3.1 Introduction

Detailed analysis of non-life insurance sector performance has not been done extensively in Indian scenario. Though some works have been there which pertains to underwriting performance of Indian non-life insurance companies like Gosalia (2008), however, it lacks a detailed study on a longer period 1999-2012. The previous work which has been done in the aspect sheds light more on the presentation of the underwriting performance rather than making statistical analysis and forecasting based on the non-life insurance performance data. Studies on the underwriting cycle have been extensively done in the US and in the European property and casualty insurance market. Some studies have also been done in the Asian market however those were more in respect of developed countries like Japan, China, South Korea and Singapore, etc. Such studies are yet to be done with respect to the Indian market. The identification of the underwriting cycle pattern remains as a virgin area in the Indian context and thus efforts are needed to be made to unfold the underwriting cycle pattern through development of appropriate models, specially, in the arena of Indian non-life insurance market.

This study is an effort to analyse the underwriting experience of non-life insurance sector in India in the post-liberalization era. The efforts have been made to analyse the non-life insurance underwriting performance data using different sophisticated parametric and smoothing models. The effort has also been made of forecasting the future behaviour of the performance pattern of the non-life insurance companies in India.

Non-life insurance may be classified into 3 major type of insurances – Fire, Marine and Miscellaneous.

Miscellaneous insurance includes the following insurances –

- Motor Insurance
- Health Insurance
- Personal Accident Insurance
- Fidelity Insurance
- Burglary Insurance
- Credit Insurance
- Workmen’s Compensation Insurance
- Travel Insurance
- Wedding Insurance
- Employee State Insurance Scheme
- Unemployment Insurance
- Personal Liability Insurance

The studies have been performed considering the combined performances of the non-life insurance companies combining the individual sectors like fire, marine and miscellaneous. Separate studies have also been undertaken on the sector-wise performance, i.e., on fire, marine and miscellaneous.

In the studies undertaken, we have included the following parameters (performance indicators) for analysis –

Earned Premium – The revenue earned by the insurance company as premium in a particular financial year.

Investment Income – The revenue earned by the insurance company from sources other than premium income, like interest from savings etc., in a financial year.

Total Income – The total revenue earned by the insurance company in a particular financial year – it is the sum of earned premium and investment income.

Incurred Claims – Claim is the expense which is made by the insurer to meet up insuree’s loss.

Operating Expenses – This expense is made by the insurer in order to run the insurance business. It includes office rent, expenditure on printing stationaries and other office expenses etc.

Underwriting Profit/Loss – It is the difference between the total premium earned in a financial year and the total expenses made in that financial year. If it is positive, the underwriting experience is profitable but if it is negative, then the business is not profitable.
Total Profit/Loss – It is the difference between the total income (earned premium + investment income) earned in a financial year and the total expenses made in that financial year. If it is positive, the business is running on profit or else it is running on loss.

Combined Ratio - Combined ratio (also defined earlier) is the ratio of paid losses and loss adjustment expenses plus underwriting expenses to the premiums, i.e., it is the ratio of the total expense to the premium earned (sometimes, it is expressed as the percentage multiplying by 100). If it is more than 1 (or 100), the underwriting experience is not profitable whereas if it is less than 1 (or 100), the underwriting experience is not profitable.

Expense/Income Percentage – It is the ratio of the total expense to the total income of the insurer in a financial year, the ratio being multiplied by 100. If the percentage is more than 100, the insurance business is profitable otherwise it is not profitable.

### 3.2 Regression and Forecasting Models

As per Sykes (Inaugural Coase Lecture note on Regression Analysis – See Ref.), regression analysis is a statistical tool for the investigation of the relationships between variables. Usually, the investigator seeks to ascertain the causal effect of one variable upon another – the effect of a price increase upon demand, for example, or the effect of changes in the money supply upon inflation rate. In statistics, regression analysis is a statistical process for estimating the relationships among variables. Regression analysis includes many techniques for modelling and analysing several variables, when the focus is on the relationship between a dependent variable and one or more independent variables. More specifically, regression analysis helps one to understand how typical value of the dependent variable (or ‘criterion variable’) changes when any one of the independent variables is varied, while the other independent variables are held fixed. Most commonly, regression analysis estimates the conditional expectation of the dependent variable given the independent variables – that is, the average value of the dependent variable when the independent variables are fixed.

A time-series is a collection of observations of well-defined data items obtained through repeated measurements over time. For example, measuring the value of retail sales each month of the year would comprise a time-series. This is because sales revenue is well defined, and consistently measured at equally spaced intervals. Data collected with irregularity or only at single time-point are not time-series.

Forecasting is the process of making statements about events whose actual outcomes (typically) have not yet been observed. A commonplace example might be estimation
of some variable of interest at some specified future date. Forecasting is the use of historic data to determine the direction of future trends.

