CHAPTER-III

NUMERICAL SIMULATION OF A TROPICAL STORM BOUNDARY LAYER

Published in Mausam* (1992), Vol. 43
*Formerly, Indian Journal of Meteorology, Hydrology and Geophysics
3.1. Introduction

Although the large scale features of tropical storms are now well known, only the relatively dominant tangential flow is well documented. In case studies there is no sufficient data density to allow an accurate determination of other parameters; as such, several aspects of tropical storm remain speculative. One of the uncertainties is the distribution of radial motion in the storm inflow layer near the boundary. Aircraft observations of the radial velocity are relatively less accurate; because this component has small magnitude. Moreover, the data have been limited to a few flight levels during an individual storm period which can not produce an accurate profile of radial winds around a single storm. Vertical variation of radial motion in the inflow layer also can not be measured. Though the vertical velocity at the top of the boundary layer is small in comparison with the radial and tangential velocities, it is very significant for the growth and maintenance of a storm.

Surface friction causes convergence in an area of positive vorticity and thereby organizes cumulus convection which leads to the development of tropical storms as suggested by Ooyama (1964). In addition, surface friction is important for determining the horizontal scale of the convective area, and in producing the eye and the eyewall as observed in tropical storm.

Lency and Roger (1960) solved the boundary layer equations numerically considering rigid body rotation; but this is not a realistic model for a tropical storm winds. Smith(1968) investigated some of the features of tropical storm surface boundary layer by using a momentum integral method. The vertical profiles
of tangential and radial velocities were taken as those at geostrophic radius. Leslie and Smith (1970) extended the work of Smith (1968) for turbulence represented by different coefficients of eddy viscosity using the same method only in the region outside the maximum tangential velocity. Carrier et al. (1971) have pointed out that the momentum integral technique is not accurate enough to predict the vertical velocity at the top of the boundary layer which is very small but a significant quantity.

If the flow above the boundary layer is represented by a steady inviscid circular vortex with zero radial flow and tangential velocity $V_{gr}$ specified, then in absence of horizontal surface boundary, a vortex of this kind can support an arbitrary radial distribution of vertical velocity. But, in presence of such a boundary and $V_{gr}$ specified, the vertical velocity $W$ is determined uniquely by the boundary layer flow. In this study the dynamics of the boundary layer determined by this external flow are governed by the Navier-Stokes equations of motion. At a large distance (about 1000 km or so) from the centre of the storm, the flow can be assumed nearly geostrophic, and at this radius the boundary layer approximates to an Ekman type of layer. But the flow in the core region is like a rigid body rotation. So in this region, boundary layer equations admit separable solutions. Keeping in view of these two types of solutions, a method has been developed in this study to find out the boundary layer solutions numerically starting from the geostrophic radius to the centre of the storm.

There is no unique method to determine the turbulent flow. It is still impossible to analyze different kinds of turbulent
flow with the aid of the same hypothesis concerning turbulent friction. In this study a few expressions for the coefficients of kinematic eddy viscosity based on different hypotheses have been tested and their results discussed.

3.2. The Basic Equations

For simplicity, we will assume a steady state. The model is intended to represent a mature and slowly moving storm. We will employ cylindrical coordinate, $R$, $\theta$, $Z$; where, $R$ is the radial distance from the origin, $\theta$ is the azimuth and $Z$ is the height above the mean sea level. The model is further simplified by neglecting derivatives with respect to the azimuth distance ($\frac{\partial}{\partial \theta}$). This implies an axisymmetric storm. Data indicate that these derivatives are usually small compared to derivatives with respect to the radial distance. In the framework of boundary layer theory, it is assumed that the same pressure gradient acts in viscous layer near the surface (Schlichting, 1968). Observations indicate that horizontal gradient of temperature in the boundary layer of tropical storm is very weak (Frank, 1977; and others); so the assumption regarding constancy of $\frac{1}{R} \frac{\partial P}{\partial R}$ with height appears to be fairly realistic for the boundary layer. Under the assumptions, the equations for turbulent flow in a rotating frame of reference with variable eddy coefficients of viscosity are:

$$
\frac{\partial U}{\partial R} + \frac{\partial U}{\partial Z} \left( \frac{V^2}{R} - fV \right) + \frac{1}{R} \frac{\partial (\rho U^2)}{\partial R} \left( \frac{\partial U}{\partial R} \right) + \frac{\partial}{\partial Z} \left( K \frac{\partial U}{\partial Z} \right) = 0
$$

$$
\frac{\partial V}{\partial R} + \frac{\partial V}{\partial Z} + \frac{\partial U}{\partial R} = \frac{\partial}{\partial R} \left( \frac{\partial V}{\partial R} \right) + \frac{\partial}{\partial Z} \left( K \frac{\partial V}{\partial Z} \right)
$$

