Chapter 2

Nuclear Structure Theory
Understanding about nuclear structure has accumulated large volume of facts and ideas since its discovery by Rutherford in 1911. Over the years, these ideas were verified experimentally and experimental inputs were used to refining the model calculation and vice versa. This dissertation focuses on studying the nuclear behavior as a function of increasing angular momentum to explore the structure of the nucleus. It is the occupation of shells by the two types of nucleons that give rise to the nucleus its structural characteristic. This occupation under certain conditions is responsible for the single particle aspects of nuclear structure and under some other conditions for its collective behavior. Understanding of these two fundamental modes and the interplay between them is one of the most important goals of nuclear structure research.

2.1. Modes of Excitation

The phenomenon spontaneous symmetry breaking is one of the main paradigms in physics. For instance, it is due to the spontaneous breaking of the particle-number symmetry, that the superconducting condensates appear in metals. Spontaneous breaking occurs if a system, in its endeavor to attain the minimal energy, chooses a symmetry-violating state even though the underlying interactions are invariant under the concerned symmetry. Nevertheless, it is the nature of the interactions that determines which symmetries are broken and under which conditions. Therefore, study of symmetry-violating states brings one closer to the understanding of the interactions which is the most fundamental goal for research. In the nuclear structure physics, one important example of spontaneous symmetry breaking is the existence of deformed nuclei. Like for molecules, violation of the spherical invariance leads to the appearance of rotational excitations, which manifest themselves in specific sequences of levels, called the rotational bands.
Figure 2.1: Collective rotation in deformed nucleus $^{158}$Er (left panel) and non-collective (single-particle excitation) corresponding to a nearly-spherical nuclei $^{146}$Gd (right panel) [1], for generation the excited states with spin ($I$).

This observation was attributed to the deviation from spherical shape. Many nucleons contribute to the rotation which is referred to as collective excitation. Depending on the conservation or violation of other symmetries, like the plane of reflection, those bands can have different structures, leading to the different types of the excitation mechanism.

2.1.1. Non Collective Single Particle Excitation

Spherical or near spherical nuclei exist near the closed shells. These kinds of nuclei generate angular momentum by re-arranging their valence nucleons. In Figure 2.1, an example of such type of excitation is shown (right panel). The total angular momentum is the sum of the individual angular momentum of valence nucleons that are not coupled to spin zero. The bulks of the nucleons form the rest of the nuclear matter (the core) and make no contribution to the angular momentum generation.
The pairing interaction favors the formation of pairs of particles coupled to a total magnetic sub-state \( M = 0 \). Therefore, near closed shells, the presence of a strong pairing interaction will inhibit the tendency to deform. The excitation of a particle to the higher excited state, through breaking of a pair, leaves a hole to the previously occupied level. In the presence of pairing the single particle and hole excitation energy is replaced by the quasi-particle energy. Thus, the particles and holes are replaced by quasi-particles representing partially filled levels. This transformation from particles to quasi-particles allows an enormous simplification in shell model calculation as the quasi-particle excitations relative to the Fermi surface are considered.

### 2.1.2. Collective Rotation

For nuclei away from closed shells the occupation of valence nucleons in specific deformation driving orbitals tends to polarize the core resulting in a deviation from spherical symmetry. For axially deformed nuclei, angular momentum can be generated by increasing the collective rotational frequency, where the rotation is about an axis perpendicular to the symmetry axis, with the bulk of the nucleons making a coherent contribution to the angular momentum. This collective motion represents a coherent motion of all the nucleons which leads to the rotational degrees of freedom. The characteristic signature of this mode of excitation is the observation of regular band sequences in level scheme (left panel Figure 2.1) with rotational energy is proportional to \( I(I + 1) \), \( I \) being the spin of the state [2].

The nuclei lying in-between the spherical and the strongly deformed regions of the nuclear chart, are termed as transitional nuclei and they exhibit both type of excitation mechanisms. These nuclei are soft with respect to deformation changes and are susceptible to polarization effects from excited nucleons. In recent years, considerable progress has been made in the spectroscopic investigation of these nuclei, since they provide a platform to study the development of collectivity and nuclear shape as a function of nucleon number. The angular momentum states up to a certain excited state can be generated due to a specific quasi-particle configuration. To generate more angular momentum states the particle from the core must be promoted to the higher orbitals \( i.e. \) with higher excitation energy. This alignment of the
quasi-particles prefers the angular momentum vector oriented along either symmetry axis or rotational axis depending on the nuclear shape of the core (prolate or oblate) and nature of the quasi-particle (hole or particle) as shown in Fig. 2.2. Such orientation occurs as it maximizes overlap of the quasi-particle with the mass distribution of core.