In the study, the regression analysis has been made first using the linear, quadratic as well as cubic model in order to understand the relationship between the dependent variable (Y) as well as the independent time variable (X). The R-square value is telling how good the regression fit is on the data-set. If it is visible that polynomial models (like quadratic, cubic etc.) are giving a good fit, then forecasting has also been done using the same model. Since determining the future values for the dependent variables is easy using the polynomial model, in case regression analysis gives good fitting with polynomial models, forecasting is done using the same model. However, in case the polynomial models are not showing a good fit result while regression analysis or the forecasted values arrived through polynomial curve fitting are not feasible enough, other forecasting models like smoothing models, ARIMA as well as trend curves with seasonal dummies have been used to arrive at better forecast values and higher fit statistics (higher values of R-square, Adjusted R-square and Root Mean Square Error).

In the above graph, dependent variable value is present till the year shown with dotted vertical line (2011). As per the smoothing model fit, the future values (2012 onwards) of the dependent variable (Royal Sundaram Premium Income) are supposed to be values shown in the graph (values of the corresponding points of Y-Axis through which the smoothing curve has passed for the corresponding years – mentioned in X-Axis). The two lines, one above the smoothing curve and one below the smoothing curve, depict the upper limit as well as lower limit of the possible values of the forecast for the corresponding year. It means that if the forecast value doesn’t become equal to the value shown in the smoothing curve for the corresponding year, it will take any value between the limits of the upper and lower line for the corresponding year with 95% probability.
Few Examples -

In the section below, both regression fit as well as forecasting are demonstrated in details to give a glimpse of our study. We have considered the performance indicators - earned premium income of GIC and combined ratio of public insurers for the demonstration below. The details of the regression and forecasting models mentioned in the section will be discussed in details in the latter part of this section.

GIC

Modelling Data on Earned Premium

Related results are shown below.

![Graph 3.2A](image1)

**Y-Axis - Premium in INR Crore - example**

**Graph 3.2A – Regression Fitting – Earned Premium over time GIC - Example**

**Table 3.1 – Fit Results - Earned Premium Values - Example**

<table>
<thead>
<tr>
<th>Curve</th>
<th>Degree (Polynomial)</th>
<th>Model DF</th>
<th>Mean Square</th>
<th>Error DF</th>
<th>Mean Square</th>
<th>R Square</th>
<th>F Stat</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>136158127</td>
<td>14</td>
<td>856633.422</td>
<td>0.9190</td>
<td>158.95</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2</td>
<td>7272930.6</td>
<td>13</td>
<td>208394.903</td>
<td>0.9817</td>
<td>348.96</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3</td>
<td>48491331.9</td>
<td>12</td>
<td>223083.282</td>
<td>0.9819</td>
<td>217.37</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

![Graph 3.2B](image2)

**Y-Axis - Premium in INR Crore - example**

**Graph 3.2B - Earned Premium-Forecast value curve (cubic regression-confidence limits)**

At the very outset, the parameter, “Earned Premium” has been considered. The best fitted model is a polynomial of third degree (though the second degree fit has almost
equal precision level, yet the third degree fit (equation) is preferred (because of its reflective maximum and minimum inherent there in the model equation theoretically), as the Graph plot (Graph 3.2A) exhibits small ups and downs. The earned premium values (predicted values) lie on the fitted third degree curve at most positions (years), with only two/three points falling outside (though very near) the curve, the pattern of movement of earned premiums is overall increasing. Graph 3.2B shows the forecasting of earned premium variable using the cubic trend (whose R-square value was found highest as per table 3.1 – the r-square value 0.9819). The succeeding graph (Graph 3.2B) shows the confidence limits for the forecast values obtained for the period, 2012 - 2022. It is seen from the Table 3.1 that the P- values corresponding to the three coefficients in the cubic polynomial are less than .0001 implying that the said coefficients are highly significant.

The actual Forecast values (in crores INR) corresponding to the years, 2012 to 2022 are given in the Table 3.2. The values corresponding to three (3) different goodness of fit criteria, namely, RMSE, R-SQUARE, AJDRSQ, are included in the Table 3.3 to adjudge the precision level of the selected curve, and the values obtained after employing other fit-criteria also claim very close fits. As the forecasted values obtained from cubic model are found feasible, the cubic model has been accepted as the best model for forecast for the parameter ‘earned premium’ of General Insurance Corporation.

Table 3.2 - Forecast Values (Crores INR) - Example

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast</td>
<td>1220</td>
<td>1356</td>
<td>1501</td>
<td>1654</td>
<td>1815</td>
<td>1984</td>
<td>2161</td>
<td>2347</td>
<td>2541</td>
<td>2743</td>
<td>2954</td>
</tr>
<tr>
<td></td>
<td>4.97</td>
<td>8.34</td>
<td>3.70</td>
<td>1.18</td>
<td>0.94</td>
<td>3.11</td>
<td>7.83</td>
<td>5.25</td>
<td>5.52</td>
<td>8.76</td>
<td>5.14</td>
</tr>
</tbody>
</table>

Table 3.3 - Fit Results (All Criteria) - Example

<table>
<thead>
<tr>
<th>Statistics of Fit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root Mean Squared Error</td>
<td>411.2951</td>
</tr>
<tr>
<td>R-Square</td>
<td>0.982</td>
</tr>
<tr>
<td>Adjusted R-Square</td>
<td>0.977</td>
</tr>
</tbody>
</table>

