CHAPTER 3

EQUIVALENT CIRCUIT AND TWO AXIS MODEL OF DOUBLE WINDING INDUCTION MOTOR

3.1 INTRODUCTION

DWIM consists of two windings on the same stator core and a squirrel cage rotor. One set of winding is connected to a three phase supply to meet mechanical load and a three phase EMF is developed in the other winding that works as an Induction Alternator (IA). This machine can either be operated in power balancing mode, at its rated capacity or in maximum efficiency mode, operated at the load current corresponding to the maximum efficiency.

An equivalent circuit has been suggested for power balancing mode of operation of DWIM. For experimental verification, a 3kW, 415V, 3-phase DWIM has been designed, fabricated and tested. Equivalent circuit has been simulated using MATLAB. Modelling of DWIM is presented using two axes theory. The model takes power source as input and gives speed and electromagnetic torque as outputs with respect to the first winding.

The second winding uses the magnetic field in the air gap as input and develops a three phase EMF by mutual induction. The equivalent circuit developed for DWIM model was verified for different slips. The simulation
results almost agree with test results of DWIM for power balancing mode of operation. Mathematical modeling of a machine helps to study the steady and transient state behavior of a machine. Double Winding Induction Motor suggested in this thesis consists of two sets of identical winding in the same stator. By adjusting the load on both the windings, the machine can be operated at its rated capacity, the load current corresponds to the maximum efficiency. An equivalent circuit and two axis model corresponds to the rated capacity of DWIM is presented.

An equivalent circuit and d-q model has been attempted using two axis theory and MATLAB. DWIM can be operated at its maximum efficiency mode and power balancing mode by an appropriate control circuit. Equivalent circuit has been determined using conventional no load and blocked rotor test readings and simulated results are verified with the equivalent circuit for the different values of slips.

Shi et al (1999) presented a generalized dynamic model of induction motor using Simulink with torque sub-model to measure electromagnetic torque developed and mechanical sub-model to measure rotor speed. The stator voltage and current are transformed using three-phase to two axis conversion. Constructional details of various sub-models for the induction motor are implemented.

Ayasun and Nwankpa (2005) described MATLAB implementation of three induction motor tests to determine equivalent circuit parameters. These simulation models are developed to support and enhance electrical machinery studies. Razik et al (2006) presented modeling and analysis of a Dual-Stator Induction Motor (DSIM). The stator windings of a DWIM can be
arranged with different shift angles between them. Dynamic model of a DWIM helps to study the effects of the shift angle between its three-phase windings.

MATLAB has been used to model three phase dual stator and conventional induction motor to study the steady state and transient behaviour of the machines. In this work, an equivalent circuit and two axes model of DWIM are presented in its power balancing mode of operation. In power balancing mode of operation, the machine is expected to run at its rated capacity. A controller continuously senses the mechanical load on the shaft and accordingly second set of winding is connected to electrical load through a contactor. Two axes model and an equivalent circuit have been suggested for a designed 3 kW, 415 V, 4-pole, 3-phase DWIM.

### 3.1.1 Test results of DWIM

Equivalent circuit parameters have been determined from the results of no load and blocked rotor tests. Efficiency and power factor determined from the equivalent circuit are verified with load test results. Equivalent circuit is also simulated using MATLAB. The current flowing through various branches is determined and verified with conventional equivalent circuit. Table 3.1 shows the no load and blocked rotor test results.

**Table 3.1 No load and Blocked rotor test**

<table>
<thead>
<tr>
<th>Test</th>
<th>Input voltage</th>
<th>Line current</th>
<th>Input power</th>
</tr>
</thead>
<tbody>
<tr>
<td>No load</td>
<td>415 V</td>
<td>2.2 A</td>
<td>560W</td>
</tr>
<tr>
<td>Blocked rotor</td>
<td>100 V</td>
<td>6.1 A</td>
<td>800 W</td>
</tr>
</tbody>
</table>
3.1.2 Determination of Equivalent circuit of DWIM

The scope of this chapter is to determine the equivalent circuit for a DWIM for power balancing mode of operation. In power balancing mode, both the windings are energized through a control circuit such that the machine is loaded to its rated capacity. In general, EMF induced in the second set of winding does not depend on slip of the machine. However, in power balancing mode of operation, the second set of winding is loaded depending on the shaft load.

An equivalent circuit is proposed for a 3kW, 415 V, 4-pole, 3-phase DWIM. In order to determine an equivalent circuit, no load, full load and blocked rotor tests were carried out. The stator of DWIM consists of two stator windings wound for similar number of poles with zero degree phase displacement between them.

