves reasoning from experience, sense perceptions, and observations to draw a general conclusion. In case of an inductive argument, the conclusion actually goes beyond the premises. For example, if 2000 crows are observed and every observed crow is black, it may be concluded that ‘All crows are black’. The conclusion is only a prediction or a conjecture. Addition of any further evidence may support or deny the conclusion; i.e. further premises might strengthen or weaken the conclusion. The 2001th crow may not be black. Therefore, in case of an inductive argument, one can affirm all the premises (2000 crows are black), but deny the conclusion (all crows are black) without leading to logical contradiction. What is said in the conclusion, is possible, or even very probable, but is nonetheless, not *necessary*. In order to ‘prove’ the conclusion, what is required is that all crows be observed at all times. But this seems to be an impractical job. Therefore it is unlikely that the conclusion will ever be proven. The conclusion can, nevertheless, be disproven, for instance, by the observation of a white crow. Then what would remain is the conclusion that "*some* crows are black". May be most crows, or nearly all crows are black. It may be pointed out here that in an inductive argument the conclusion states more than the premises actually necessitate.

In an inductive argument, the premises are intended only to be so strong that, if they are true, then it is *unlikely* that the conclusion is false. Belief in the truth of the premises, provides good reason to believe that the conclusion is true. Thus, in an inductive argument, the conclusion follows only with some probability; the premises being true it is probable that the conclusion will also be true; to state differently the premises being true it is improbable that the conclusion will be false; the premises provide some *support* or evidence for the truth of the conclusion, but in no way ascertain or guarantee its truth. Let us give an example in the form of an argument:

Crows in India are black (Premise)

Crows in U.S.A are black (Premise)

Crows in Europe are black (Premise)

∴ All Crows are black (Conclusion)

Here the premises provide support for the truth of the conclusion "All crows are black". However, in this example, even if all the premises are true, it is still possible for the conclusion to be false (may be there are white crows, albino crows, for example).

What is claimed in an inductive argument is not that the premises provide conclusive evidence or guarantee for the truth of the conclusion, but only that the premises provide some *support* for the probability that the conclusion is true. Inductive arguments therefore cannot be "valid" or "invalid", in the sense in which deductive arguments can be. They however, may be evaluated as better or worse, according to the degree of support provided by their premises to their conclusions.

Moreover, unlike deductive arguments in which no amount of additional premises can make it more certain, the addition of further premises can strengthen or weaken an inductive argument. For example in case of the following argument:

Richard is 70 years old

<u>Richard has people to take care of him</u>

∴ Richard will live for another 20 years

We can always add the premise that "Richard has a good health", or "Richard is in the best of health" etc. to make it more probable. On the other hand, addition of premises like "Richard is suffering from some disease" etc. weakens the argument to a great extent. So inductive arguments can be more or less probable, but never valid or invalid; deductive argument alone can be valid or invalid. Because, unlike deductive arguments, inductive reasoning permits the possibility of the conclusion being false, while all the premises at the same time being true.

1.2.1.3 Induction and Deduction Compared

The difference between deduction and an induction depends upon "the *intended strength of the connection* between premises and conclusion".[4] In case of a deductive argument the premises are intended to give full support for the conclusion, while, in case of an inductive argument the premises give only some degree of support for the conclusion. In the latter case the conclusion is only probable; the premises being true, the conclusion does not follow with absolute certainty. In fact, it is quite possible that the conclusion turns out to be false, despite all the premises being true. In Virginia Klenk's words, "there is always some "slippage" or "logical gap" between the premises and the conclusion".[5]

In a valid deductive argument, on the other hand, the premises being true, it follows with absolutely certainty that the conclusion will turn out to be true. The premises offer absolute support for the truth of the conclusion. In all cases where the premises provide such a support, the argument turns out to be valid, otherwise they are invalid. Again, the 'goodness' of an inductive argument is always a matter of degree. The premises being true, one may be 99% sure of the truth of the conclusion, for example, from the premise that 1000 men are mortal to the conclusion that 1001th man will be mortal; or the premises may even provide very little support for the truth of the conclusion. In inductive arguments, the strength varies from nearly 100%

probable, as in case of 1001th man being mortal, to almost impossible, as in the case of predicting a lottery win of a person against 10 million other contenders. In inductive arguments, the relation of support between the premises and conclusion admits of degrees, it may be more strong or less strong or even weak. In other words, inductive arguments vary in strength; they are either *strong* or *weak,* but never valid or invalid. They are either more probable or less probable. Deductive arguments alone are valid or invalid. For, deductive arguments cannot vary in strength; they cannot be more or less strong. In case of deductive arguments, the premises guarantee the truth of the conclusion, and being a guarantee is something which does not admit of degrees.

Again, deductive arguments have the property of monotonicity, whereas inductive arguments lack this property. By the property of monotonicity is meant that if a conclusion follows from a set of premises then, addition of a further premise, will not affect the truth of the conclusion. The conclusion will still follow from the premises. In case of a deductive argument, if a conclusion follows from the premises, addition of a further premise does not augment the truth of the conclusion; it does not affect the validity of the argument. The truth of the conclusion follows from the same set of premises. For example:

All men are mortal

Socrates is a man

∴ Socrates is mortal

If we add another premise 'Socrates is a great philosopher', the conclusion follows from the same set of premises. Addition of the further premise does not make the deductive argument valid or invalid.