As the value of the R-square is very near to 1.0 (= 0.98), also, the Adjusted R-square value being 0.977, the fit is adjudged excellent.
Public Non-Life Insurers

Performance of the Public Non-life Insurers (Combined Data): Combined Ratio

Graph 3.3 - Linear/Quadratic/Cubic Model Fitting in Combined Ratio Data - example

Table 3.4 - Linear/Quadratic/Cubic Fit Table – Combined Ratio Data

<table>
<thead>
<tr>
<th>Curve</th>
<th>Degree(Polynomial)</th>
<th>DF</th>
<th>Mean Square</th>
<th>DF</th>
<th>Mean Square</th>
<th>R-Square</th>
<th>F Stat</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>73.0024</td>
<td>11</td>
<td>41.9372</td>
<td>0.1366</td>
<td>1.74</td>
<td>0.2138</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2</td>
<td>148.8047</td>
<td>10</td>
<td>23.6702</td>
<td>0.5570</td>
<td>6.29</td>
<td>0.0171</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3</td>
<td>100.3155</td>
<td>9</td>
<td>25.9295</td>
<td>0.5632</td>
<td>3.87</td>
<td>0.0498</td>
</tr>
</tbody>
</table>

Forecasting - Graph

Y-Axis – Combined Ratio / X-Axis - Year

Graph 3.4 - Forecasting on Combined Ratio Data - example

We have tried different linear and non-linear models for forecasting and found the ARIMA (1, 1, 1) model to be most suitable for modelling the combined ratio data. The fit is poor (R-square = 0.3).
Table 3.5 – Fit Results for Combined Ratio Data

<table>
<thead>
<tr>
<th>Statistics of Fit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-Square</td>
<td>0.3</td>
</tr>
<tr>
<td>Root Mean Square Error</td>
<td>6.944782214</td>
</tr>
<tr>
<td>Adjusted R-Square</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 3.6 – Forecast Values - Combined Ratio Data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast</td>
<td>121.8</td>
<td>118.8</td>
<td>120.8</td>
<td>121.1</td>
<td>122.0</td>
<td>122.6</td>
<td>123.3</td>
<td>124.0</td>
<td>124.7</td>
<td>125.5</td>
<td>126.2</td>
<td></td>
</tr>
</tbody>
</table>

We have tried combined cubic trend for the same data and the result is given below.

**Comparison with Cubic Trend**

![Graph 3.5 - Forecasting on Combined Ratio Data with cubic trend - example](image)

Table 3.7 – Fit Results for Combined Ratio Data with Cubic Trend

<table>
<thead>
<tr>
<th>Statistics of Fit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-Square</td>
<td>0.56</td>
</tr>
<tr>
<td>Root Mean Square Error</td>
<td>4.26</td>
</tr>
<tr>
<td>Adjusted R-Square</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Table 3.8 – Forecast Values - Combined Ratio Data

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreca</td>
<td>110.8</td>
<td>106.8</td>
<td>101.4</td>
<td>95.6</td>
<td>89.2</td>
<td>82.2</td>
<td>74.5</td>
<td>66.1</td>
<td>57.0</td>
<td>47.3</td>
<td>36.9</td>
</tr>
</tbody>
</table>

The forecast values obtained from the cubic model (shown in table 3.8) are not feasible in long run as it is showing a downward trend and will be negative after 15 years or more. Hence, we cannot use cubic trend (for forecasting) whose R-square fit value (0.56 – Table 3.7) came more than ARIMA(1,1,1) model fit whose R-square fit value came to be just 0.3 (Table 3.5). Feasibility of the forecast values have also been considered at the time of selecting the model for forecast and not just the best fit regression model has been used to make the forecasting.

The details of the models mentioned above will be discussed in details in the below section.

The following models have been used in the study –

- Linear Trend Model
- Quadratic model
- Cubic model
- Log Linear Trend Model
- Log Cubic Trend Model
- Log Linear (Holt) Exponential Smoothing Model
- ARIMA (1,1,1) Model
- Log Double (Brown) Exponential Smoothing Model
- Square-Root Linear Model
- Simple Exponential Smoothing
- Log Damped Trend Exponential Smoothing Model
- Logarithmic Trend + Seasonal Dummies
- Linear Trend + Seasonal Dummies
- Double (Brown) Exponential Smoothing
Linear Trend Model

The relationship between a response variable and a set of explanatory variables can be studied through a regression model

\[ y_i = f(x_i) + \epsilon_i, \]

where \( y_i \) is the \( i \)-th observed response value, \( x_i \) is the \( i \)-th vector of explanatory values, and \( \epsilon_i \) are uncorrelated random variables with zero mean and a common variance.

