Chapter IV

Chapter - IV

ELECTRICAL RESISTIVITY METHOD

Introduction

Among all geophysical methods, electrical resistivity method is most widely used in groundwater exploration. This method originated during 1912 and CONRAD SCHLUMBERGER perfected the technique and it was his idea that remains to this day. F. WENNER also developed the same idea approximately at the same time in analysing the properties of a measuring configuration which bears his name. In the electrical resistivity method, an artificially generated current is introduced into the earth through two current electrodes and the resulting potential differences are measured at the surface through the two potential electrodes. Actually, one measures the resistivity of the earth, which is known as apparent resistivity. The concept of apparent resistivity comes from the fact that the measured resistivity is the resistivity of a fictitious homogeneous earth upto certain depth. Resistivity is a electrical property of a substance and it is independent of size and shape of the substance and is defined as the resistance offered during flow of electric current by unit cube of the substance, when voltage is applied across the opposite faces. Normal unit is "ohm" for resistance and the "metre" for length. Thus, the unit of resistivity is "ohm-metre"; One ohm-m is equal to 100 ohm-centimetres. Also, natural occurring electrical fields (Telluric) are used for the same purpose, especially for exploring deeper horizons. Similarly, very low frequency electromagnetic (VLF-EM) field in the range of 15-25 KHz can also be used for exploring shallow horizons especially in hard rock areas. The electrical resistivity method can also be used in stratigraphic correlations in mineral and oil prospecting.

Electric Conduction in Continuous Media

The electrical resistivity method is based on elementary form of OHM'S LAW. Let the conductor be a bar (rectangular body) of length "dl" and area of cross-section "A". The potential difference "dV" between the end faces is given by ohm's law as follows :

$$I = \frac{dV}{R} \qquad \ldots\ldots \qquad (4.1)$$

where "I" is the current flowing perpendicular to the area of cross section "A" and "R" is the resistance of the conductor (rectangular body). Since "R" is directly proportional to the length (dl) and inversely proportional to the cross-section "A" of the conductor,

$$R = \rho . \frac{dl}{A} \qquad \ldots\ldots \qquad (4.2)$$

where "ρ" is the resistivity of the material of the conductor. From equations 4.1 and 4.2, one gets

$$\frac{I}{A} = \frac{1}{\rho} . \frac{dV}{dl} \qquad \ldots\ldots \qquad (4.3)$$

The ratio $\frac{I}{A}$ of equation 4.3 is the current density, while $\frac{dV}{dl}$ on right hand side (r.h.s.) of the equation 4.3 is the electric field or potential gradient in the direction of current density vector. This is abbreviated as E with a minus sign in Electrical geophysics.

Potential Field due to Point Source

Let us consider a point source of current on the surface of a homogeneous isotropic ground, then the current flow is radial and the equipotential surfaces are concentric hemispheres. If the radial distance between two concentric hemispherical shells is "dr", then the potential gradient at any point between them is $\frac{dV}{dr}$. If the total current passing from the point source into the ground is "I", the potential drop across the two hemispheres can be written using equation 4.3 as :

$$dV = I.\rho. \frac{dr}{2\pi r^2}$$

Integrating both sides, one gets for the potential at a distance "r" from a point source

$$V(r) = \frac{I.\rho}{2\pi} . \frac{1}{r} + C$$

where "C" is an arbitrary constant. If $V \rightarrow 0$, when $r = \infty$, therefore $C = 0$, one gets

$$V(r) = \frac{I.\rho}{2\pi} . \frac{1}{r} \quad .... \quad (4.5)$$

For the two electrodes, the resultant potential at any point "P" on the ground will then be :

$$V = \frac{I\rho}{2\pi} \left(\frac{1}{r} - \frac{1}{r'}\right) \quad .... \quad (4.6)$$

where "r" and "r´" are the distances of point "P" from the two electrodes.

Buried Current Electrode in Homogeneous, Isotropic Earth

Since, the current flow is horizontal, it is evident that "V" satisfies the condition $\left(\frac{1}{\rho}\right) \left(\frac{\partial V}{\partial z}\right) = 0$ at the ground surface and can be expressed as Laplace´s equation :

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \ldots \qquad (4.7)$$

in rectangular coordinates. The solution of equation 4.7 that satisfies the above boundary conditions, is :

$$V = \frac{I\rho}{2\pi} \left(\frac{1}{r_1} + \frac{1}{r_1'}\right) \qquad \ldots \qquad (4.8)$$

where r_1 and r_1' are the distances of the point "P" from the current electrode and its image in the ground surface respectively. With two buried electrodes, a positive and a negative one as usual, the potential becomes,

$$V = \frac{I.\rho}{2\pi} \left(\frac{1}{r_1} + \frac{1}{r_1'} - \frac{1}{r_2} - \frac{1}{r_2'}\right) \qquad \ldots \qquad (4.9)$$

where r_2 and r_2' are the distances of "P" from the negative electrode and its image respectively.