There exist two extreme coupling limits in which the valence nucleons of a nucleus can be coupled to the rotating deformed core. The two couplings limits are illustrated in Figure 2.3. This model assumes the motion of the valence nucleon is affected by the Coriolis force and the deformed nuclear field only, and does not include the affects of a short range pairing force. At low rotational frequencies, the Coriolis force may not be strong enough to affect the motion of the valence nucleon. With a sufficiently deformed nuclear field the nucleonic motion will be coupled to the deformation of the core, as depicted in Figure 2.3 (a). The angular momentum of the valence nucleon will subsequently be aligned with the symmetry axis. This coupling is known as the strong coupling limit.
As the nucleus rotates faster, the Coriolis force becomes strong enough to overcome the pairing force for a specific pair of nucleons. The nucleons motion can be coupled to the rotation of the core, as shown in Figure 2.3 (b), hence the angular momentum will be aligned along the the rotation axis. The energy of the Coriolis force can be express as [3],

\[ E_{Cor} = \frac{\hbar^2}{\mathfrak{I}} J \]  \hspace{1cm} (2.1)

where, \( J \) and \( \mathfrak{I} \) are the angular momentum of the valence nucleon and the moment of inertia of the core, respectively. Consequently high -\( j \) orbitals are predicted to be aligned first, as they are especially sensitive to the Coriolis force. The gain in angular momentum from the breaking of a pair of nucleons results in the nucleus decreasing its angular frequency and a backbend is observed when the spin is plotted against the rotational frequency.

The different characteristic features exhibited in the excitation mechanism due to the movement of the nucleons inside the nucleus were well described by various simplified models.
Chapter 2: Theoretical Models

Comparison of Experimental results

<table>
<thead>
<tr>
<th>Shell Model</th>
<th>Nilsson Model</th>
<th>Cranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[N, l, j]$</td>
<td>$[N, n, \Lambda] \Omega^n$</td>
<td>$[\pi, \alpha]$</td>
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<tr>
<td>$1g_9/2$</td>
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Figure 2.4: The effect of deformed nuclear potential and core rotation on a spherical shell model state ($1g_{9/2}$ for example). $N$, $l$ and $j$ are the total oscillator quantum number, orbital angular momentum and total angular momentum, respectively. In case of deformed nuclei, $n_z$, $\Lambda$ and $\Omega$ are the components of $N$, $l$ and $j$, along the symmetry axis, where $\pi$ and $\alpha$ are the parity signature quantum number.

Simplified picture of the particle motion in the hierarchy of various nuclear potentials as described in Fig. 2.4. The simplest assumption is the motion of nucleons in a 3-D harmonic oscillator potential that generate the single-particle energies of the nuclear orbits. The degeneracy of each shell model state, $(2j + 1)$ where $j$ is the single particle angular momentum, can be removed when an axially deformed Nilsson potential has been considered. Each Nilsson state can be occupied by a maximum of two particle since the states with a projection of the particle spin $j$ on the symmetry axis with $+\Omega$ and $-\Omega$ are indistinguishable. When the deformed nucleus experiences a fast rotation, Coriolis and centrifugal forces act on the individual nucleons in the rotating frame and an extra term $-\hbar \omega j_x$ must be added to the intrinsic Hamiltonian for independent particle motion. Here, $j_x$ represents the spin projection on the axis of rotation. This lifts the two fold degeneracy of the Nilsson levels and each level being split into two components as shown in the right panel of the Fig. 2.4. Thus, the fast nuclear rotation offers the possibility of studying the single particle spectrum in its full complexity where all degeneracy’s are removed.
2.2. Comparison of Experimental results with the Theoretical Calculations

A rotational band is characterised experimentally by the transition energies ($E_{\gamma}$) and their intensities ($I_{\gamma}$) between the $I$ and $I-2$ states and the angular correlation between these transitions. To compare the experimental results with theoretical predictions, several experimental quantities have been defined as,

I. The information on the excited single particles is extracted by subtracting a rigid rotation reference ($E_{RLD}$) from the experimentally observed energy ($E_{exc}$), as a function of spin, with the reference given by,

$$E_{RLD} = \frac{1}{2j_{rig}} I(I + 1), \quad \frac{1}{2j_{rig}} \approx 32.32A^{-5/3} \text{ MeV}/\hbar^2 \quad (2.2)$$

II. The kinematic moment of inertia ($j^{(1)}$) and dynamic moment of inertia ($j^{(2)}$) as a function of spin ($I$), or rotational frequency ($\omega$), used to describe collective rotational structures and defined them as

$$j^{(1)} = \frac{I}{\omega} \quad (2.3)$$

and

$$j^{(2)} = \frac{dI}{d\omega} \quad (2.4a)$$

$$\approx \frac{4\hbar}{\Delta E_{\gamma}} \quad \text{for the } \Delta I = 2 \text{ band} \quad (2.4b)$$
Here, we note that the dynamic moment of inertia depends only on the difference in energy of transitions between two consecutive decays and has no dependence on the absolute spin of the levels. It is therefore a very useful quantity in cases where the decay out of a band is not observed and the spin of the states is not well established. The variation of the dynamic moment of inertia with rotational frequency can give important information about the structural changes that are taking place within a rotational band.