If the form of the regression function \( f \) is known except for certain parameters, the model is called a parametric regression model. Furthermore, if the regression function is linear in the unknown parameters, the model is called a linear model. In the case of linear models with the error term \( \epsilon_i \) assumed to be normally distributed, classical linear models can be used to explore the relationship between the response variable and the explanatory variables. A nonparametric model generally assumes only that \( f \) belongs to some infinite-dimensional collection of functions. For example, \( f \) may be assumed to be differentiable with a square-integrable second derivative. When there is only one explanatory \( X \) variable, one can use nonparametric smoothing methods, such as smoothing splines, kernel estimators, and local polynomial smoothers. One can also request confidence ellipses and parametric fits (mean, linear regression, and polynomial curves) with a linear model. When there are two explanatory variables in the model, one can create parametric and nonparametric (kernel and thin-plate smoothing spline) response surface plots. With more than two explanatory variables in the model, a parametric profile response surface plot with two selected explanatory variables can be created.

When the response \( y_i \) has a distribution from the exponential family (normal, inverse Gaussian, gamma, Poisson, binomial), and the mean \( \mu_i \) of the response variable \( y_i \) is assumed to be related to a linear predictor through a monotone function \( g \):

\[ g(\mu_i) = X_i'\beta, \]

where \( \beta \) is a vector of unknown parameters, the relationship can be explored by using generalized linear models.
The linear trend curve –

\[ Y_i = A + B \cdot X_i + \epsilon_i, \] where A and B are the parameters.

**Quadratic Model**

The quadratic trend model can be described as –

\[ Y_i = A + B \cdot X_i + C \cdot X_i^2 + \epsilon_i, \] where A, B and C are the parameters.

**Cubic Model**

The cubic model can be described as –

\[ Y_i = A + B \cdot X_i + C \cdot X_i^2 + D \cdot X_i^3 + \epsilon_i, \] where A, B, C and D are the parameters.

**Log-linear Trend Model**

A **log-linear model** is a mathematical model that takes the form of a function whose logarithm is a first-degree polynomial function of the parameters of the model, which makes it possible to apply (possibly multivariate) linear regression. That is, it has the general form

\[ \exp \left( c + \sum_i w_i f_i(X) \right), \]

In which the \( f_i(X) \) are quantities that are functions of the variables \( X \), in general a vector of values, while \( c \) and the \( w_i \) stand for the model parameters. (Log-Linear Model – Wikipedia)

**Log Cubic Trend Model**

A **log-cubic model** is a mathematical model that takes the form of a function whose logarithm is a third-degree polynomial function of the parameters of the model, which makes it possible to apply regression.
Square-Root Linear Model

This is similar to linear model where the linear regression line is fitted to the square-root of the dependent variable observations.

Equations for the Smoothing Models

Simple Exponential Smoothing

The model equation for simple exponential smoothing is

\[ Y_t = \mu_t + \epsilon_t \]

The smoothing equation is

\[ L_t = \alpha Y_t + (1 - \alpha) L_{t-1} \]

The error-correction form of the smoothing equation is

\[ L_t = L_{t-1} + \alpha \epsilon_t \]

(Note: For missing values, \( \epsilon_t = 0 \).)

The k-step prediction equation is

\[ \hat{Y}_t(h) = L_t \]

The ARIMA model equivalency to simple exponential smoothing is the ARIMA(0,1,1) model
The moving-average form of the equation is

\[(1 - B)Y_t = (1 - \theta B)\epsilon_t\]

\[\theta = 1 - \alpha\]

The moving-average form of the equation is

\[Y_t = \epsilon_t + \sum_{j=1}^{\infty} \alpha \epsilon_{t-j}\]

For simple exponential smoothing, the additive-invertible region is \[\{0 < \alpha < 2\}\]

The variance of the prediction errors is estimated as

\[\text{var}(\epsilon_t(k)) = \text{var}(\epsilon_t) \left[1 + \sum_{j=1}^{k-1} \alpha^2\right] = \text{var}(\epsilon_t)(1 + (k - 1)\alpha^2)\]

**Double (Brown) Exponential Smoothing**

The model equation for double exponential smoothing is

\[Y_t = \mu_t + \beta_t \delta_t + \epsilon_t\]
The smoothing equations are

\[ L_t = \alpha Y_t + (1 - \alpha) L_{t-1} \]

\[ T_t = \alpha (L_t - L_{t-1}) + (1 - \alpha) T_{t-1} \]

This method may be equivalently described in terms of two successive applications of simple exponential smoothing:

\[ S_{t}^{[1]} = \alpha Y_t + (1 - \alpha) S_{t-1}^{[1]} \]

\[ S_{t}^{[2]} = \alpha S_{t}^{[1]} + (1 - \alpha) S_{t-1}^{[1]} \]

where \( S_{t}^{[1]} \) are the smoothed values of \( Y_t \) and \( S_{t}^{[2]} \) are the smoothed values of \( S_{t}^{[1]} \). The prediction equation then takes the form:

\[ \hat{Y}_t(k) = (2 + \alpha k/(1 - \alpha)) S_{t}^{[1]} - (1 + \alpha k/(1 - \alpha)) S_{t}^{[2]} \]