Two Current Electrodes at Surface

The potential at any nearby surface point is affected by both the current electrodes when the distance between the current electrodes is finite. Hence, potential due to C_1 at P_1 is

$$V_1 = - \frac{A_1}{r_1}, \text{ where } A_1 = - \frac{I\rho}{2\pi}$$

Similarly, potential due to C_2 at P_1 is

$$V_2 = - \frac{A_2}{r_2}, \text{ where } A_2 = \frac{I\rho}{2\pi} = - A_1$$

Since the current at the two electrodes are equal and opposite

$$V_1 + V_2 = \frac{I\rho}{2\pi} \left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

Finally, by introducing a second potential electrode at P_2, one can measure the potential difference between P_1 and P_2

$$V = \frac{I\rho}{2\pi} \cdot G \quad \ldots \quad (4.10)$$

Where "G" is known as GEOMETRIC FACTOR and is given as

$$G = \left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right).$$

Mathematical expression to determine the potential difference in a layered medium has been given in appendix 4.I.

The DAR ZARROUK Parameters

Let us consider a prism of unit cross section which is characterised by its thickness "h" and resistivity "ρ". Than the resistance (T) perpendicualr to the face of prism and the conductance (S) parallel to the face of prism can be written as

$$T = h.\rho \quad \ldots \quad (4.11)$$

$$S = \frac{h}{\rho} \quad \ldots \quad (4.12)$$

From equation 4.11, one can write

$$\log \rho = -\log h + \log T \quad \ldots \quad (4.13)$$

which defines a straight line inclined at an angle 135° to the h-axis and cutting at a distance T from the origin, if ρ is plotted against h on a double logarithm graph sheet. Similarly, from equation 4.12, one gets

$$\log \rho = \log h - \log S \quad \ldots \quad (4.14)$$

which again defines a straight line inclined at an angle of 45° with the h-axis and meeting at a distance S from the origin. The point of intersection of the two straight lines given by equations 4.13 and 4.14 will be the resistivity and thickness of a particular column of T and S and known as DAR ZARROUK parameters (Maillet, 1947).

Now, consider that the prism consists of n layers and completely characterised by its thickness $h_1, h_2, \ldots, h_n$ and resistivities $\rho_1, \rho_2, \ldots, \rho_n$ respectively (Fig. 4.1). Then the total resistance to the current flowing perpendicular to the layers will be the sum of resistances offered by each layers and can be expressed as follows :

$$T = T_1 + T_2 + \ldots + T_n$$

$$T = h_1.\rho_1 + h_2.\rho_2 + \ldots + h_n.\rho_n = \sum_{i=1}^{n} h_i.\rho_i \qquad \ldots \qquad (4.15)$$

which is the "transverse resistance". The transverse resistivity to the current flowing perpendicular to the layers is given by

$$\rho_{\perp} = \frac{T}{H}, \text{ where } H = \Sigma h_i .$$

Similarly, the total conductance to the current flowing parallel to the layers will be the sum of conductances through each layers and can be expressed as

$$S = S_1 + S_2 + \ldots + S_n$$

$$S = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2} + \ldots + \frac{h_n}{\rho_n} = \sum_{i=1}^{n} \frac{h_i}{\rho_i} \qquad \ldots \qquad (4.16)$$ 16)

which is the "longitudinal conductance". The average conductivity to the current flowing parallel to the layers is given by

$$\sigma_{\parallel} = \frac{S}{H}$$

The longitudinal conductance ($\sigma_{\parallel}$) is always less than the transverse resistance ($\rho_{\perp}$), unless the medium is uniform. In that case the values ($\rho_\perp$ and $\sigma_{..}$) will be equal. This dependence of resistivity on the direction of current flow is known as anisotropy. The whole medium acts like a anisotropy though the individual layers are isotropic. This phenomenon is termed as "Pseudoanisotropy". The coefficient (λ) of pseudoanisotropy is given by

$$\lambda = \sqrt{\rho_\perp . \sigma_{..}} = \sqrt{\frac{T}{H} . \frac{S}{H}} = \sqrt{\frac{TS}{H^2}} = \frac{\sqrt{TS}}{H}$$

and the geometric average resistivity is

$$\rho_m = \sqrt{\rho_\perp . \frac{1}{\sigma_{..}}} = \sqrt{\frac{T}{H} . \frac{H}{S}} = \sqrt{\frac{T}{S}}$$