Equivalent circuit of DWIM consists of two equivalent load resistances, one in the rotor and the other in one of the stator windings to which an external electrical load is connected. The main challenge is to determine the equivalent load resistance of second stator winding depending on the value of slip. Equivalent circuit under discussion is considered only for the power balancing mode of operation. The equivalent circuit of DWIM is shown in Figure.3.1.

In order to determine equivalent load resistance of second set of winding of DWIM, the load test was carried out for various load currents. No load test results help to find out the core loss and magnetizing components
and the blocked rotor test results help to determine copper loss component of an equivalent circuit.

Figure 3.1 Equivalent Circuit of DWIM

The approximate value of the equivalent load resistance of the second set of winding is estimated as $r_{ls} = K \cdot \frac{V_{ph}}{I_2'}$, where, $K$ is constant which varies between 1.5 and 1.8. The slip of motor influences the current flowing through second set of winding, load resistance in the second winding varies with respect to load torque.
3.1.3 Analysis of equivalent circuit

The watt component of no load current is responsible for core losses of the machine. The stator current $I_1s$ has three components namely no load current $I_0$, current through the rotor circuit $I_2'$ and current delivered to the electrical output by the second stator winding $I_2s$. With respect to the equivalent circuit diagram shown in Figure 3.1, the voltage across the rotor circuit and second stator winding are given by the following equations:

$$V_2 = V_1 - I_{1s} (r_{1s} + jX_{1s})$$ (3.1)

$$I_2' = \frac{V_2}{(r_2' + r_L' + jX_{2s}')\)}$$ (3.2)

$$I_{2s} = \frac{V_2}{(r_{2s} + r_{Ls} + jX_{2s})}$$ (3.3)

$$I_{1s} = I_0 + I_2' + I_{2s}$$ (3.4)

From the equivalent circuit, the performance of DWIM has been evaluated. The details of various power stages are given below:

**Electrical power input**

$$= 3 \ V \ I_{1s} \ \cos\phi$$

**Stator copper loss**

$$= 3 \ (I_{1s}^2) \ r_{01}$$

**Iron losses**

$$= 3 \ V \ I_0 \ \cos\theta$$

**Rotor copper loss**

$$= 3 \ I_2'^2 \ r_2'$$

**Mechanical power output**

$$= 3 \ I_2'^2 \ r_L'$$

**Electrical power output**

$$= 3 \ I_{2s}^2 \ r_{Ls}$$

The various combinations of electrical and mechanical load have been applied to the machine to find the equivalent load resistance of second
set of stator winding. Using the test results, a constant has been determined to find out the equivalent load resistance of second set of winding which almost suits for all the values of slips. In order to validate the equivalent circuit, the pre determined performance from the equivalent circuit has been verified with load test result. Experimental verification for two slip values has been presented.

No load test details

No load supply voltage $V_0$ = 415 V

No load current = 2.2 A

No load input power = 560 W

Working component of no load current $I_w$ = 0.77 A

Magnetizing component of no load current $I_{jl}$ = 2.06 A

Core loss component $R_0 = \frac{V_{ph}}{I_w}$ = 311 $\Omega$

Magnetizing reactance $X_0 = \frac{V_{ph}}{I_{ll}}$ = j116 $\Omega$

Blocked rotor test details

Applied voltage $V_{sc}$ = 100 V

Current drawn from the mains $I_{sc}$ = 6.1 A

Power drawn $W_{sc}$ = 800 W

Total resistance referred to stator/phase $R_{01}$ = 7.16 $\Omega$

Total impedance referred to stator/phase $Z_{01}$ = 9.47 $\Omega$
Total reactance referred to stator/phase $X_{01}$ = 6.18 $\Omega$

Stator resistance/phase $R_1$ = 4.24 $\Omega$

**Trial I**

Verification of equivalent circuit for a load current of 6.1A with 720W of electrical power in the second set of winding and the corresponding rotor speed is 1426 rpm.