III. The transformation of the experimental excitation energies, $E_{\text{exc}}$, and the experimental spin ($I$), to the intrinsic rotating frame of the nucleus, makes it possible to compare the experimental results to the theoretical calculations which are calculated in the rotating frame. The rotational frequency $\omega$, can be deduced from the experimentally observed values of energy ($E$) and spin ($I$) as,

$$\hbar \omega(I) = \frac{dE(I)}{dI_x} = \frac{E(I + 1) - E(I - 1)}{I_x(I + 1) - I_x(I - 1)}$$

(2.5)

where the quotient of the finite differences has been used and $I_x$,

$$I_x = \sqrt{I(I + 1) - K^2},$$

(2.6)

is the $x$-component of the total angular momentum. The variable $K$ refers to the projection of the total angular momentum onto the symmetry axis and is equal to the angular momentum at the band head. This yields an experimental Routhian value equivalent to

$$E'(\omega) = [E(I + 1) + E(I - 1)] - \hbar \omega I_x(I).$$

(2.7)
To account for the relative nature of the energy of the quasi-particle, a reference must be subtracted, \( i.e. \) the quasi-particle Routhian energy and the aligned angular momentum are obtained by subtracting the reference from their absolute values as,

\[
e'(\omega) = E'(\omega) - E_{ref}(\omega) \tag{2.8}
\]

and

\[
i(\omega) = I_x(\omega) - I_{x,ref}(\omega), \tag{2.9}
\]

respectively. The reference alignment angular momentum \([I_{x,ref}(\omega)]\) and the energy reference \([E_{ref}(\omega)]\) are calculated using the following relations

\[
I_{x,ref}(\omega) = (J_0 + \omega^2 J_0)\omega \tag{2.10}
\]

and

\[
E_{ref}(\omega) = -\frac{1}{2}\omega^2 J_0 - \frac{1}{4}\omega^4 J_1 + \frac{\hbar^2}{8 J_0}, \tag{2.11}
\]

where, \(J_0\) and \(J_1\) are the Harris parameters. The values of these parameters are adjusted in such a way as to produce zero initial alignment for the reference configuration of a particular nucleus. This is justified since, at low rotational frequencies, angular momentum is generated by collective rotation only.

IV. The electromagnetic transition rates or reduced transition probabilities gives us important structural information regarding the structure of the nucleus. The transition probability, for a \(\gamma\)-ray decay from an initial state of spin \(I_i\) to a final state of spin \(I_f\) by emitting a photon
of angular momentum, $L$, is given by,

$$T_{fi}(\lambda L) = \frac{\ln 2}{T_{1/2}^{\gamma L}} = \frac{8\pi(L + 1)}{\hbar L[(2L + 1)!!]^2} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} B(\lambda L : I_i \rightarrow I_f),$$  \hspace{1cm} (2.12)

where, $B(\lambda L)$ is the reduced transition probability for a $\gamma$-ray transition of energy $E_\gamma$ and multipolarity $\lambda$ and which carries $L$ units of angular momentum. Therefore, one can obtain the reduced transition probability if the lifetime and branching ratio of the decay are measured.

The reduced transition probabilities of the electric and magnetic transitions are given by,

$$B(EL : I_i \rightarrow I_f) = \frac{1}{2I_i + 1} | < f | \hat{Q} | i > |^2$$  \hspace{1cm} (2.13)

and

$$B(ML : I_i \rightarrow I_f) = \frac{1}{2I_i + 1} | < f | \hat{M} | i > |^2,$$  \hspace{1cm} (2.14)

respectively. Here $\hat{Q}$ and $\hat{M}$ are the electric and magnetic multipole operators, respectively.

To obtain the relative magnitudes of the transition rates or lifetimes, the probabilities can be expressed in Weisskopf units. The Weisskopf single-particle estimate for the reduced electric transition strength for the $\lambda^{th}$ multipole is given by,

$$B(EL)_w = \frac{1.22L}{4\pi} \left( \frac{3}{L + 3} \right)^2 A^{2L/3} \left[ e^2(fm)^{2L} \right]$$  \hspace{1cm} (2.15)

and for the magnetic transition as
where, $A$ represents the atomic mass. The Weisskopf partial half life ($T_{1/2}^w$) and the experimentally measured partial half-life ($T_{1/2}^{exp}$) are related through the equation,

$$T_{1/2}^w(i) = T_{1/2}^{exp} \left(1 + \alpha_i \frac{BR(\gamma_i)}{2L^2} \right).$$  \hspace{1cm} (2.17)

Here, $i$ represents the transition of interest whereas $\alpha_i$ is the internal conversion coefficient for the transition, $i$, and $BR(\gamma_i)$ is the total branching ratio for the transition $i$.