Inductive arguments on the other hand, lack the property of monotonicity. In case of inductive arguments, addition of a further premise can make the conclusion false. For example, suppose, we have observed 2000 crows to be black, on the basis of that we have inferred that 'all crows are black'. Suppose, we spot a crow that is white. In the 2001^{th} case, then, the conclusion will no more follow from the premises. The conclusion will be proved to be false.

In case of a deductive argument the premises *entail* the conclusion. The relation between the premises and the conclusion is one of *entailment*. In an inductive argument on the other hand, the premises *probabalify* the conclusion. That is to say, the relation between the premises and the conclusion is one of *probabilification.*

1.3 The Notion of Validity

It is clear that only deductive arguments are capable of being valid or invalid. Let us now see what is meant by "validity", and "invalidity". Validity can be defined in two ways— syntactically and semantically, i.e., in terms of the axioms and rules of the system, and in terms of its interpretation.

1.3.1 Semantic Validity

An argument is semantically valid when the conclusion is true in all interpretations in which all the premises are true.

To formulate it differently;

$A_1 \ldots A_{n-1}$, A_n is valid-in-L if A_n is true in all interpretations in which $A_1 \ldots A_{n-1}$ are true.

This can be represented as under:

$A_1 \ldots A_{n-1}, \models_L A_n$.

Again,

$A_1 \ldots A_{n-1}$, A_n is invalid-in-L if A_n is false in at least one interpretation in which $A_1 \ldots A_{n-1}$ are true.

In other words, the argument is invalid if the conclusion is false in at least one interpretation in which all the premises are true. As for example:

Jack is a bachelor

∴ Jack is unmarried

This argument is valid, as there is no possible situation where Jack is a bachelor but not unmarried, i.e. there is no possible situation where the premise is true and the conclusion is false. On the other hand, consider the following argument

The president of U.S.A. is tall

John is tall

∴ John is the president of U.S.A.

This argument is invalid as there are possibilities of several situations where the premise is true but the conclusion is false.

1.3.2 Syntactic Validity

An argument on the other hand, is syntactically valid when the conclusion is derivable from the set of premises and axioms of the system, by the rules of inference of that system.

To represent it formally:

$A_1 \ldots A_{n-1}$, A_n is valid in –L, if A_n is derivable from $A_1 \ldots A_{n-1}$, and the axioms of L, by the rules of inference of L; it is invalid otherwise; (where $A_1 \ldots A_{n-1}$ are the premises, and A_n is the conclusion).

This can be shown as follows:

$A_1 \ldots A_{n-1} \vdash_L A_n$.

However, it should be kept in mind that that these two conceptions of validity are system relative as is indicated by the 'L' in '$\vdash_L$' and '$\models_L$'.

Thus, 'validity', syntactically defined, is the relation of derivability or deductibility within a system. But then, to say that a proposition q is deducible from (or follows from) another proposition p, is the same as saying that the proposition p entails the proposition q. That is to say, an argument is valid when it is impossible for the premises to be true and the conclusion to be false. In other words, if an argument is valid then the truth of the premises entails the truth of the conclusion. In the case where the premises are true and the conclusion is false, the conclusion is not deducible from the premises; the premises do not entail the conclusion. Let us consider the following argument:

Socrates was a Philosopher

Plato was a Philosopher

∴ Socrates was Plato

This is an invalid argument, as both the premises, i.e. "Socrates was a Philosopher" and "Plato was a Philosopher" are true, but the conclusion "Socrates was Plato" is false.

While the classical example of an argument:

All men are mortal

Socrates is a man

∴ Socrates is mortal

is a valid one, for it is impossible for the premises to be true and the conclusion to be false. In this case, given the truth of the premises, the conclusion must be true; the truth of the conclusion follows from that of the premises; in other words, the premises entail the conclusion.

Thus, it is clear that the notions of validity and invalidity of an argument are based on the notion of deducibility which in turn is dependent on the notion of entailment. Thus the problem of entailment arises only in case of deductive arguments and not in case of inductive arguments.

In the chapters that follow definitions of entailment[6] as given by logicians are explained and examined. We start with Russell's theory of entailment.

NOTES AND REFERENCES

1. G.E., Moore, 'External and Internal Relations', in *Proceedings of the Aristotelian Society*, New Series, Vol. 20, 1919 - 1920, p. 40-62; also in *Philosophical Studies*, 1922, pp. 276-309.
2. G. E. Moore, 'External and Internal Relations' in *Proceedings of the Aristotelian Society, op. cit.*, p. 53, (of 40-62).
3. G. E. Moore, 'External and Internal Relations' in *Philosophical Studies*, 1922, p.291, (of 276-309).
4. Virginia Klenk, Understanding Symbolic Logic, Prentice Hall, 1983, p. 6.
5. *Ibidem*.
6. Implication according to Diodorus and Philo:

   The notion of Entailment, one of the central topics of logic, has a very early origin. It finds its roots in Megarian logic in late 4th century to early 3rd century B.C, when there was a tussle between Philo and his mentor Diodorus Cronus, regarding the nature of conditional. Philo accepted the truth-functional 'if-then' i.e. the material implication of the modern classical propositional calculus as the correct interpretation of conditional, while Diodorus thought of a modalized conditional as an appropriate one. Diodorus argued that implication contained an element of necessity and that it ought not to be considered true simply on the basis of its antecedent being false or its consequent true.