Predetermination of performance from the Equivalent circuit

Supply voltage = 415 V

Current drawn from the mains = 6.1A

Input power = 3640 W

Torque measured = 14.2Nm

Speed of the machine = 1426 rpm

Output power in the second winding = 720 W

Slip = 0.049

Equivalent rotor resistance = 57 $\Omega$

Current through rotor circuit = 3.5-j0.71 A

Current through second winding = 1.18-j0.17A

Equivalent second winding resistance = 1670hm

Total load current = 4.6-j0.87A
Input current = Total load current + No load current = 5.4-j2.9 A

Equivalent mechanical output
= 3 x I_2'^2 x r_{L'} Watts
= 3 x 3.5^2 x 57
= 2095 W

Equivalent electrical output
= 3xI_{2s}^2 x r_{Ls} Watts
= 3 x 1.2^2 x 167
= 721 W

Total output (Equivalent circuit) = 2816 W

Mechanical Output (load test)
= (2\pi x N x T)/60 Watts
(2 x \pi x 1426 x 14.2) / 60 = 2119W

Electrical output (second winding) = 720 W

Total output (direct testing) = 2839 W

**Trial II**

Verification of equivalent circuit for a load current of 6.1A with 1380 W of electrical power in the second set of winding and the corresponding rotor speed is 1440 rpm.

Supply voltage = 415 V

Current drawn from the mains = 6.1 A

Input power = 3980 W

Speed of the machine = 1440 rpm
Output power in the second winding = 1380 W

Slip = 0.04

Equivalent rotor resistance = 70 Ω

Current through rotor circuit = 3.18-j0.33A

Current through second winding = 1.99-j0.18A

Equivalent second winding resistance = 115Ω

Total load current = 5.17-j0.52 A

Input current = Total load current + No load current = 5.93-j2.56 A

Equivalent mechanical output = 3 x I₂₂'² x r_L' Watts

= 3 x 3.2² x 70

= 2150 W

Equivalent electrical output = 3 x I₂s² x r_Ls Watts

= 3 x 2² x 115

= 1380 W

Total output (Equivalent circuit) = 3530W

Mechanical Output (load test) = (2 x π x 1440 x 13.8) / 60

= 2080W

Electrical output (second winding) = 1380 W

Total output (direct testing) = 3460 W
Trial III

Verification of equivalent circuit for a load current of 6.1A with 1990 W of electrical power in the second set of winding and the corresponding rotor speed is 1456 rpm.

Supply voltage = 415 V
Current drawn from the mains = 6.1A
Input power = 4000
Speed of the machine = 1456 rpm
Output power in the second winding = 1990W
Slip = 0.029
Equivalent rotor resistance = 109.4 Ω
Current through rotor circuit = 2.01-j0.21A
Current through second winding = 2.87-j0.86A
Equivalent second winding resistance = 70Ω
Total load current = 4.89-j1.07 A
Input current = Total load current + No load current
= 5.66-j 3.1A
Equivalent mechanical output = 3 x I_2^2 x r_L Watts
= 3 x 2.03^2 x 109.4
= 1380 W
Equivalent electrical output

\[ = 3I_{2s}^2 \times r L_s \text{ Watts} \]
\[ = 3 \times 3^2 \times 70 \]
\[ = 1890 \text{ W} \]

Total output (Equivalent circuit)
\[ = 3270 \text{ W} \]

Mechanical Output (load test)
\[ = (2 \times \pi \times 1456 \times 9) / 60 \]
\[ = 1371 \text{ W} \]

Electrical output (second winding)
\[ = 1880 \text{ W} \]

Total output (direct testing)
\[ = 3251 \text{ W} \]

Trial –I Input voltage: 415 V, Line Current: 6.0A Speed: 1426 rpm,

Input power
\[ = 3640 \text{ W} \]

Output power measured by conventional load test
\[ = 2839 \text{ W} \]

Output power verified by equivalent circuit
\[ = 2816 \text{ W} \]

Trial –II Input voltage: 415 V, Line current 6.1A, Speed: 1440 rpm,

Input power
\[ = 3980 \text{ W} \]

Output power measured by conventional load test
\[ = 3460 \text{ W} \]

Output power verified by equivalent circuit
\[ = 3530 \text{ W} \]

Trial –III Input voltage: 415 V, Line current 6.1A, Speed: 1456 rpm,

Input power
\[ = 4000 \text{ W} \]

Output power measured by conventional load test
\[ = 3251 \text{ W} \]

Output power verified by equivalent circuit
\[ = 3270 \text{ W} \]
It is verified that the output power determined using equivalent circuit of DWIM almost agree with the results obtained from direct loading for the given value of slip.

### 3.1.4 Simulation of Equivalent Circuit

An equivalent circuit is proposed for power balancing mode of operation and in this mode, mechanical and electrical loads are adjusted such that machine is operated at its rated capacity. Equivalent load resistance depends on the shaft load similar to equivalent load resistance of induction motor.