Finally, the reduced transition probabilities of the electric and magnetic transitions in Weisskopf units, by using Eqs. 2.12, 2.15, 2.16 and 2.17, can be expressed as,

$$B(EL) = \frac{(ln2)L[(2L + 1)!!]^2 h \times BR}{1.2^{2L}(L + 1)A^{2L/3}T_{1/2}^{exp} (1 + \alpha)} \left(\frac{L + 3}{3}\right)^2 \left(\frac{hc}{E_\gamma}\right)^{2L+1} \left[ W.u. \right]$$  \hspace{1cm} (2.18)

and

$$B(ML) = \frac{(ln2)L[(2L + 1)!!]^2 h \times BR}{80(1.2)^{2L}(L + 1)A^{2L-2/3}T_{1/2}^{exp} (1 + \alpha)} \left[ \frac{e^2 (fm)^{2L-2}}{\mu N (fm)^{2L-2}} \right] \left(\frac{L + 3}{3}\right)^2 \left(\frac{hc}{E_\gamma}\right)^{2L+1} \left[ W.u. \right],$$  \hspace{1cm} (2.19)

respectively. The measured transition strengths for the single particle states $\sim 1$ W.u. whereas a significant departures from these values are observed, compared to the Weisskopf estimate, when the nucleus exhibits collective behaviour.
The $B(M1)/B(E2)$ ratios are extremely sensitive to the quasi-particle configuration involved and are very useful in assigning band configurations. The experimental $B(M1)/B(E2)$ ratios can be extracted from the measured $I_\gamma(M1)/I_\gamma(E2)$ branching ratios, as,

$$\frac{B(M1)}{B(E2)} = 0.697 \frac{E_2^3}{E_1^3} \frac{1}{1 + \delta^2 I_\gamma(\Delta I = 2)}.$$  

Here, $E_1$ and $E_2$ are the energy of the M1 ($\Delta I = 1$) and E2 ($\Delta I = 2$) $\gamma$-ray transitions. The value of $\delta$ determines the E2/M1 mixing ratio of the $\Delta I = 1$, M1, transition.

### 2.3. Magnetic Rotation

In the early 1990s, rotational-like patterns of $\gamma$ rays were discovered in nuclei that were thought to be almost spherical (Fig. 2.5). The $\gamma$ rays were found to be magnetic dipole (M1) transitions, implying that the states in these bands had the same parity but differed by only

![Figure 2.5:](image-url)

**Figure 2.5:** Experimental observation of the (a) band-like structure of magnetic dipole transitions (b) $\gamma$-ray energy spectrum of the band in $^{199}$Pb [11, 12].
one unit of angular momentum [4–7]. At first, the sequences were mistakenly considered as the member of the superdeformed bands as the observed energy spectra were very similar to those for superdeformed bands [8, 9] (Fig. 2.6). But the angular correlation measurements confirmed the dipole character of the in-band $\gamma$ rays. Later, electron conversion and $\gamma$-ray polarization measurements confirmed the magnetic dipole nature of the transitions.

General features of these rotational-like dipole bands are:

I. The level energies in a band nearly follow the $I(I + 1)$ behaviour (ignoring band crossings and signature splitting). It has been suggested that they follow the pattern of $\Delta E \sim (I - I_0)^2$, where $\Delta E$ is the energy difference ($E - E_0$) with respect to the bandhead energy ($E_0$) and $I$ and $I_0$ are the spin of the state in the band and the bandhead, respectively [10].

II. The intraband transitions are $\Delta I = 1$ magnetic dipole ($M1$) in nature and the cross-over $E2$ transitions are either weak or absent implying small deformation ($\beta$) of the nucleus. The dipole transition strengths [$B(M1)$] are large generally of the order of $1 - 10 \, \mu_N^2$. The quadrupole transition rates [$B(E2)$] lie in the range of $1 - 0.01 \,(eb)^2$ or less.
III. The $B(M1)$ values decrease with increasing angular momentum. The ratios of $B(M1)/B(E2)$ values are quite large [$\geq 20\mu_N^2/(eb)^2$] and decrease with angular momentum.

IV. The dynamical moment of inertia ($\mathcal{J}^2$) is small, of the order of 10 - 25 $\hbar^2\text{MeV}^{-1}$, correspondingly, the ratio $\mathcal{J}^2/B(E2)$ is larger than 150 $\hbar^2\text{MeV}^{-1}/(eb)^2$ compared with well-deformed [$\sim 10\hbar^2\text{MeV}^{-1}/(eb)^2$] or superdeformed [$\sim 5\hbar^2\text{MeV}^{-1}/(eb)^2$] structures.