Resistivity Interpretation

The interpretation of resistivity data is done in two stages. In the first stage, the data are interpretated in terms of physical parameters, i.e. resistivities and corresponding thicknesses. In the later stage, the interpreted results are corrrelated with the available geologic knowledge to arrive at the realistic picture of the sub-surface. There are many ways to interpret the resistivity data into physical properties starting from usual method of curve matching and recent application of fast computers.

The most widely used method of interpretation techniques is curve matching. This is an indirect type of interpretation. The field curves are matched against the standard curves prepared from Stefanesco's expression (1930) for the potential due to point source of current on stratified earth. An album of theoretical curves for different combinations of resistivities and corresponding thicknesses are prepared for two, three and four layered earth models, for matching with the field curves, (Mooney and Wetzel, 1956; Orellana and Mooney, 1966; Lazreg, 1972; Rijkswatersttat, 1975). The number of theoretical curves required increases with the number of layers and it becomes extremely difficult to handle if the number of layers exceeds three. Flathe (1955a), Baranov and Tassencourt (1959), Deppermann (1961), Baranov (1962) and Vandam (1965) suggested methods for calculation of exact theoretical curves for given parameters to match with the field curves. Ebert (1943), Kalenov (1957), Ohio (1959), Zhody (1965), Orellana and Mooney (1966) suggested auxiliary point methods for use with two and three layer theoretical curves for interpreting multi-layer cases. However, the curve matching procedure requires a great deal of practice and time, and at times, can become very difficult task even to the experienced interpreter.

Methods of directly interpreting the field data were also attempted by many workers. The idea of obtaining the layer parameters directly from the field measurements was suggested theoretically by Slichter (1933), who showed that the layer parameters can be obtained directly from field measurements in two steps. The first step was determination of "Kernel function" in the Stefanesco's integral from the field measurements and the second step was deduction of layer parameters from Kernel function. Pekeris (1940) gave an extremely useful method of carrying out Slichter's second step. These suggestions are of little use because of the difficulty in carrying the first step, namely deriving the Kernel fucntion from the field data. Koefoed (1968) suggested a method to obtain Kernel function from the field data. He also suggested a modified method to carry out the second step by introducing a new function called "Resistivity Transform". Koefoed's methods are useful to obtain the layer parameters from the field curves, using the knowledge of sampling and filter techniques common in the field of communication theory. Ghosh (1971) suggested a speedy method of obtaining the Kernel function from the field data. Das and Verma (1977) have written a monograph describing the theory of linear filtering and its application to the interpretation of resistivity sounding measurements containing computer programmes.

Zohdy (1975) suggested another procedure for calculating the layer parameters directly from the field curves by use of DAR ZARROUK curves. In his method, the layer parameters, the thicknesses and resistivities are obtained to the first approximation by digitizing the field curves and considering them to be the points on a modified DAR ZARROUK curves. The vertical electrical sounding (VES) curve is calculated theoretically for the above stratified earth and then the same is compared with the field curve. A second approximation to the "MODIFIED DAR ZARROUK" (MDZ) curve is obtained utilising the difference between calculated and field curves. The iteration is continued until good match is obtained between calculated and field curves.

Usefulness of Electrical Resistivity Surveys

It is well known that resistivity methods can be successfully employed for groundwater investigations where good electrical resistivity contrasts exists between the water bearing formations and the surrounding strata (Zohdy, 1974). In general, the matrix minerals in rocks are normally high resistive. In rocks containing fluids, current is conducted electrolytically by the interstitial fluids and resistivity is contorolled by porosity, water content as well as quantity of dissolved salts and clay minerals. However, conduction in clay minerals is electrical as well as electrolytic (Zohdy, 1974). Thus, weathered and fractured rocks, when water bearing, show low values of resistivity as compared to high resistivity values for dry rocks.

In general, this method is useful and has been widely used in various geological situation for groundwater exploration. In the present study area, this method has also yielded very useful information for groundwater prospecting.

