This thesis entitled "Step-Stress Accelerated Life Testing Designs" is submitted to the Aligarh Muslim University, Aligarh, India, to supplicate the degree of Doctor of Philosophy in Statistics. It embodies the research work carried out by me in the Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh.

This thesis deals with the analysis of accelerated life test. First, we provide related concepts and second, we provide detailed information of Inverse Weibull distribution. Various methods are considered which are plausible in this context and estimation procedure is outlined. Finally, a numerical example is considered using various models introduced earlier.

The term "Accelerated life test" applies to the type of study where failure times can be accelerated by applying higher "stress" to the component. This implies that the failure time is a function of the so called "stress factor" and higher stress may bring early failure.

Models and methods of accelerated life testing are useful when technical systems under test tend to have very long lifetimes. Most of the accelerated life test models have two assumptions, first a life distribution for a unit's behavior at a constant stress level, and second is an assumed relationship between the life characteristics and the stress factor which expresses the effect of changing factors like temperature, voltage, humidity on a products failure time. Based on these assumptions, the life distribution under normal stress levels can be estimated. Commonly used methods are constant stress accelerated life test, step-stress accelerated life test and progressive stress accelerated life test. Recently, step stress test plans have drawn attention in the discipline of accelerated life test. Step stress accelerated life test is one specific type of accelerated life test, in which all the test units are tested at a common level of stress. The level of stress, however, can increase or decrease at points in time. The stress for survival units is generally changed to a higher stress level at a predetermined time.

This thesis consist different designs of accelerated life tests such as step stress accelerated life test, step-stress partially accelerated life test with progressive data and constant stress accelerated life test using geometric process. The method of maximum
likelihood estimation is used to obtain the estimates of model parameters because it provides estimators that have both a reasonable intuitive basis and many desirable statistical properties. In order to get the asymptotic variance of the maximum likelihood estimator, the Fisher information matrix is constructed. In particular, this thesis focused on the derivation of different accelerated life test plans and designs for the lifetimes of units that are assumed to follow Frechet life distribution at use stress with different types of schemes. The concluding remarks of the investigations in implementation of proposed optimization techniques are stated as follows:

This thesis consists of six chapters. Chapter 1 is the usual introductory chapter. It provides a detailed introduction to accelerated life test designs and their analysis. An overview of the available literature is also given in this chapter.

In Chapter 2, we focus on the Step Stress Accelerated Life Test design problem or inverse Weibull failure data because of its broad application in industry. The optimal simple step stress accelerated life test, which involves only two stress levels, is first derived. Log-linear life stress relationship is assumed. In this Khamis Higgings model is introduced for step-stress test instead of commonly used cumulative exposure model. Optimal step stress accelerated life test plan is proposed by minimizing the asymptotic variance of the maximum likelihood estimates of given 00 pth percentile at design stress. Here, optimal time is obtained by graphical method or the purpose of accuracy. Asymptotic variance and covariance matrix of the estimators is then obtained by using the Fisher information matrix. Confidence intervals for parameters and respective errors are also obtained. A simulation study is performed to illustrate the statistical properties of the parameters and the confidence bounds.

Chapter 3 is an extension of Chapter 2. In this experiment three stress levels are established and are subjected to two types of relationship between lifetimes and stress (linear and Quadratic). The scale parameter of the baseline distribution at a constant stress level is assumed to have log-linear and quadratic relationship with stress and a Khamis-Higgings model holds. Numerical examples are presented to illustrate all the methods of inference developed here and a comparison of the maximum likelihood estimates for different sample sizes is shown. The optimum test plan specifies the optimal stress switching point which is determined by minimizing the generalized asymptotic variance of the maximum likelihood estimates for the model parameters. Tables of optimum times of changing stress level for both plans are
also obtained. In addition, asymptotic interval estimates of the parameters of the distribution using fisher information matrix are also evaluated. The performance of the estimators is evaluated by a simulation study with different pre-fixed parameters.

Accelerated Life Testing, generally, deals with the log linear function between life and stress to obtain the estimates of parameters of the life distribution. Therefore, it is preferable from statistical point of view that we work with the original parameters instead of developing inferences for the parameters of the log-linear link function. In this situation, the use of geometric process may be a good alternative in accelerated life testing to obtain the estimates of original parameters of life distribution directly.

Chapter 4 introduces the geometric process model for the analysis of accelerated life testing with an inverse Weibull life distribution under constant stress. By assuming that the lifetimes under increasing stress levels forms a geometric process, the estimates of the parameters are obtained by using the maximum likelihood method. In addition, asymptotic interval estimates of the parameters of the distribution using fisher information matrix are also evaluated. The statistical properties of estimates and confidence intervals are examined through a simulation study.

In accelerated life testing analysis, there are situations where life stress relationship is not known or cannot be assumed. In such situations, partially accelerated life tests are used. In partially accelerated life tests, test units are run at both use and accelerated conditions. Constant stress partially accelerated life test and step stress partially accelerated life test are two commonly used methods in partially accelerated life tests analysis. In Constant stress partially accelerated life test products are tested at either normal use or accelerated condition only until the test is terminated. In step stress partially accelerated life test, a sample of test items is run at use condition and, if it does not fail for a specified time, then it is run at accelerated condition until test is terminated.

Chapter 5 integrates features of step stress partially accelerated life test and progressive Type-II censored data based on random removal. The lifetimes of units are assumed to follow a Inverse Weibull distribution while the number of removals at each inspection is assumed to follow a binomial and uniform distribution. The optimal accelerated life test plans are determined by minimizing the asymptotic variance of an
estimated quantile at use condition. Several numerical examples are provided for illustrative purposes.

Finally, Chapter 6 summarizes future directions of research problems in the field of accelerated life test that arise as extensions of what we currently achieved in this thesis.

A comprehensive list of references is provided at the end of the thesis.
LIST OF PUBLICATIONS USED IN THIS THESIS

Chapter 2 based on

Chapter 3 based on

Chapter 4 based on

Chapter 5 based on