V. The bandhead excitation energies and spins are relatively high. The minimum bandhead spin observed is $13/2\hbar$ [13]. These bands have a multi-quasiparticle configuration consists of high-$j$ protons and neutrons, and correspond to small quadrupole deformations ($\beta < 0.15$).

So far, more than 175 bands possessing such properties have been observed in more than 60 nuclides [14]. They group in four islands situated near the shell closure of the nuclear chart.

Figure 2.7: Reduced M1 transition probabilities, $B(M1)$, as a function of transition energy for MR bands in Pb isotopes [10]. The solid curves represent results of TAC calculations.
Amongst the various theoretical approaches which provide insight into the phenomenon of the $\Delta I = 1$ bands and their properties is the pioneering work of Frauendorf within the framework of the tilted axis cranking (TAC) calculations [15, 16], as shown in Fig. 2.7. Later, Macchiavelli et al. [17] presented a semi-classical calculation for the generation of angular momentum in these bands and predictions of reduced transition probabilities $B(M1)$ and $B(E2)$ based on the effective interaction between the neutron and the proton quasi-particles. An integrated view of these ideas is contained in a review article by Clark and Macchiavelli [10]. The reduced transition probabilities $B(M1)$ and $B(E2)$ values have been calculated as a function of spin($I$) and the calculated values are compared with the experimental results. Though MR bands have been observed for weakly deformed systems, the contribution of small core rotation plays a significant role in the excitation mechanism of these bands.

In an alternative approach [18–20], this competition between the core rotation and the quasi-particles coupling have been considered and suggested as a possible mechanism for the generation of angular momentum in the case of $\Delta I = 1$ band. This framework is known as shears mechanism with principal axis cranking (SPAC) model.

### 2.3.1. Structure of the Dipole Bands: Shears Mechanism

An important observation that helped to understand the structure of the $M1$ bands is that they are only built on high-spin states and the band heads are always produced due to the coupling of particle and hole states. They are never found on any other structures, in particular not on the individual particle or hole excitations. As an example, the two proton-particle $I^\pi = 11^-$ state has similar or larger deformation than the bandhead of the rotational-like dipole cascades observed in the Pb isotopes, but no $M1$ bands built on the $11^-$ states have been observed. This fact is the key feature to understand that the $M1$ bands are a new type of nuclear excitation in which the symmetry of the quantal system is broken by anisotropic currents of a few high-spin particles and holes, and not primarily by a deformed core of the nucleus [Fig. 2.8 (a)].
The coupling of the angular momentum vectors produced by the nucleons is the basic mechanism for generating the total spin $I$ of the nucleus. For the weakly deformed nuclei, few protons quasi-particles occupy the high $j$ particlelike orbitals while the neutrons quasi-particles fill up the high $j$ holelike orbitals, or vice versa. A perpendicular coupling of their angular momenta is energetically favored because it maximizes the overlap of the spatial density distribution, which is torus-like for particle orbitals and dumbbell-like for hole orbitals [Fig. 2.9 (a) and (b)]. Energy and angular momentum are increased by a decrease of the angle between the particle and hole spins, $j_\pi$ and $j_\nu$, respectively. Finally, the particle and hole spins are fully aligned. This corresponds to the highest-spin state that can be produced in such a configuration and at that spin the band terminates [Fig. 2.9 (c)]. Since this process can be assimilating to the closing of the blades of a pair of shears, the observed M1 bands are called “shears bands” [10, 13]. This coupling of the particle and hole orbitals, as illustrated in Fig. 2.10, both with high angular momentum ($j_\pi$ or $j_\nu$), results in a substantial transverse component ($\mu_\perp$) of the magnetic dipole moment ($\vec{\mu}$) and gives rise to large magnetic dipole transition probabilities. As the magnetic dipole moment ($\vec{\mu}$) rotates about the axis of the
Chapter 2: Theoretical Models

Structure of the Dipole Bands: Shears Mechanism

Figure 2.9: Density distribution of the (a) high-\(j\) particle and (b) high-\(j\) hole orbitals. In figure (c) the vector diagrams show how the hole and the particle angular momentum vectors couple with increasing spin.

For large angular momenta this mode has been called “magnetic rotation” (MR) [10, 13].