The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + T_{t-1} + \alpha e_t \]

\[ T_t = T_{t-1} + \alpha^2 e_t \]
The k-step prediction equation is

\[
\hat{Y}_t(k) = L_t + ((k - 1) + \frac{1}{2}) \epsilon_t
\]

The ARIMA model equivalency to double exponential smoothing is the ARIMA(0,2,2) model

\[
(1 - B)^2 Y_t = (1 - \theta B)^2 \epsilon_t
\]

\[
\theta = 1 - \alpha
\]

The moving-average form of the equation is

\[
Y_t = \epsilon_t + \sum_{j=1}^{\infty} (2\alpha + (j - 1)\alpha^2) \epsilon_{t-j}
\]

For double exponential smoothing, the additive-invertible region is

\[
\{0 < \alpha < 2\}
\]

The variance of the prediction errors is estimated as
The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + T'_{t-1} + \alpha e_t \]

\[ T_t = T'_{t-1} + \alpha \gamma e_t \]

(Note: For missing values, \( e_t = 0 \).)

The k-step prediction equation is

\[ \text{var}(e_t(k)) = \text{var}(e_t) \left[ 1 + \sum_{j=1}^{k-1} (2\alpha + (j-1)\alpha^2) \right] \]
\[ \hat{Y}_t(k) = L_t + kT_t \]

The ARIMA model equivalency to linear exponential smoothing is the ARIMA(0,2,2) model

\[ (1 - B)^2 Y_t = (1 - \theta_1 B - \theta_2 B^2) \epsilon_t \]

\[ \theta_2 = 2 - \alpha - \alpha \gamma \]

\[ \theta_2 = \alpha - 1 \]

The moving-average form of the equation is

\[ Y_t = \epsilon_t + \sum_{j=1}^{\infty} (\alpha + j \alpha \gamma) \epsilon_{t-j} \]

For linear exponential smoothing, the additive-invertible region is

\[ \{ 0 < \alpha < 2 \} \]

\[ \{ 0 < \gamma < 4/\alpha - 2 \} \]
The variance of the prediction errors is estimated as

\[
\text{var}(\hat{c}_t) = \text{var}(c_t) \left[ 1 + \sum_{j=1}^{s-1} (\alpha + j \gamma)^2 \right]
\]

Damped-Trend Linear Exponential Smoothing

The model equation for damped-trend linear exponential smoothing is

\[
Y_t = \mu_t + \beta_t \hat{L}_t + \epsilon_t
\]

The smoothing equations are

\[
\hat{L}_t = \alpha Y_t + (1 - \alpha)(\hat{L}_{t-1} + \phi \hat{T}_{t-1})
\]

\[
\hat{T}_t = \gamma(\hat{L}_t - \hat{L}_{t-2}) + (1 - \gamma)\phi \hat{T}_{t-1}
\]

The error-correction form of the smoothing equations is

\[
\hat{L}_t = \hat{L}_{t-1} + \phi \hat{T}_{t-1} + \alpha \epsilon_t
\]

\[
\hat{T}_t = \phi \hat{T}_{t-1} + \alpha \gamma \epsilon_t
\]

(Note: For missing values, \( \epsilon_t = 0 \).)
The k-step prediction equation is

$$\hat{Y}_t(k) = L_t + \sum_{i=1}^{k} \phi^i T_i$$

The ARIMA model equivalency to damped-trend linear exponential smoothing is the ARIMA(1,1,2) model

$$(1 - \phi R)(1 - R) Y_t = (1 - \theta_1 R - \theta_2 R^2) \epsilon_t$$

$$\theta_2 = 1 + \phi - \alpha - \alpha \gamma \phi$$

$$\theta_2 = (\alpha - 1) \phi$$

The moving-average form of the equation (assuming $|\phi| < 1$) is

$$Y_t = \epsilon_t + \sum_{j=1}^{\infty} (\alpha + \alpha \gamma \phi \left(\phi^j - 1\right) / \left(\phi - 1\right))) \epsilon_{t-j}$$

For damped-trend linear exponential smoothing, the additive-invertible region is

$$\{0 < \alpha < 2\}$$

$$\{0 < \phi \gamma < 4 / (\alpha - 2)\}$$
The variance of the prediction errors is estimated as

$$\text{var}(e_t) = \text{var}(z_t) \left[ 1 + \sum_{j=1}^{k-1} (\alpha + \alpha \gamma \phi^j) ((\phi - 1)^j/(\phi - 1)^j)^2 \right]$$

**Seasonal Exponential Smoothing**

The model equation for seasonal exponential smoothing is

$$Y_t = \mu_t + s_p(t) + \epsilon_t$$

The smoothing equations are

$$L_t = \alpha (Y_t - S_{t-p}) + (1 - \alpha) L_{t-1}$$

$$S_t = \delta (Y_t - L_t) + (1 - \delta) S_{t-1}$$

The error-correction form of the smoothing equations is

$$L_t = L_{t-1} + \alpha \epsilon_t$$

$$S_t = S_{t-p} + \delta (1 - \alpha) \epsilon_t$$

(Note: For missing values, \( \epsilon_t = 0 \).)