49
\[
\frac{\partial}{\partial r}(UR) + \frac{\partial}{\partial z}(VR) = 0
\]  
(3.3)

where, \(U, V\) and \(W\) are the velocity components in the radial, tangential and vertical directions; \(f\) is the Coriolis parameter \(=5\times10^{-5}\) sec\(^{-1}\). According to the assumptions the pressure gradient term in equation (3.1) has been replaced by the prescribed tangential velocity field \(V_{gr}(R)\) at the top of the friction layer. \(K_r\) and \(K_z\) are eddy coefficients of viscosity in the radial and vertical directions respectively.

### 3.2.1 Boundary conditions and other functions

We will assume a no slip condition at the surface. This implies:

\[
U = V = W = 0; \quad Z = 0 \quad \text{(3.4a)}
\]

and \(V = V_{gr}, U = 0; \quad Z = \infty \text{ (i.e. in the free atmosphere, where friction is negligible.)} \quad \text{(3.4b)}\)

The tangential velocity component \(V\) is assumed as:

\[
V = V_{gr}G(r, \eta) \quad \text{(3.5a)}
\]

where, \(G\) is a function of non-dimensional radial and vertical distances, \(r\) and \(\eta\) respectively. It is zero at \(\eta = 0\) and unity where \(V = V_{gr}\).

The radial and vertical velocity components \((U\ and\ W)\) may be expressed by a Stoke's stream function \(\psi\), which is taken as:

\[
\psi = Z_0RV_{gr}(1 + V_{gr}/Rf \ F(r, \eta)) \quad \text{(3.5b)}
\]

where \(F\) is a function of \(r\) and \(\eta\); \(Z_0\) is a scaling factor for \(Z\), (Sect 3.2.2). \(F\) is zero at \(\eta = 0\); and \(\partial F/\partial \eta\) is zero at \(\eta = 0\) and at large \(\eta\). The term \((1 + V_{gr}/Rf)\) in (3b) is the absolute angular velocity divided by the Coriolis parameter \(f\). The Stoke's stream function \(\psi\) has been assumed in such a way that it can represent flows both in the interior and exterior regions of a tropical
storm. At a large distance from the storm's centre the term $V_{gr}/Rf$ is negligible and the boundary layer equations approximate to an Ekman layer formulation. In the core region, if the superimposed flow is in rigid body rotation, the boundary layer equations admit separable solutions (Schlichting, 1968). Using the equation of continuity (3.3), $U$ and $W$ may be expressed by:

$$U = V_{gr} \left( 1 + \frac{V_{gr}}{Rf} \right) \frac{\partial F}{\partial \eta}$$  \hspace{1cm} (3.5c)

$$W = -Z_0 \left( \frac{dV_{gr}}{dR} \frac{R_f}{R} + 2 \frac{V_{gr}}{Rf} \frac{dV_{gr}}{dR} \right) F - \frac{Z_0 V_{gr}}{R} \left( 1 + \frac{V_{gr}}{Rf} \right) \frac{\partial F}{\partial R}$$  \hspace{1cm} (3.5d)

Boundary conditions on $\theta$ and $F$, which are consistent with the equation (3.4) are defined in Section 3.2.2.