A schematic model has been developed that describes the coupling of two long angular momentum vectors (\(\vec{j}_\pi\) and \(\vec{j}_\nu\)). Using the nomenclature introduced in Fig. 2.10, \(\theta_\pi\) and \(\theta_\nu\) are defined as the angles of the proton and the neutron angular momentum vectors with respect to the total angular momentum, \(\vec{I} = \vec{j}_\pi + \vec{j}_\nu\). The angle between the proton and the neutron angular momentum vectors (\(\vec{j}_\pi\) and \(\vec{j}_\nu\)) is called the shears angle (\(\theta\)). The shears angle (\(\theta\)) for a given state in the band can be derived using the semiclassical expression [10, 13],

\[
\cos \theta = \frac{\vec{j}_\pi \cdot \vec{j}_\nu}{|\vec{j}_\pi||\vec{j}_\nu|} = \frac{[I(I+1) - j_\pi(j_\pi + 1) - j_\nu(j_\nu + 1)]}{2\sqrt{j_\pi(j_\pi + 1)j_\nu(j_\nu + 1)}}. \tag{2.21}
\]

For large angular momenta this can be approximated by

\[
\cos \theta \approx \frac{(I^2 - j_\pi^2 - j_\nu^2)}{2j_\pi j_\nu}. \tag{2.22}
\]
Figure 2.10: Illustration of the spin coupling scheme of a shears state. The perpendicular coupling of the proton particle (hole) and neutron hole (particle) orbitals, each with high spin $J$, results in a large transverse component of the magnetic moment vector, $\mu_\perp$, that rotates around the total angular momentum, $I$, and creates the enhanced $M1$ transitions between the shears states.

The dependence of the magnetic dipole strength $[B(M1)]$ for the transition between the states having spin $I$ and $I - 1$ to the proton angle ($\theta_\pi$) can be expressed as [10, 13],

$$B(M1, I \rightarrow I - 1) = \frac{3}{8\pi} \mu_\perp^2 = \frac{3}{8\pi} g_{\text{eff}} j_\pi^2 \sin^2 \theta_\pi$$ \hspace{1cm} (2.23)

where $g_{\text{eff}} = g_{\pi} - g_\nu$ is the effective gyromagnetic factor and $j_\pi$ is the angular momentum due to the proton quasi-particles.

The $B(E2, I \rightarrow I - 2)$ value for the shears configuration which is proportional to the square of the electric quadrupole tensor can be expressed as [10, 13],

$$B(E2, I \rightarrow I - 2) = \frac{5}{16\pi} (eQ)^2_{\text{eff}} \frac{3}{8} \sin^4 \theta_\pi [e^2 b^2]$$ \hspace{1cm} (2.24)
where, \((eQ)_{eff} = e_{\pi}Q_{\pi} + \frac{j_{\pi}^{2}}{j_{\nu}^{2}} e_{\nu}Q_{\nu}\) which takes into account the contribution from protons and neutrons. The \(B(E2)\) values also decrease with the closing of the shears and became zero because the charge distribution becomes symmetric around the rotation axis for the complete alignment.

From the experimental spins, parities and excitation energies of the states in the MR bands in Pb \((A \sim 190)\) region it was concluded that they are built on proton particles, in the \(h_{9/2}\) and / or \(i_{13/2}\), coupled to the \(\nu_{13/2}\) holes. In the \(A \sim 100\) region, the main components of the structure of the shears bands are the \(\pi g_{9/2}^{n}\) proton holes coupled to \(\nu h_{11/2}^{n}\) neutron-particle excitations \([10, 13]\). The experimental results of the dipole bands have been well reproduced assuming the above configurations in mass \(A \sim 190\) and 100 regions \([10, 13, 21, 22]\). For the \(A \sim 140\) region \(\pi h_{11/2}^{n}\) proton particles coupled to \(\nu h_{11/2}^{n}\) neutron and / or \(\pi (g_{7/2}d_{5/2})^{-n}\) proton holes have been suggested for the structure of the \(M1\) bands \([10, 13]\). It has been suggested \([13]\) that the dipole bands in \(A \sim 140\) region are based on considerable deformation of the core, compared to other mass region \([10]\), which is not included in the coupling scheme of the shears model.

### 2.3.2. Shears Mechanism with the Principal Axis Cranking

In general, the conditions required for magnetic rotation is that the coupling of high-spin particles and holes has to be dominate over the electric rotation of the weakly deformed core. In recent studies, several MR bands have been observed in which the states have significant contribution of deformed rotational core angular momentum to the total angular momentum vector. Several attempts were made to take into account of the core rotational contribution of the shears mechanism. Recently, a geometric model called Shears mechanism with the Principal Axis Cranking (SPAC) becomes very successful to describe the intrinsic properties of the MR bands in mass \(A \sim 140\) region.