Ambiguities in Electrical Resistivity Interpretation

In the electrical resistivity interpretation, at times it is difficult to determine the characteristics of layers whose thicknesses are much smaller compared to the thicknesses of neighbouring layers. If the $\rho_n h_n$ for any layer is much smaller compared to the same of its immediate neighbouring layers such as $\rho_{n-1} h_{n-1}$ and $\rho_{n+1} h_{n+1}$, such layers would have no appreciable influence on the apparent resistivity curves. But when the value of $\rho_n h_n$ increases and is comparable to either ρ_{n-1} h_{n-1} or ρ_{n+1} h_{n+1}, the signature of ρ_n h_n on the apparent resistivity curve would be significant. This phenomenon is known as principle of equivalence. Similar properties of principle of equivalence also holds good if longitudinal conductance $(\frac{h_n}{\rho_n})$ is taken instead of transverse resistance $(h_n \rho_n)$. Fig 4.2 shows two examples illustrating the principle of equivalence and its limit.

However, if $\rho_{n-1} > \rho_n > \rho_{n+1}$ or $\rho_{n-1} < \rho_n < \rho_{n+1}$, then such intermediate layer (h_n), as long as the same does not have sufficient thickness, has practically no effect on the apparent resistivity curve. But when the thickness of such a layer (h_n) increases, the influence of the same on the apparent resistivity curve is evident. This phenomenon is known as principle of suppression. Fig. 4.3 shows two examples illustrating the principle of suppression and its limit. In some places of the study area, the configuration of lithologs representing successive conductive layers (dry alluvium, wet alluvium both overlying a highly thick conductive clay layer) is a typical case of principle of suppression posing at times difficulties in estimating accurate thickness of the clay bed.

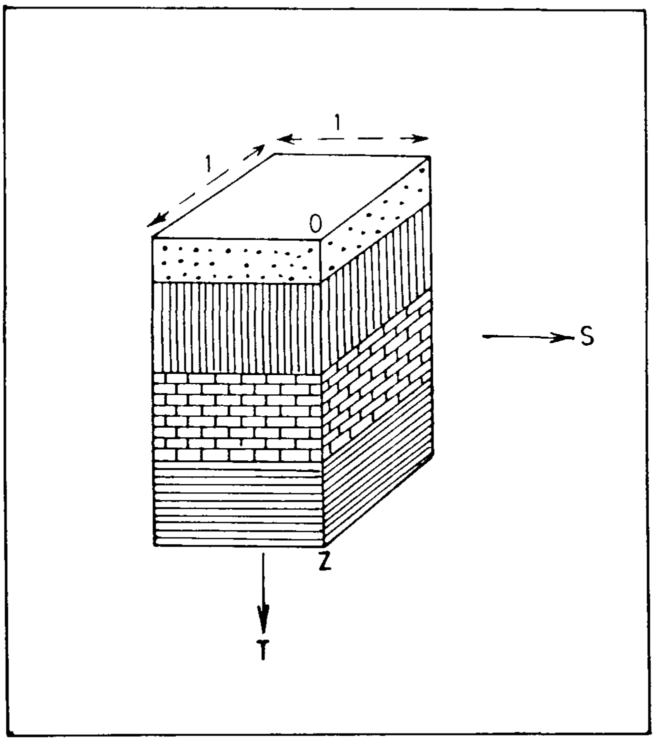

Fig 4.1 : Illustrating the Transversal resistance and the Longitudinal conductance.