### 3.2.2 Scaling of the variables

Following scalings are used to make the equations non-dimensional:

$$Z_0 = \gamma (K_z^* f^*), \quad Z = Z_0 \eta, \quad R = R_0 r, \quad U = R_0 f u, \quad V = R_0 f v, \quad W = Z_0 f w$$

$$V_{gr} = R_0 f v_{gr}, \quad K_z = K_z^* k(r, \eta), \quad K_r = K_r^* f, \quad \text{and} \quad k = k^* h(r, \eta)$$  \hspace{1cm} (3.6)

where, $f = 5 \times 10^{-5} \text{sec}^{-1}$ is Coriolis parameter; $R_0$ is 50km (radius of maximum tangential velocity of a typical storm); $K_z^*$ is 50 $m^2\text{sec}^{-1}$ (a characteristic value of vertical eddy coefficient); $K_r^*$ is $1.25 \times 10^5 m^2\text{sec}^{-1}$ (a characteristic value of radial eddy coefficient defined in terms of $R_0$ and $f$); $u$, $v$, $w$ and $v_{gr}$ are respectively the nondimensional radial, tangential, vertical velocities and superimposed gradient wind; $k(r, \eta)$ and $h(r, \eta)$ are non-dimensional vertical and radial eddy coefficients of viscosity.

Using (3.6) and omitting common factors, equations (3.1) and (3.2) reduce to the following non-dimensional form with variable coefficients:
\[ a_1 \frac{\partial^3 F}{\partial \eta^3} + a_2 \frac{\partial^2 F}{\partial \eta^2} + a_3 \frac{\partial F}{\partial \eta} + a_4 \left( \frac{\partial F}{\partial \eta} \right)^2 + a_5 \frac{\partial^2 F}{\partial \eta^2} \frac{\partial^2 F}{\partial \eta^2} + a_6 - 1.0 + G + a_6 G^2 \\
+ a_7 \frac{\partial^2 F}{\partial \eta \partial \tau} + a_8 \left( \frac{\partial F}{\partial \tau} \frac{\partial^2 F}{\partial \eta \partial \tau} - \frac{\partial^2 F}{\partial \tau \partial \eta \partial \eta} \right) = 0 \]  
(3.7)

\[ b_1 \frac{\partial^2 G}{\partial \eta^2} + b_2 \frac{\partial G}{\partial \eta} + b_3 G \frac{\partial F}{\partial \eta} + b_4 \frac{\partial G}{\partial \eta} - b_5 \frac{\partial F}{\partial \eta} + b_6 G + b_7 \frac{\partial G}{\partial \eta} \\
+ b_8 \left( \frac{\partial F}{\partial \tau} \frac{\partial G}{\partial \eta} - \frac{\partial F}{\partial \eta} \frac{\partial G}{\partial \tau} \right) + b_9 \frac{\partial^2 G}{\partial \eta^2} = 0 \]  
(3.8)

where, the coefficients \( a_1 \ldots a_8 \) and \( b_1 \ldots b_9 \) are functions of one or more of the variables \( r, k(r, \eta), h(r, \eta), \)

\( \frac{\partial k(r, \eta)}{\partial \eta}, \frac{dv_{gr}}{dr} \) and \( \frac{d^2 v_{gr}}{dr^2} \).

The boundary conditions consistent with those of 3.4(a) and 3.4(b) are:

\[ \eta = 0 : G = 0; F = 0; F' = 0 \]  
(3.9a)

\[ \eta = \infty : G = 1; F = 0; F' = 0 \]  
(3.9b)

where prime(’) denotes partial differentiation with respect to \( \eta \).

3.3. Numerical procedure

Equations (3.7) and (3.8) are non-linear. The boundary conditions for \( F \) and \( G \), are to be applied at \( \eta = 0 \), and \( \eta = \infty \) (at large \( \eta \)). The solutions have natural oscillation making it difficult to obtain an initial guess for the finite difference method. Even for a single differential equation of second order, the two point problem can be troublesome. If the problem is not well behaved, it is usually very difficult or even worthless to apply shooting method (Milne, 1970). A finite difference method does have a chance for convergence as it tends to keep a firm hold on the entire solutions at once. Considering this, a finite difference method has been used to solve the simultaneous non-linear equations.
We introduce a grid of \( N \) intervals of equal step size by choosing points from \( \eta = \Delta \eta \) to \( \eta = \Delta N \) in the vertical. The initial conditions at \( \eta = 0 \) and \( \eta = \infty \) (\( \text{Nth point} \)) are introduced directly in the finite difference equations. The non-linear equations (3.7) and (3.8) are replaced by linearised finite difference equations for, \( i = 1, \ldots, (N-1) \), giving \( 2N-2 \) equations, for \( 2N-1 \) unknowns (since, \( \theta_N = 1 \)). An extra equation is obtained by taking the derivative of (3.7) with respect to \( \eta \), at \( \eta = 0 \), as below:

\[
\frac{\partial^4 F(r,0)}{\partial \eta^4} + c_2 \frac{\partial^2 F(r,0)}{\partial \eta^2} + c_3 \frac{\partial^3 F(r,0)}{\partial \eta^3} + \frac{\partial G(r,0)}{\partial \eta} = 0 \tag{3.10}
\]

where, \( c_1 = a_1(0) \); \( c_2 = 2a_2(0) \); \( c_3 = a_2(0) + a_3(0) \).

Equations (3.7), (3.8) and (3.10) represent a complete set for the solution of the problem, giving \( 2N-1 \) equations for \( 2N-1 \) unknowns.

3.3.1. Linearization

If, \( F \) be a dependent variable, and \( \tilde{F} \) be assumed to be an approximate guess, or a previous step solution, we may set \( F = \tilde{F} + \varepsilon \), where \( \varepsilon \) is the correction required to produce the correct solution \( F \). We need not only \( \varepsilon \), but also \( \varepsilon', \varepsilon'', \varepsilon''', \varepsilon^{(iv)} \) in order to form the new quantities \( F' = \tilde{F}' + \varepsilon' \), \( F'' = \tilde{F}'' + \varepsilon'' \) and so on. The term like \( FG' \), are linearised by putting,

\[
FG' = (\tilde{F} + \varepsilon)(\tilde{G}' + \delta') = \tilde{F}G' + \varepsilon\tilde{G}' + \delta'\tilde{F}' \quad \text{(neglecting \( \varepsilon\delta' \))}
\]

\[
= FG' + \tilde{F}G' - FG' \tag{3.11}
\]

3.3.2. Finite differences

The differentials, are replaced by following finite difference scheme:
where, $\phi$ is any variable, 'i' is the grid point and 's' is the grid length.

When the finite difference scheme is applied on the lower boundary i.e. at $\eta=0$, the values of the variables at two grid points below the boundary are required.

Putting the boundary conditions of $F$ and $G$, at $\eta=0$, first equation of motion (3.7) reduces to:
\[
 k(r,\phi)F''(r,\phi) + \frac{\partial k(r,\phi)F'(r,\phi)}{\partial \eta} - 1 = 0
\]  (3.13)

Expanding $F$ in the neighbourhood of $\eta=0$ (at $\eta=\pm \Delta \eta$ and $\pm 2\Delta \eta$) by a Taylor's series and putting the boundary condition of $F$, the values of $F''(r,\phi)$ and $F'''(r,\phi)$ can be obtained in terms of $F_1$, $F_2$, $F_{-1}$, $F_{-2}$ and $\Delta \eta$. Replacing the differentials in (3.13) with these finite differences, we get:
\[
 F_{-1} = c_{a}F_1 - c_{ca} \\
 F_{-2} = c_{b}F_2 - c_{cb}
\]  (3.14)

where, $c_{a}$, $c_{ca}$, $c_{b}$, $c_{cb}$ are functions of $k(r,\phi)$, $\frac{\partial k(r,\phi)}{\partial \eta}$ and $s(=\Delta \eta)$. Putting the boundary conditions of $F$ and $G$, at $\eta=0$ in second equation of motion(3), we get:
\[
 G'(r,\phi) + \rho G'(r,\phi) = 0
\]
where, $\rho = \frac{\partial k(r,\phi)}{k(r,\phi)}$

In the neighbourhood of $\eta=0$, $G$ has a solution of the form:
\[
 G = 1.0 - e^{-\rho \eta}
\]
Expanding the exponent at \( \eta = \pm 2 \Delta \eta (= \pm s) \) and \( \eta = \pm 2 \Delta \eta (\neq \pm s) \); and since, \( |\rho s| < 1.0 \); after simplification, we get:

\[
G_1 = G_1 + \rho^2 s^2 + \rho^4 s^4 / 12 + \ldots \ldots \ldots \ldots
\]

\[
G_2 = G_2 + \rho^2 s^2 + \rho^4 s^4 / 3 + \ldots \ldots \ldots \ldots
\]