In this model a part of the core angular momentum \((\vec{R})\) is coupled to the shears angular momentum \((\vec{j}_{sh})\) to generate the total angular momentum \(I\) of the observed states \([18–20, 23]\).
(Fig. 2.11). The total energy of an excited state with spin $I$ can be expressed as,

$$E(I) = E(\text{core}) + E(\text{shears}) + \text{constant}. $$

Here,

$$E(\text{core}) = \frac{R^2(I, \theta_1, \theta_2)}{2J(I)}$$

represents the contribution of the core to the energy $E(I)$, and

$$E(\text{shears}) = v_2 P_2(\cos(\theta_1 - \theta_2))$$

is the interaction energy due to the shear blades $\vec{j}_1$ and $\vec{j}_2$, and, $\theta_1$ and $\theta_2$ are the angle of $\vec{j}_1$ and $\vec{j}_2$ with respect to rotational axis $\hat{x}$. The reduced transition probabilities of dipole and quadrupole transitions, with the classical approximation for Clebsch-Gordon coefficients, can be written as,

$$B(M1) = \frac{3}{8\pi}[j_1g_1^*\sin(\theta_1 - \theta_I) - j_2g_2^*\sin(\theta_1 - \theta_2)]^2$$

and

$$B(E2) = \frac{15}{128\pi}[Q_{eff}\sin^2\theta_1 + Q_{coll}\cos^2\theta_I]^2,$$
respectively, where, $g_1^* = g_1 - g_R$, $g_2^* = g_2 - g_R$ and $g_R = \frac{Z}{A}$. Here, $Q_{eff}$ and $Q_{coll}$ are the quasi-particle and collective quadrupole moments, respectively. Here, $\theta_I$ is the angle of the total angular momentum vector ($I$) with respect to the rotational axis ($\hat{x}$). For each value of $I$, $\theta_1$ and $\theta_2$ can be found from the energy minimization condition,

$$\frac{\partial^2 E(I, \theta_1, \theta_2)}{\partial \theta_1 \partial \theta_2} = 0.$$ 

The detail of the model is discussed in Appendix A.

The experimental energy ($E_\gamma$), spin ($I$), rotational frequency ($\omega$) and $B(M1)$ values of the dipole bands for weakly deformed nuclei in $A \sim 140$ region have been well reproduced in the framework of the SPAC model. An illustration of the comparison of the experimental results of the MR bands in $^{142}$Gd with the SPAC model calculations is shown in Fig. 2.12.
2.4. Antimagnetic Rotation

A special case of rotational-like spectra in weakly deformed nuclei may occur when the symmetry is broken with respect to the total angular momentum vector produced by two shears-like configurations [Fig. 2.8 (b)]. There are evidences for this type of shears structure in the $A \sim 110$ region. The following coupling will give rise to $E2$ bands. However, unlike the case of magnetic rotation, these two shears are arranged in such a way that there is practically no component of the magnetic dipole moment perpendicular to the total angular momentum and, therefore, the $B(M1)$ values vanish. This type of shears coupling has been called “antimagnetic rotation” [16] due to it’s resemble to the cancellation of magnetic moments in an antiferromagnetic substance. For the antimagnetic rotation, the magnetic moment of each of the two shears specifies the orientation like the magnetization of one of the sublattices in an antiferromagnet. For the antimagnetic-rotational bands the angular momentum is generated by the simultaneous step-by-step closing of the two shears. Since the antimagnetic rotor is symmetric with respect to a rotation by $180^\circ$ about the angular momentum axis, the bands should consist of sequences of energy levels differing in spin by $2\hbar$. Due to the small deformation of the core they should decay by weak $E2$ transitions, with $B(E2)$ values decreasing with increasing spin [16].

There were first tentative suggestions for antimagnetic-rotational bands in $^{100}$Pd [24]. In this isotope the sequences of $E2$ transitions were observed which could have antimagnetic character. In a recent work, lifetimes have been measured for states in the lowest-lying positive-parity band in $^{106}$Cd [25, 26]. The deduced $B(E2)$ values are small and decrease with increasing spin as shown in Fig. 2.13. As can be seen in Fig. 2.13, the TAC calculations are in good agreement with the experimental $B(E2)$ values. The TAC calculations yield $\beta_2 \approx 0.16$ at spin $I \sim 17$ and there is a rapid decrease in $\beta_2$ with increase in spin. The experimental results are in good agreement with the cranked Nilsson-Strutinsky (CNS), TAC and the semiclassical model (SCM) calculations (Fig. 2.13). The observed rapid approach to a spherical shape is well explained by the shears mechanism [13]. The neutron particles drive the core towards oblate deformation, while the proton holes are prolate-driving. When the
Chapter 2: Theoretical Models

Antimagnetic Rotation

The number of investigations in antimagnetic rotation is not very large, and so there is a need to investigate and find more evidence for this new type of shears coupling through the measurement of lifetimes in particular, in this and the other mass regions.

References

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Figure 2.13: Comparison of the experimental $B(E2)$ values of the antimagnetic rotational band in $^{106}$Cd with the cranked Nilsson-Strutinsky (CNS), tilted axis cranking (TAC) and the semiclassical model (SCM) calculations [25]. The arrows in the figure show the relative orientation of angular momentum of the $g_{9/2}$ proton holes as predicted by the TAC calculations.