The k-step prediction equation is
\[ \hat{Y}_t(k) = L_t + S_{t-p+k} \]

The ARIMA model equivalency to seasonal exponential smoothing is the ARIMA\((0,1,p+1)(0,1,0)_p\) model

\[(1 - B)(1 - B^p)Y_t = (1 - \theta_1 B - \theta_2 B^p - \theta_3 B^{p+1})\epsilon_t \]

\[\theta_1 = 1 - \alpha \]

\[\theta_2 = 1 - \delta(1 - \alpha) \]

\[\theta_3 = (1 - \alpha)(\delta - 1) \]

The moving-average form of the equation is

\[ Y_t = \epsilon_t + \sum_{j=1}^{\infty} \psi_j \epsilon_{t-j} \]

\[ \psi_j = \begin{cases} 
\alpha & \text{for } j \mod p \neq 0 \\
\alpha + \delta(1 - \alpha) & \text{for } j \mod p = 0 
\end{cases} \]

For seasonal exponential smoothing, the additive-invertible region is
\[
\{ \max(-p\alpha, 0) < \delta(1 - \alpha) < (2 - \alpha) \}
\]

The variance of the prediction errors is estimated as

\[
\text{var}(e_t | \hat{s}_t) = \text{var}(s_t) \left[ 1 + \sum_{j=1}^{k-1} \psi_j^2 \right]
\]

**Winters Method -- Additive Version**

The model equation for the additive version of Winters method is

\[
Y_t = \mu_t + \beta_t s_t + \gamma_t T_t + e_t
\]

The smoothing equations are

\[
L_t = \alpha(Y_t - S_{t-p}) + \beta_t(1 - \alpha)(L_{t-1} + T_{t-1})
\]

\[
T_t = \gamma(L_t - L_{t-1}) + (1 - \gamma)T_{t-1}
\]

\[
S_t = \delta(Y_t - L_t) + (1 - \delta)S_{t-1}
\]

The error-correction form of the smoothing equations is

\[
L_t = L_{t-1} + T_{t-1} + \alpha e_t
\]
\[ T_t = T_{t-1} + \alpha \gamma e; \]

\[ S_t = S_{t-p} + \delta (1 - \alpha) e; \]

(Note: For missing values, et=0.)

The k-step prediction equation is

\[ \hat{Y}_t(k) = L_t + k T_t + S_{t-p+k} \]

The ARIMA model equivalency to the additive version of Winters method is the ARIMA(0,1,p+1)(0,1,0)_p model

\[
(1 - B)(1 - B^p)Y_t = \left[ 1 - \sum_{i=1}^{p+1} \theta_i B^i \right] e;
\]

\[
\theta_j = \begin{cases} 
1 - \alpha - \alpha \gamma & j = 1 \\
-\alpha \gamma & 2 \leq j \leq p - 1 \\
1 - \alpha \gamma - \delta (1 - \alpha) & j = p \\
(1 - \alpha)(\delta - 1) & j = p + 1
\end{cases}
\]

The moving-average form of the equation is

\[ Y_t = \varepsilon_t + \sum_{j=1}^{\infty} \psi_j \varepsilon_{t-j} \]
For the additive version of Winters method (see Archibald 1990), the additive-invertible region is
\[
\psi_j = \begin{cases} 
\alpha + j\alpha\gamma & \text{for } j \mod p \neq 0 \\
\alpha + j\alpha\gamma + \delta(1 - \alpha) & \text{for } j \mod p = 0
\end{cases}
\]

where \( \psi \) is the smallest non-negative solution to the equations listed in Archibald (1990).

The variance of the prediction errors is estimated as
\[
\text{var}(e_t) = \text{var}(\hat{\epsilon}_t) \left[ 1 + \sum_{j=1}^{k-1} \psi_j^2 \right]
\]

**Winters Method -- Multiplicative Version**

In order to use the multiplicative version of Winters method, the time series and all predictions must be strictly positive.

The model equation for the multiplicative version of Winters method is
\[
Y_t = (\mu_t + \beta_t \hat{\epsilon}) s_p(t) + \epsilon_t
\]
The smoothing equations are

\[ L_t = \alpha(Y_t / S_{t-p}) + (1 - \alpha)(L_{t-1} + T_{t-1}) \]

\[ T_t = \gamma(L_t - L_{t-1}) + (1 - \gamma)T_{t-1} \]

\[ S_t = \delta(Y_t / L_t) + (1 - \delta)S_{t-p} \]

The error-correction form of the smoothing equations is

\[ L_t = L_{t-1} + T_{t-1} + \alpha e_t / S_{t-p} \]

\[ T_t = T_{t-1} + \alpha \gamma e_t / S_{t-p} \]

\[ S_t = S_{t-p} + \delta(1 - \alpha)e_t / L_t \]

(Note: For missing values, \( e_t = 0 \).)