### 3.1.5 Simulation model and output wave forms

An equivalent circuit has been developed using MATLAB for DWIM for power balancing mode of operation, with the machine expected to operate at its rated capacity. Depending upon the mechanical load demand, second set of winding is connected to the external electrical load. Equivalent circuit has been verified for three sets of loading arrangements.

The equivalent circuit obtained with 720W of electrical power in the second set of winding. The total load current is maintained at 6.1A. Mechanical power delivered is 2119 Watts and corresponding slip is 4.9%. Figure 3.2 shows no load current, stator current and rotor current for slip of 4.9%.

The equivalent circuit obtained with 1380W of electrical power in the second set of winding. The total load current is maintained at 6.1A. Mechanical power delivered is 2150 Watts and corresponding slip is 4%. Figure 3.3 shows no load current, stator current and rotor current for slip of 4%.
Figure 3.2 Current through stator and rotor for 4.9% slip
Figure 3.3 Current through stator and rotor for 4% slip
The equivalent circuit obtained with 1990W of electrical power in the second set of winding. The total load current is maintained at 6.1A. Mechanical power delivered is 1380 Watts and corresponding slip is 2.6%. Figure 3.4 shows no load current, stator current and rotor current for a slip of 2.6%.

![Figure 3.4 Current through stator and rotor for 2.6 % slip](image)
An equivalent circuit of DWIM has been determined using MATLAB. Current flowing through both stator windings and rotor circuit has been simulated. The simulated results were verified with load test results. Pre determined results obtained from the equivalent circuit for various values of slips almost agree with test results obtained from direct loading.

### 3.2 TWO AXES MODEL OF DOUBLE WINDING INDUCTION MOTOR

Ansari and Deshpande (2010) presented a hybrid model of an induction motor which includes d,q models and abc models which allow inclusion of various forms of impedance and voltage unbalance. A generalized dynamic model of an induction motor consists of electrical sub-model to implement three-phase to two-phase transformation of stator voltage and current, a torque sub-model to calculate the developed electromagnetic torque, and a mechanical sub-model to yield the rotor speed. Generalised two axes model of a DWIM is shown in Figure 3.5.

![Figure 3.5 Two axes model DWIM](image-url)
Voltage and current relations are derived from basic generalised two axes model is shown in equation 3.1 of the machine. Two coils are represented in both direct and quadrature axes. The direction of rotation of the machine is assumed to be clockwise.

Voltage equation

\[
\begin{bmatrix}
V_{d1} \\
V_{d2} \\
V_{q1} \\
V_{q2} \\
V_{dr} \\
V_{qr}
\end{bmatrix} =
\begin{bmatrix}
R_p + L_d P & L_m P & 0 & 0 & L_m P & 0 \\
L_m P & R_q + L_q P & 0 & 0 & L_m P & 0 \\
0 & 0 & R_p + L_d P & L_m P & 0 & L_m P \\
0 & 0 & L_m P & R_q + L_q P & 0 & L_m P \\
L_m P & L_m P & -\omega L_m & -\omega L_m & R_q + L_r P & L_q \omega r \\
\omega L_m & \omega L_m & L_m P & L_m P & L_q \omega r & R_q + L_r P
\end{bmatrix}
\begin{bmatrix}
I_{d1} \\
I_{d2} \\
I_{q1} \\
I_{q2} \\
I_{dr} \\
I_{qr}
\end{bmatrix}
\]

\[ (3.1) \]

Torque equation:

Considering self inductances of the stator coil, the torque developed in the machine is shown in equation 3.2

\[
T = \begin{bmatrix}
I_{d1} & I_{q1} & I_{dr} & I_{qr}
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & L_m & 0 & L_r \\
L_m & 0 & L_r & 0
\end{bmatrix}
\begin{bmatrix}
I_{d1} \\
I_{q1} \\
I_{dr} \\
I_{qr}
\end{bmatrix}
\]

\[ (3.2) \]

Simulink model of three phase to two phase conversion is shown in Figure 3.6. Current flowing through first set of stator winding, Current flowing through second set of stator winding, Rotor Current, Torque, Mechanical model including moment of inertia, No-load current sub-models are shown from Figure 3.7 to Figure 3.14 respectively. Conversional induction motor model consist of \( V_{ds}, V_{qs} \) \( V_{ds} \) and \( V_{qr} \) to represent the stator
and rotor winding. In DWIM, in addition to these components $V_{ds2}$ and $V_{qs2}$ are added to represent second set of stator winding. Electrical and mechanical outputs are separately calculated.