(3.15)

Beyond the upper boundary \( i = N \), the values of \( F \) and \( G \) are also required at the point \( i = N+1 \). Using boundary conditions, these can be taken as:

\[
F_{N+1} = -10 F_{N+3} + 6 F_{N-1} - 2 F_{N-2} + F_{N-3} / 3
\]

\[
G_{N+1} = 1.0
\]

(3.16)

Finally, applying linearization method (3.11), finite difference scheme (3.12) and relations, (3.14), (3.15) and (3.16); equations (3.7) and (3.8) can be expressed by the following difference form at the grid point \( 'i' \):

\[
\theta_i F_{i-2} + \alpha_i F_{i-1} + \beta_i F_i + \gamma_i F_{i+1} + \delta_i F_{i+2} + \xi_i G_i = \lambda_i
\]

(3.17)

\[
e_i F_{i-2} + \xi_i F_{i-1} + l_i F_i + c_i F_{i+1} + d_i F_{i+2} +

m_i G_{i-2} + p_i G_{i-1} + q_i G_i + r_i G_{i+1} + n_i G_{i+2} = t_i
\]

(3.18)

for, \( i = 1, 2, \ldots, N-1 \). Similarly, Eqn (3.10) reduces to:

\[
cb_1 F_1 + cb_2 F_2 + cb_3 G_1 + cb_4 G_2 = cb_5
\]

(3.19)

where, the coefficients \( \theta_i, \alpha_i, \beta_i, \gamma_i, \delta_i, \xi_i, \epsilon_i, \xi_i, l_i, c_i, d_i, m_i, p_i, q_i, r_i, n_i, \lambda_i \), and \( t_i \) \( (i = 1, N-1) \); \( cb_1, cb_2, cb_3, cb_4 \) and \( cb_5 \) are functions of one or more of the variables: \( r \), \( v_r \), \( k(r, \eta) \), \( h(r, \eta) \), \( f(r, \eta) \) and \( \bar{g}(r, \eta) \).

Equations (3.17), (3.18) and (3.19) have \( 2N-1 \) equations for \( 2N-1 \) unknowns \( (F_1, F_2, F_3 \ldots, F_N; G_1, G_2, G_3 \ldots, G_{N-1}) \); which are solved at each step by a direct method using double
precision arithmetic. In our numerical calculations, $N=40$; so, 79 equations are solved at a time.

At geostrophic radius (large $r$), first step initial values of $F$ and $G$ are taken from the known Ekman layer solutions, i.e.

$$F = e^{-\eta/\sqrt{2}} \left[ \cos(\eta/\sqrt{2}) + \sin(\eta/\sqrt{2}) - 1 \right]/\sqrt{2}$$

$$G = 1 - e^{-\eta/\sqrt{2}} \left[ \cos(\eta/\sqrt{2}) \right]$$

At successive steps (at same $r$), $F$ and $G$ are taken from previous step solutions. The process is repeated until the variations of $F$ and $G$ at consecutive steps at each point of $\eta$ become negligible. At the next smaller value of $r$, the initial guesses for $F$ and $G$ are taken from the previous step solutions and the same process is repeated until the solutions converge. Repeating the process, solutions up to the nearest point of storm's centre are obtained.

3.4. Numerical Experiment

We feel that a more quantitative description of the structure of turbulence is needed for a satisfactory formulation of the inflow layer. This could be achieved after more observations become available; but in absence of sufficient data, it is better to test different semi-empirical hypothesis and compare with available informations.

One of the objectives of this study is to investigate how the turbulent structure could be represented in the boundary layer. The following different formulations of the eddy coefficient of viscosity for turbulent flow are tested and compared with available informations.
3.4.1(a) Constant $K_z$ (Case A)

The simplest way to study the turbulent flow is to represent it by a constant kinematic eddy coefficient of viscosity $K_z$, as in laminar flow. In this case, it is difficult to estimate what should be the proper value of $K_z$. A value of the order of $50\text{m}^2/\text{s}$ was suggested by budget studies of hurricanes by Kasahara (1961) and Syono (1951). Rosenthal (1962), Smith (1968) and Haurwitz (1936) used the same value. But, some assume a value as low as $10\text{m}^2/\text{sec}$, while other consider it in the range of $100-150\text{m}^2/\text{sec}$. Here we put $K_z=50\text{m}^2/\text{sec}$ with no variation in any direction.