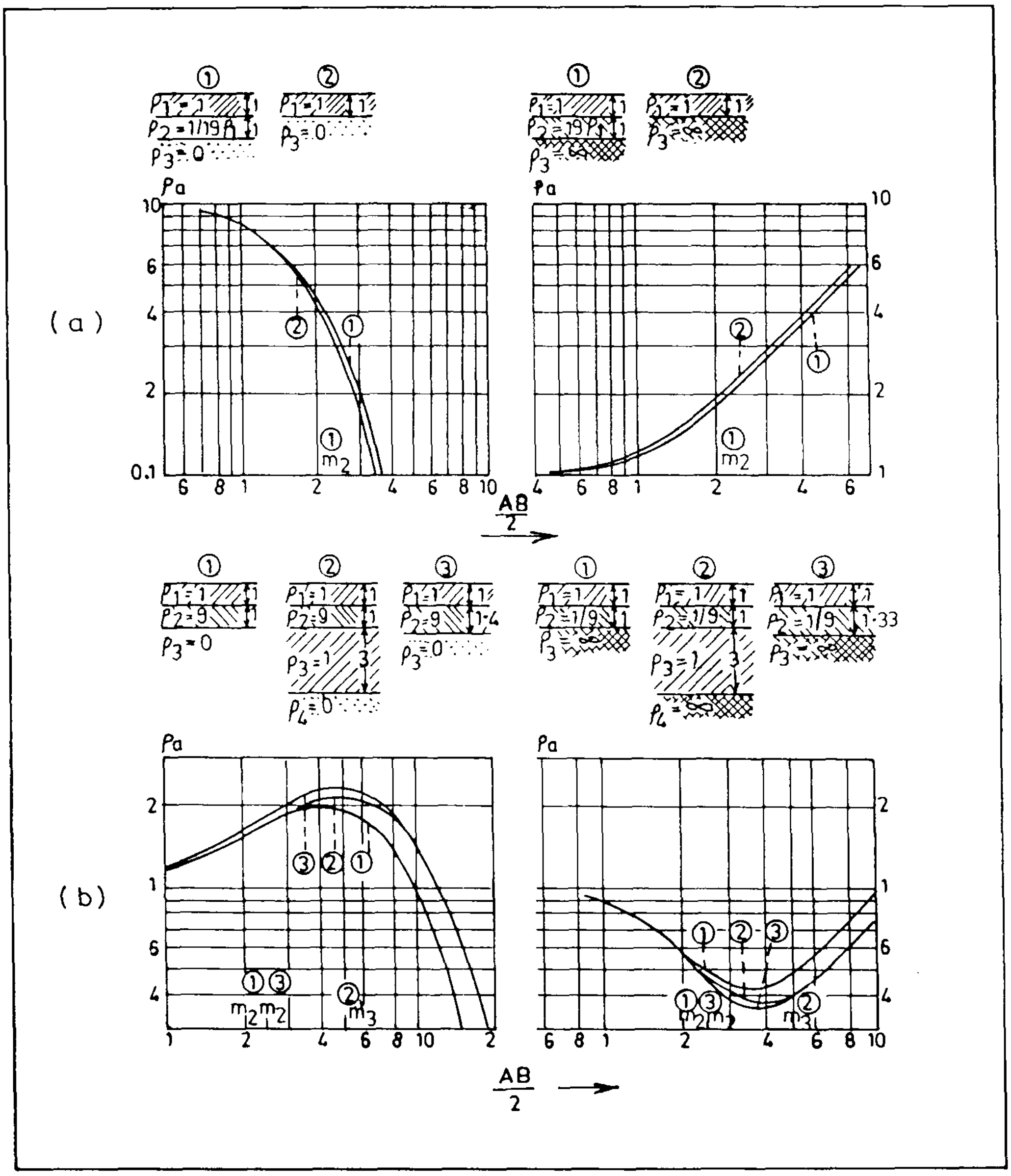

Fig 4.2 : Principle of equivalence and its limits (after Kunetz, 1966)