The k-step prediction equation is

\[ \hat{Y}_t(k) = (L_t + kT_t)S_{t-p+k} \]

The multiplicative version of Winters method does not have an ARIMA equivalent; however, when the seasonal variation is small, the ARIMA additive-invertible region of
the additive version of Winters method described in the preceding section can approximate the stability region of the multiplicative version.

The variance of the prediction errors is estimated as

\[ \text{var}(\epsilon_t) = \text{var}(\epsilon_t) \left\{ \sum_{i=0}^{\infty} \sum_{j=0}^{p-1} (\psi_{j+i} \phi_{i+p} S_{t+i+k} / S_{t+i+k-j})^2 \right\} \]

where \( \psi_j \) are as described for the additive version of Winters method, and \( \psi_j = 0 \) for \( j > k \).

**ARIMA Model**

A dependent time series that is modeled as a linear combination of its own past values and past values of an error series is known as a (pure) ARIMA model.

**Non-seasonal ARIMA Model Notation**

The order of an ARIMA model is usually denoted by the notation ARIMA\((p,d,q)\), where

- \( p \) is the order of the autoregressive part
- \( d \) is the order of the differencing (rarely should \( d > 2 \) be needed)
- \( q \) is the order of the moving-average process

Given a dependent time series \( \{Y_t: 1 \leq t \leq n\} \), mathematically the ARIMA model is written as

\[ (1 - B)^d Y_t = \mu + \frac{\theta(B)}{\phi(B)} \epsilon_t, \]

where

- \( t \) indexes time
µ is the mean term

B is the backshift operator; that is, \( BX_t = X_{t-1} \)

\( \phi(B) \) is the autoregressive operator, represented as a polynomial in the back shift operator: \( \phi(b) = 1 - \phi_1 B - \ldots - \phi_p B^p \).

\( \theta(B) \) is the moving-average operator, represented as a polynomial in the back shift operator: \( \theta(b) = 1 - \theta_1 B - \ldots - \theta_p B^p \).

\( \epsilon_t \) is the independent disturbance, also called the random error.

For example, the mathematical form of the ARIMA(1,1,2) model is

\[
(1 - B)Y_t = \mu + \frac{(1 - \theta_1 B - \theta_2 B^2)}{(1 - \phi_1 B)} \epsilon_t
\]

Linear Trend + Seasonal Dummies

This series is modeled using an unobserved component model called the basic structural model (BSM). The BSM models a time series as a sum of three stochastic components: a trend component \( \mu_t \), a seasonal component \( \gamma_t \), and random error \( \epsilon_t \). Formally, a BSM for a response series \( y_t \) can be described as

\[
y_t = \mu_t + \gamma_t + \epsilon_t
\]

Each of the stochastic components in the model is modeled separately. The random error \( \epsilon_t \), also called the irregular component, is modeled simply as a sequence of independent, identically distributed (i.i.d.) zero-mean Gaussian random variables. The trend and the seasonal components can be modeled in a few different ways. The model for trend used here is called a locally linear time trend. This trend model can be written as follows:

\[
\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \quad \eta_t \sim i.i.d. \ N(0, \sigma^2_{\eta})
\]
These equations specify a trend where the level $\mu$ as well as the slope $\beta_t$ is allowed to vary over time. This variation in slope and level is governed by the variances of the disturbance terms $\eta_t$ and $\xi_t$ in their respective equations. Some interesting special cases of this model arise when these disturbance variances are manipulated. For example, if the variance of $\xi_t$ is zero, the slope will be constant (equal to $\beta_0$); if the variance of $\eta_t$ is also zero, $\mu_t$ will be a deterministic trend given by the line $\mu_0 + \beta_0 t$.

**Logarithmic Trend + Seasonal Dummies**

This is similar to linear model where logarithmic trend model is used instead of linear trend model.

**ARCH/GARCH Models**

In econometrics, **AutoRegressive Conditional Heteroskedasticity** (ARCH) models are used to characterize and model observed time series. They are used whenever there is reason to believe that, at any point in a series, the error terms will have a characteristic size, or variance. In particular ARCH models assume the variance of the current error term or innovation to be a function of the actual sizes of the previous time periods' error terms: often the variance is related to the squares of the previous innovations.

**ARCH(q) model Specification**

Suppose one wishes to model a time series using an ARCH process. Let $\varepsilon_t$ denote the error terms (return residuals, with respect to a mean process) i.e. the series terms. These $\varepsilon_t$ are split into a stochastic piece $z_t$ and a time-dependent standard deviation $\sigma_t$ characterizing the typical size of the terms so that

$$ \varepsilon_t = \sigma_t z_t $$

The random variable $z_t$ is a strong White noise process. The series $\sigma_t^2$ is modelled by
An ARCH(q) model can be estimated using ordinary least squares. A methodology to test for the lag length of ARCH errors using the Lagrange multiplier test was proposed by Engle (1982). This procedure is as follows:

1. Estimate the best fitting autoregressive model AR(q) -

\[ y_t = a_0 + a_1 y_{t-1} + \cdots + a_q y_{t-q} + \epsilon_t = a_0 + \sum_{i=1}^{q} a_i y_{t-i} + \epsilon_t \]

2. Obtain the squares of the error \( \hat{\epsilon}^2 \) and regress them on a constant and \( q \) lagged values:

\[ \hat{\epsilon}_t^2 = \hat{\alpha}_0 + \sum_{i=1}^{q} \hat{\alpha}_i \hat{\epsilon}_{t-i}^2 \]

where \( q \) is the length of ARCH lags.