Figure 3.6 Simulink model of 3-phase to 2-phase conversion
Figure 3.7 Sub model of current flowing through first stator winding

Figure 3.8 Sub model of current flowing through second stator winding

Figure 3.9 Sub model of current flowing through rotor
Figure 3.10 Torque sub model

Figure 3.11 Mechanical sub model

Figure 3.12 No load current sub model
Figure 3.13 Main model of DWIM
Figure 3.14 Sub model of DWIM
3.2.1 Simulation Results

Performance of DWIM has been simulated for power balancing mode of operation in which the machine is expected to be operated at its rated capacity. The induction motor chosen for the simulation studies has the following parameters:

- Capacity of the machine = 3 kW, 3-phase
- Stator resistance per phase = 4.24Ω
- Rotor resistance per phase = 2.92Ω
- Stator inductance per phase = 0.015 H
- Mutual Inductance = 0.0135 H
- Moment of inertia = 0.05 kg m²
- Pairs of poles = 2

3.2.2 Simulation Results for 4% Slip

Two axes model of a DWIM has been developed by integrating sub models representing stator voltage, current through various branches, second stator voltage and torque respectively. Output of DWIM was varied for various slip values. Simulated outputs for 4% slip are shown from Figure 3.15 to Figure 3.21. For 4% slip, 2A electrical load is connected in the second set winding and total current is maintained at 6.5A. The input current drawn from the main is sum of current through rotor, second stator and no load current.
Figure 3.15 Two phase input voltage
Figure 3.16 Current through first stator winding for 4% Slip

Figure 3.17 Speed in rps for 4% Slip
Figure 3.18 Torque in Nm for 4% Slip

Figure 3.19 Current through second stator winding for 4% Slip
Figure 3.20 Current through rotor for 4% Slip

Figure 3.21 No load current for 4% Slip
3.2.3 Simulation Results for 4.9 % Slip

Simulated outputs for 4.9% slip are shown in figures form Figure 3.22 to Figure 3.27. For 4% slip, 1.2A electrical load is connected in the second set winding and total current is maintained at 6.1A. The input current drawn from the main is the sum of current through rotor, second stator and no load current.

Figure 3.22 Current through first stator winding for 4.9 % Slip
Figure 3.23 Speed in rps for 4.9 % Slip

Figure 3.24 Torque in Nm for 4.9 % Slip
Figure 3.25 Current through second stator winding for 4.9 % Slip

Figure 3.26 Current through rotor for 4.9 % Slip
Two axes model of DWIM has been developed and verified for different values of slips. Table 3.2 shows the comparison of simulated results with results obtained from direct loading of the machine for various combinations of electrical and mechanical loadings. Simulated results almost agree with the results obtained from direct loading.
Table 3.2 Comparison of simulated and load test result

<table>
<thead>
<tr>
<th>%Slip</th>
<th>First stator current I$_{s1}$ (A)</th>
<th>Second stator current I$_{s2}$ (A)</th>
<th>Rotor current I$_{2}'$ (A)</th>
<th>No load current I$_{0}$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tested</td>
<td>Simulated</td>
<td>Tested</td>
<td>Simulated</td>
</tr>
<tr>
<td>4.0</td>
<td>6.5</td>
<td>6.98</td>
<td>2</td>
<td>1.3</td>
</tr>
<tr>
<td>4.9</td>
<td>6.10</td>
<td>6.29</td>
<td>1.2</td>
<td>1.07</td>
</tr>
<tr>
<td>6.0</td>
<td>5.80</td>
<td>5.56</td>
<td>1</td>
<td>0.85</td>
</tr>
</tbody>
</table>

3.3 SUMMARY

Equivalent circuit of an induction machine helps to predetermine its performance for various values of slip. Second set of winding has been assumed to be parallel to the rotor circuit and uses same magnetic field established by energising first set of winding. Equivalent circuit has been demonstrated for power balancing mode of operation and simulation results almost agree with test results.

The main task of equivalent circuit is the load resistance with respect to the second set of winding where electrical load is connected. Since it is aimed to operate the machine either in power balancing mode or in energy efficiency mode, resistance corresponding to the second set of winding depends upon the value of slip. The value of load resistance corresponding to the second set of winding is given by $r_{ls} = K.V_{ph}/I_{2}'$. The value of ‘K’ is obtained by load test results for various combinations of electrical and
mechanical loads. Equivalent circuit has been demonstrated for power balancing mode of operation.

Two axes model of a DWIM has been developed for power balancing mode of operation. Output results are obtained from the simulated model for three different values of slips. The current flowing through stator windings, rotor and no load current are verified. Simulated results almost agree with direct testing of machine for the same load current.