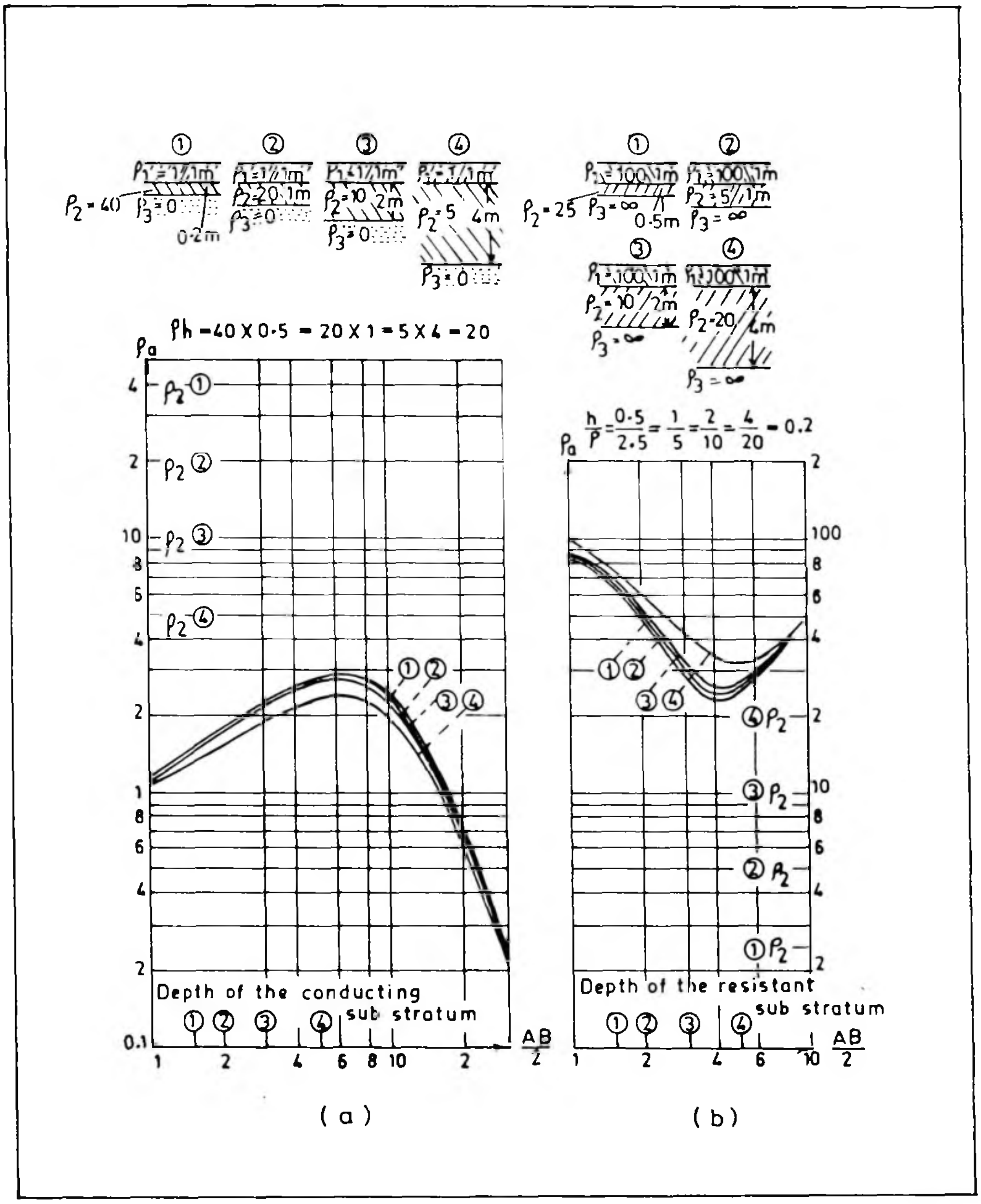

Fig 4. . . Principle of suppression and its limits (after Kunetz, 1966).

APPENDIX - 4

4.I. Determination of Potential distribution in a layered media

Let us consider a succession of horizontal, homogeneous and isotropic beds, whose resistivities are $\rho_1, \rho_2, \ldots, \rho_n$ and thicknesses are $h_1, h_2, \ldots, h_n$ respectively and the thickness of n+1 bed is infinite.

Now, the potential of each bed must satisfy Laplace's equation which is in rectangular coordinates is given in equatnion 4.7 :

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \ldots \qquad (4.7)$$

In cylindrical coordinate system, it can be written as

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad \ldots. \qquad (4.I.1)$$

4.1.

It is assumed that the z-axis is positive downward and the source is at the origin. Now assume a solution that has the form

$$V(r,z) = U(r).W(z) \qquad \ldots \qquad (4.I.2)$$

where U is a function of r only and W is a function of z only. Substituting equation 4.I.2. in equation 4.I.1, one obtains,

$$W.\frac{d^2 U}{dr^2} + \frac{W}{r}\frac{dU}{dr} + U\frac{d^2 W}{dz^2} = 0$$

$$\text{or, } \frac{1}{U}\frac{d^2 U}{dr^2} + \frac{1}{Ur}\frac{dU}{dr} = -\frac{1}{W}\frac{d^2 W}{dz^2} \qquad \ldots \qquad (4.I.3)$$

The above equation can be written as follows :

$$\frac{1}{W}\frac{d^2 W}{dz^2} = \lambda^2 \text{ or } \frac{d^2 W}{dz^2} = \lambda^2 W \qquad \ldots \qquad (4.I.4)$$

$$\text{and} \quad \frac{1}{U}\frac{d^2U}{dr^2} + \frac{1}{Ur}\frac{dU}{dr} = -\lambda^2$$

$$\text{or,} \quad r\frac{d^2U}{dr^2} + \frac{dU}{dr} + \lambda^2 rU = 0 \quad \ldots \quad (4.I.5)$$

where λ may have any value.