3.4.1(b) Variable $K_z$ (Case B)

Since eddy viscosity is not a property of fluid like viscosity of Stoke's law in laminar flow; but depends upon itself on the mean flow. It was unlikely that $K_z$ could be regarded as a constant over the whole extent of the storm. Prandtl hypothesizes that lumps of fluid on the whole preserve their velocity in turbulent motion and hence their momentum in the principal flow direction. According to his hypothesis the coefficient of kinematic eddy viscosity $K_z=l^2 \left( \frac{\partial V}{\partial z} \right)$, where $V$ is the velocity of mean motion and $'l'$ is mixing length which is unknown. It is known from experimental evidence that turbulent drag is roughly proportional to the square of velocity and the same result is obtained from Prandtl's hypothesis, if mixing length is assumed to be independent of magnitude of the velocity. This consideration leads us to take $K_z$ proportional to $V_{gr}$. So we put $K_z=1.25V_{gr}$ with no vertical variation. The choice of constant (1.25) is to make the maximum value of $K_z=50\text{m}^2/\text{sec}$, where $V_{gr}$ becomes maximum.
3.4.1(c) Exponential vertical variation of $K_z$ (Case C)

Under neutral stability and for moderate wind speeds (less than about 15 m/sec), it is well established observationally that $K_z = k v^* z$ in the sublayer, where shearing stress remains constant. The height of this layer ranges from 50 m to 200 m. In this formula, $k$ is Von Karman's constant which is approximately 0.4 and $v^*$ is friction velocity defined in terms of the surface stress $\tau_0$ by $v^* = |\tau_0/\rho|$. At a large height the variation of $K_z$ with $Z$ is an open question; but one can say that the variation is much slower than in the sublayer. Moreover, $K_z$ cannot increase indefinitely. Consequently, we put $K_z = K_z^* \left[ 1 - \exp\left( -kv^* z / K_z^* \right) \right]$, where $K_z^* = 0.4$ and $v^* = 5$ m/sec. With this representation, $K_z$ varies nearly linearly with height near the surface, and less rapidly with increasing height. At large height, it will approach to $50 m^2/sec$. In this case, there is no radial variation of $K_z$.

3.4.1(d) Radial and vertical variation of $K_z$ (Case D)

Here, $K_z$ is considered from Prandtl's mixing length hypothesis as discussed in section 3.4.1(b), with a constant mixing length and variation of $\frac{\partial V}{\partial Z}$ in the boundary layer. In addition, a constant value is added to it, so that at the top of the boundary layer, $K_z$ does not become insignificant, where $\frac{\partial V}{\partial Z}$ is very small. Thus, we put $K_z = 25 m^2/sec + 10^3 m^2 \left( \frac{\partial V}{\partial Z} \right)$. Anthes(1977) in his model for hurricane used such an expression for $K_z$ from empirical consideration. Here, $K_z$ will vary both radially and vertically.

3.4.2 External flow above the boundary layer

The external flow above the boundary layer is assumed to be axisymmetric and in gradient balance without any radial flow. In
a tropical storm, one does not know the exact swirl distribution and it need not be the same for all storms. However, it is sufficient to take a profile which has the general form of those observed in a typical tropical storm. Rosenthal (1962) and Smith (1968) derived the velocity profile of the external flow by assuming a suitable pressure profile in gradient balance. In this problem the profile has been taken from those obtained in the Chapter II. Fig. 3.1 shows its variation with radial distance $r$. It increases rapidly from zero at the centre to its maximum value of 40.5 m/sec at $r=1.4(R=70\text{km})$ and then decreases gradually with increasing $r$ to about 5 m/sec at $r=16.0 (R=800 \text{ km})$. In the core region, the profile has slightly been modified to represent exact rigid body rotation up to $r=0.9$. This enables us to compare the behaviour of our solutions with those having constant angular velocity.