Angular momenta of the proton holes are perpendicular to those of the neutrons, i.e. near the band head, they generate a small prolate deformation. The symmetry axis of this charge distribution is perpendicular to the rotational axis. This yields the $B(E2)$ values near the band head. However, when the angular momentum of the protons and neutrons are aligned, their contributions to the deformation cancel, resulting in a near-spherical shape with very small $B(E2)$ values.

Another type of regular band-like structure with small decreasing features of the $B(E2)$ transition strength has been observed for several nuclei in the mass $A \sim 110$ and 160 regions and has been interpreted as smoothly terminating bands [2, 28]. In a smoothly terminating band, the angular momenta of the valence particles are gradually aligned and the band terminates when all quasi-particle vectors are aligned along the axis of symmetry, just as the antimagnetic rotors terminate when the shears blades are aligned. However, the magnitude of the quadrupole deformation is larger than that expected for antimagnetic rotors, which start with a small deformation that is substantially reduced or even becomes zero when approaching termination. Moreover, the number of valence particles which generate the angular momentum increment in smoothly terminating bands (typically $> 10$ in the $A \sim 110$ region) is larger.
Figure 2.14: Plot of the $J^{(2)}/B(E2)$ ratio vs spin for the antimagnetic bands in $^{105,106}$Cd and the smoothly terminating bands in $^{108}$Sn and $^{109}$Sb [25–27].

than for the antimagnetic rotors, which makes the rotation more collective. For antimagnetic rotational bands, the experimental dynamic moment of inertia $J^{(2)}$ remain fairly constant as a function of spin and rotational frequency. This contrasts with the behavior seen in the smoothly terminating bands where a definite decrease is observed as the spin increases in the region of interest. In the case of antimagnetic rotation, since the $B(E2)$ values are expected to decrease rapidly with increasing spin, the $J^{(2)}/B(E2)$ ratio should show a rapid increase in contrast to a constant character in the case of a smoothly terminating band. This contrast characteristic of the antimagnetic rotational bands in $^{105,106}$Cd and the smoothly terminating bands in $^{108,109}$Sb has been demonstrated in Fig. 2.14.

The observation of conjugate shear structure responsible for the generation of angular momentum in near spherical systems in the form of AMR is also associated with the possibility of a similar complementary excitation mode called magnetic rotation due to single shear structure [16]. Since both these two types of quantized rotation are the consequence of the shear mechanism, it is expected to observe both of them in the mentioned mass regions. However, until today, simultaneous occurrence of these two phenomena have been observed only in mass $\sim 100$ region where firm experimental evidence of antimagnetic rotation has been reported in several Cd [25–27, 29–31] isotopes and in the $^{104}$Pd nucleus [32] along with the observation of
Chapter 2: Theoretical Models

SPR model

MR bands [33, 34]. These bands have been interpreted in the fully self-consistent microscopic tilted axis cranking method based on covariant density functional theory [35] and as well as in the framework of simple geometric semi-classical particle plus rotor (SPR) model [31, 36].

2.4.1. Semi-classical particle plus rotor model

In order to explore underlying characteristics of a AMR band the geometric model called semi-classical particle plus rotor model has become a powerful tool in the nuclear structure studies. This model has been interpreted successfully the experimental signatures of the AMR bands in Cd-isotopes in $A \sim 110$ mass region. In this model, the energy $E(I)$ of a state with spin $(I)$ can be expressed as,

$$E(I) = \frac{(I - j_\pi - j_\nu)^2}{23} + \frac{V_{\pi\nu}}{2} \left( \frac{3\cos^2\theta - 1}{2} \right) + \frac{V_{\pi\nu}}{2} \left( \frac{3\cos^2(-\theta) - 1}{2} \right) + \frac{V_{\pi\pi}}{n} \left( \frac{3\cos^2(2\theta) - 3}{2} \right)$$

(2.25)

where, the first term is the core rotational contribution to the shears energy. Here, $\theta$ is the angle between the proton and neutron quasi-particles angular momentum vectors, $j_\pi$ and $j_\nu$, respectively. The rest of the terms arise due to effective interaction ($V_{\pi\nu}$ and $V_{\pi\pi}$) between the vectors. The number “n” is the scaling factor between $V_{\pi\nu}$ and $V_{\pi\pi}$ and is determined by the actual number of particle-hole pairs for a given single-particle configuration. The angular momentum state for a given energy and the corresponding $B(E2)$ transition strength can be derived from the energy minimization w.r.t. $\theta$.

The details of the SPR model have been described in Appendix B.


Chapter 2: Theoretical Models

Bibliography