Equation 4.I.4 has two solutions

$$W = Ce^{-\lambda z} \text{ and } W = Ce^{+\lambda z} \quad \ldots \quad (4.I.6)$$

where C is an arbitrary constant.

The solution of equation 4.I.5 is

$$U = C.J_o(\lambda r) \quad \ldots \quad (4.I.7)$$

where J_o is the Bessel function of zero order.

By combining equation 4.I.2, 4.I.6 and 4.I.7, one can get two solutions for the potential field in any of the layers :

$$\left.\begin{aligned} V &= C.J_o(\lambda r)e^{-\lambda z} \\ V &= C.J_o(\lambda r)e^{+\lambda z} \end{aligned}\right\} \quad \ldots \quad (4.I.8)$$

where both C and λ have arbitrary values.

Since both C and λ may assume any value, the most general expression of such a linear combination is

$$V = \int_0^{\infty} \left[A_i(\lambda)e^{-\lambda z} + B_i(\lambda)e^{+\lambda z}\right] J_o(\lambda r).d\lambda \quad \ldots \quad (4.I.9)$$

The functions $A_i(\lambda)$ and $B_i(\lambda)$ will be determined in such a way that the solution will satisfy the required boundary conditions :

(a) in the neighborhood of the current electrode; and

(b) at the boundaries of the beds such as the earth's surface, interfaces between beds and infinity.

(a) The primary potential for the first medium which is homogeneous and infinite can be expressed through Webber's integral, when the potential tends to infinity at near the current electrode, as :

$$\frac{1}{U} = \frac{1}{\sqrt{r^2+z^2}} = \int_0^{\infty} e^{-\lambda|z|} J_0(\lambda r) d\lambda \qquad \ldots \quad (4.I.10)$$

The potential in the first medium can be written as :

$$V_1 = C\left\{ \frac{1}{U} + \int_0^{\infty} \left[(A_1-1)e^{-\lambda z} + B_1 e^{+\lambda z} \right] J_0(\lambda r).d\lambda \right\} \qquad \ldots (4.I.11)$$

(b) At the earth's surface, the vertical component of the current must be zero everywhere except at the electrode. Since the primary potential already satisfies that condition, the coefficients of second term also satisfies the condition. Thus,

$$A_1 - 1 = B_1.$$

At the interfaces between bed i and bed i+1, the continuity of the potential requires that

$$A_i(\lambda)e^{-\lambda p_i} + B_i(\lambda)e^{+\lambda p_i} = A_{i+1}(\lambda)e^{-\lambda p_i} + B_{i+1}(\lambda)e^{+\lambda p_i}.$$

where the depth of this interface is $p_i = h_1 + h_2 + \ldots + h_i$.

The normal component of the current flow across the boundary must also be continuous, so that,

$$\sigma_i \left[-A_i(\lambda)e^{-\lambda p_i} + B_i(\lambda)e^{+\lambda p_i} \right] = \sigma_{i+1}\left[-A_{i+1}(\lambda)e^{-\lambda p_i} + B_{i+1}(\lambda)e^{+\lambda p_i} \right]$$

In the last bed, the potential tends to zero, so that, $B_n(\lambda)$ must be zero.

Now, one can make the computations for two beds - a bed of thickness h and resistivity ρ_1, resting on an infinitely thick sub-stratum of resistivity ρ_2. Then,

$$A_1 - B_1 = 1$$

$$A_1 . e^{-\lambda h} + B_1 e^{+\lambda h} = A_2 e^{-\lambda h}.$$

$$-\frac{1}{\rho_1} . A_1 \ e^{-\lambda h} + \frac{1}{\rho_1} . B_1 \ e^{+\lambda h} = -\frac{1}{\rho_2} . A_2 \ e^{-\lambda h}.$$

By putting $k = \frac{(\rho_2 - \rho_1)}{(\rho_2 + \rho_1)}$, one gets,

$$A_1 - 1 = B_1 = k . \left(\frac{e^{-2\lambda h}}{1 - k^{-2\lambda h}} \right).$$

The expression for the potential at the earth's surface is,

$$V_1 \ (r, 0) = C \left[\frac{1}{r} + 2 \int_0^\infty J_0(\lambda r) \left(\frac{k e^{-2\lambda h}}{1 - k e^{-2\lambda h}} \right) d\lambda \right] \quad \ldots \quad (4.I.12)$$