3.4.3 Grids and numerical computations

From thermodynamic consideration, Palmen and Riehl (1957) concluded that the inflow must be restricted to the lower 3 km of the storm. This conclusion is consistent with the mean hurricane composite obtained from a standard data network by Jordan (1952) and Miller (1958). Anthes (1982) pointed out that the strongest inflow occurs within an elevation of about 500 m; although weak inflow often occurs at an elevation of about 3 km or more. The radial wind component is a significant component of flow below 1.0 km. Carrier et al. (1971) with their work beyond the core region of the storm, have shown that thickness of inflow layer is of the order of 1.6 km with an outflow layer of very weak
radial velocity above it. Frank(1977) in his composite studies of hurricanes mentioned that there is a cyclonic maximum between 900 and 800 mb, at the effective top of the friction boundary layer. It is possible that mid-level inflow results from an upward extension of frictional convergence by cloud momentum fluxes or from net entrainment into deep clouds.

In our numerical calculations we have kept the upper boundary at a height of 1.6 km ($\eta=1.6$) with equal grid spacing of 40 grid points, which reduces the grid length $\Delta \eta$ to 0.04. In radial direction the grid length is variable. From outer region, it is decreased step by step to the centre of the storm.

In tropical storm the ratio of its vertical scale to horizontal scale is very small; in consequence of it, the horizontal gradients of the stress terms are much smaller than the vertical gradients. So, in the present study the radial coefficient of eddy viscosity is not considered for numerical computation. It will be considered in the next study.

3.5. Results and discussions

Fig.3.2 shows the radial variations of maximum (at a particular $r$) inflow velocity $U$ for four cases. These occur at different heights at different radial distances. In all cases, it increases with decreasing radius, slowly at first and then more rapidly as the region of maximum tangential velocity is approached; thereafter, it decreases sharply to zero at the centre. These are typical profiles of inflow winds associated with a moderate storm. Its highest value shifts slightly to the centre from the maximum superimposed velocity. This shifting may be the result
of an inward turbulent transfer of angular momentum from higher wind region. In A and B, it has same value of 16.8 m/sec; while it is smallest (15.3 m/sec) in D and highest (20.5 m/sec) in C. Far away from the centre, A has lesser value than B; C and D are in between them. In B, it is nearly one third of V_{sr} (superimposed).

The variation of maximum radial velocity with the same external flow can probably be explained by comparing it with experimental results of laminar flow. In laminar flow, the boundary layer thickness of rotating fluid on solid surface is proportional to $\sqrt{K_z/\Omega}$, where $\Omega$ is the angular velocity of external flow (Schlichting, 1968). With same $\Omega$ the depth through which $V$ changes from zero at the surface, to $V_{sr}$ is proportional to $\sqrt{K_z}$. But, as in case C, if $K_z$ increases with height, the value of $V$ remains appreciably lower than $V_{sr}$ through greater depth than with constant $K_z$. Thus the radial inflow which is caused by the deficit of centrifugal force in the rotational component of velocity will continue to maintain its increasing tendency through greater depth causing higher value. The maximum value of $U$ highest in C and lowest in D, may probably be due to this reason.

The maximum values of $U$ obtained in cases A, B and D appear to be consistent with those found in other models and real storm. In Smith's (1968) model with 40 m/sec maximum superimposed velocity, same surface boundary conditions and constant $K_z$ as in A, the maximum inflow velocity was 15.25 m/sec; while in Rosenthal's (1962) model with maximum superimposed velocity of 46 m/sec same value of $K_z$, and different surface boundary condition, the maximum inflow velocity was slightly more than 14 m/sec. Anthes (1982) has pointed out that for a wind speed of 50 m/sec and
typical cross isobaric flow angle of 25 degrees, the radial component would be 21 m/sec.