3. The null hypothesis is that, in the absence of ARCH components, we have \( \alpha_i = 0 \) for all \( i = 1, 2, \ldots, q \). The alternative hypothesis is that, in the presence of ARCH components, at least one of the estimated \( \alpha_i \) coefficients must be significant. In a sample of \( T \) residuals under the null hypothesis of no ARCH errors, the test statistic \( TR^2 \) follows \( \chi^2 \) distribution with \( q \) degrees of freedom. If \( TR^2 \) is greater than the Chi-square table value, we reject the null hypothesis and conclude there is an ARCH effect in the ARMA model. If \( TR^2 \) is smaller than the Chi-square table value, we do not reject the null hypothesis.

GARCH
If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a **generalized autoregressive conditional heteroskedasticity** (GARCH, Bollerslev (1986)) model.

In that case, the GARCH (p, q) model (where p is the order of the GARCH terms $\sigma^2$ and q is the order of the ARCH terms $\varepsilon^2$) is given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

Generally, when testing for heteroskedasticity in econometric models, the best test is the White test. However, when dealing with time series data, this means to test for ARCH errors (as described above) and GARCH errors (below).

EWMA is an alternative model in a separate class of exponential smoothing models. It can be an alternative to GARCH modelling as it has some attractive properties such as a greater weight upon more recent observations but also some drawbacks such as an arbitrary decay factor that introduce subjectivity into the estimation.

**GARCH(p, q) model specification**

The lag length p of a GARCH(p, q) process is established in three steps:

1. Estimate the best fitting AR(q) model

   $$y_t = a_0 + a_1 y_{t-1} + \cdots + a_q y_{t-q} + \varepsilon_t = a_0 + \sum_{i=1}^{q} a_i y_{t-i} + \varepsilon_t$$

2. Compute and plot the autocorrelations of $\varepsilon^2$ by

   $$\rho = \frac{\sum_{t=i+1}^{T} (\varepsilon_t^2 - \hat{\sigma}_t^2)(\varepsilon_{t-1}^2 - \hat{\sigma}_{t-1}^2)}{\sum_{t=1}^{T} (\varepsilon_t^2 - \hat{\sigma}_t^2)^2}$$

3. The asymptotic that is for large samples standard deviation of $\rho(i)$ is $1/\sqrt{T}$. Individual values that are larger than this indicate GARCH errors. To estimate the total number of lags, use the Ljung-Box test until the value of these are less than, say, 10% significant. The Ljung-Box Q-statistic follows $\chi^2$ distribution with n degrees of freedom if the squared residuals $\varepsilon_t^2$ are uncorrelated. It is recommended to consider up to T/4

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values of $n$. The null hypothesis states that there are no ARCH or GARCH errors. Rejecting the null thus means that such errors exist in the conditional variance.

### 3.3 Data Source

As mentioned earlier, the Indian insurance market became liberalized and was open for competition with the formation of IRDA and entrance of the private players in the sector 2000 onwards. The study of the behaviour of the Indian non-life insurance market after 2000 is necessary to understand the market scenario and future growth opportunities. This has been done through statistical analyses of different performance indicators for non-life insurance like, gross and net premiums earned, income from investment, claims incurred, administrative expenses, underwriting profit and loss, profit after tax and combined ratio based on the secondary data collected from GIC Annual Reports, IRDA Annual Reports, IRDA Journals as well as the Annual Reports of the public (1996 to 2012) and private (as available since 2000) non-life insurance companies. As mentioned earlier, the study of combined ratio has been done to understand the presence of underwriting cycle in the sector. The statistical modelling has been done for non-life insurance sector as a whole as well as for different constituents of the non-life sector like, fire, marine and miscellaneous insurance etc. The studies have also been done separately on the performances of the different public sector companies like General Insurance Corporation, National Insurance Company, New India Assurance Company, Oriental Insurance Company, United India Insurance Company and private insurance companies like, Royal Sundaram Insurance Company, Reliance General Insurance Company, IFFCO-Tokyo Insurance Company, TATA-AIG Insurance Company, Bajaj-Alliance Insurance Company, ICICI, Cholamandalam, Future Generali and Universal Sompo.

The underwriting indices, like, underwriting profit/Loss, combined ratio and profit/loss percentage etc. have been calculated based on the above-mentioned data.
3.4 Tools

The following tools have been used in the study –

- Microsoft Excel 2010
- SAS BASE 8.2
- SAS Stat 8.2
- SAS ETS 8.2
- SAS ANALYST 8.2
- E-View 8.0 (for ARCH/GARCH Model Fitting)