The apparent resistivity with the Schlumberger configuration is proportional to

$$r^2 \frac{\partial V_1}{\partial r} .$$

In case of three beds consisting of two upper layers of thicknesses h_1 and h_2 and resistivities ρ_1 and ρ_2 and a sub-stratum of resistivity ρ_3, the potential is

$$V_1 = C \left[\frac{1}{r} + 2 \int_0^\infty J_0(\lambda r) \ \frac{k_1 e^{-2\lambda h_1} + k_2 e^{-2\lambda (h_1 + h_2)}}{1 - k_1 e^{-2\lambda h_1} - k_2 e^{-2\lambda (h_1 + h_2)} + k_1 k_2 e^{-2\lambda h_2}} d\lambda \right]$$
(4.I.13)

where $k_1 = \frac{(\rho_2 - \rho_1)}{(\rho_2 + \rho_1)}$ and $k_2 = \frac{(\rho_3 - \rho_2)}{(\rho_3 + \rho_2)}$

The fraction by which the Bessel function is multiplied under the integral has been known as "Stefanesco's function".

4.II. Derivation of Bessel Function

An explicit expression for $J_n(z)$ in the form of ascending series of powers of z is obtainable by considering the series for $\exp\left(\frac{1}{2} zt\right)$ and $\exp\left(-\frac{1}{2}.\frac{z}{t}\right)$, then

$$\exp\left[\frac{1}{2}.z\left(t-\frac{1}{t}\right)\right]=\sum_{r=o}^{\infty}\left\{\frac{\left(\frac{1}{2}.z\right)^{r}.t^{r}}{r!}\right\}.\sum_{m=o}^{\infty}\left\{\frac{\left(-\frac{1}{2}.z\right)^{m}.t^{-m}}{m!}\right\}$$

when n is a positive integer or zero, the only term of the first equation on the right hand side which, when associated with the general term of the second equation gives rise to a term involving t^n is the term for which r=n+m; and since $n \geq 0$, there is always one term for which r has this value. On associating these terms for all the values of m, one can see that the coefficient of t^n in the product is

$$\sum_{m=o}^{\infty}\frac{\left(\frac{1}{2}z\right)^{n+m}\left(-\frac{1}{2}z\right)^{m}}{(n+m)!m!}.$$

Therefore, the result is

$$J_n(z)=\sum_{m=o}^{\infty}\frac{(-)^{m}\left(\frac{1}{2}z\right)^{n+2m}}{m!(n+m)!} \quad \ldots \qquad (4.II.1)$$

where n is a positive integer or zero. The first few terms of the equation are

$$J_n(z)=\frac{z^n}{2^n.n!}\left[1-\frac{z^2}{2^2.1.(n+1)}+\frac{z^4}{2^4.1.2(n+1)(n+2)}-\ldots\right].$$

In particular

$$J_o(z) = 1 - \frac{z^2}{2^2} + \frac{z^4}{2^2.4^2} - \frac{z^6}{2^2.4^2.6^2} + \ldots$$

To obtain the Bessel coefficients of negative order, one selects the terms involving t^{-n} in the product of the equation representing $\exp(\frac{1}{2}.\ zt)$ and $\exp(-\frac{1}{2}.\ \frac{z}{t})$, where n is positive integer. The term of the second equation which, when associated with the general term of the first series gives rise to a term in t^{-n} is the term for which $m = n+r$; and so one has

$$J_{-n}(z) = \sum_{r=o}^{\infty} \frac{(\frac{1}{2}\ z)^r (-\frac{1}{2}.z)^{n+r}}{r!\ (n+r)!} \qquad \ldots \quad (4.II.2)$$ .2)

or,

$$J_{-n}(z) = (-)^n J_n(z).$$

It is observed that in the equation 4.II.1, the ratio of the (m+1)th to the mth term is $\frac{-\frac{1}{4}z^2}{\{m(n+m)\}}$, and this tends to zero as $m \to \infty$, for all values of z and n. By D'Alembert's ratio test for convergence, it follows that the equation representing $J_n(z)$ is convergent for all values of z and n and hence it is an integral function of z when $n = 0, \pm 1, \pm 2, \pm 3, \ldots$, (Watson, 1944).

Step by step procedure for numerical analysis of resistivity for a horizontally layered earth by solving "Kernel function" has been described by Onodera (1963).