The radial velocity varies with height and radial distance. Fig. 3.3 shows the variation of $U$ with height at different radial distances in case B. The profiles away from the centre are more flat and gradually become steeper as the radial distance decreases. The variation is steeper near the surface boundary as would be expected. At higher levels it slowly decays to a vanishing value. The height of maximum value also decreases with decreasing radial distance. In the core region, there is a weak outflow above the inflow region ($r \leq 1.3$). Composite studies by Shea and Gray (1973) support this. Extremely low sea level pressure in the centre of a fully developed storm far exceeds the value that could be caused by the hydrostatic effect of the temperature rise in the wall cloud by the release of latent heat of condensation. The development of the central eye with its considerably higher temperature probably due to the subsidence caused by the radial outflow of air in the lower level during deepening process. Gray and Shea (1973) mentioned that very strong horizontal wind shear at the boundary of the eye and eye wall region induces a high degree of turbulent mixing and horizontal momentum transfer into the eye where pressure gradient is small. The winds on the inner edge of the eye, thus must become supergradient and be accelerated outward into the wall clouds inducing sinking of air from higher levels and warming in the eye.

The vertical variation of $G$, and hence $V$ at different radial distances are shown in Fig. 3.4. Here also, the variation is steeper near the surface and tends asymptotically to unity at
higher level. In the upper part of the core region, the value of \( G \) exceeds unity. As discussed in previous paragraph, this may be due to horizontal momentum transfer into the eye.

The function \( F \) is such that its value is always negative and zero at the surface. With increasing height, its absolute value increases rapidly near the surface, then slowly and tends to a constant value at a large height. With the decrease of radial distance \( r \), its absolute value decreases (Fig. is not shown).

The profiles of vertical velocity \( W \), at the top of the inflow layer for the four cases A, B, C and D are shown in Fig.3.5. The circumferential velocity distribution in the boundary layer must transport fluid radially inwards, and the fluid must be supplied by a very slow downward motion in the exterior region. In all cases there is a large region of slowly varying weak down draft surrounding a small region of updraft around the centre. The updrafts are strongest close to and slightly inside of maximum tangential velocity. The downdraft extends from the edge of the storm, gradually increases with decreasing radius, then again decreases and becomes an updraft. Upto 125km the updraft is very small; but inward of this position, it increases very sharply. Maximum value of downdrafts in cases A, B, C and D are \(-0.65\), \(-0.25\), \(-0.52\) and \(-0.25\)cm/sec respectively. In Carrier's(1971) work for exterior region \((K_z=10m^2/sec)\) it was \(-0.26\)cm/sec; while in Smith's(1968) model \((K_z=50m^2/sec)\), it was \(-0.50\)cm/sec. In A and C, the maximum upward velocity in the core region remains nearly constant with decreasing \( r \); while in B and D, after attaining highest value it decreases towards the centre, more sharply in B than in D. The depth of the inflow layers in the
core region are shown in Fig. 3.6. In A, the maximum value of $W$ is 32.5 cm/sec with nearly constant thickness of the inflow layer in the core; the case C is nearly of the same type, but with lesser values. In B, the maximum value of $W$ is 34.5 cm/sec. Both $W$ and thickness of the inflow layer gradually decreases towards the centre. In D, the maximum value of $W$ is 35.2 cm/sec, which is the highest of all cases. The inflow layer also decreases towards the centre but less rapidly than B. The variation of $W$ and the thickness of the inflow layer in the core region in all cases seem to be consistent with the variation of the coefficients of eddy viscosity.

The flows near a stationary wall, when the fluid at a large distance above it rotates with constant angular velocity and constant viscosity are discussed in Schlichting (1968). It is found that the vertical velocity component, $W$ does not depend upon the radial distance $r$, and at all points it is upward. In this study the angular velocity of the superimposed flow is nearly constant in the core region, and in A we have got same type of results as above. In Rosenthal's (1962) model, with same $K_z$ and boundary condition as in A, and maximum absolute vorticity $\zeta = 650 \times 10^{-5}$/sec, the maximum upward velocity was 22 cm/sec near the maximum superimposed velocity, $V_{max} = 46$m/sec. In Smith's (1968) model, it was more than 100 cm/sec at a distance of 12 km from the centre with $\zeta = 205 \times 10^{-5}$/sec and $V_{max} = 40$m/sec at 40 km from centre. In Leslie and Smith's (1970) model, with $K_z = 7.8 \text{ m}^2$/sec, and same $V_{gr}$ profile as in Smith's (1968), maximum $W$ was 37 cm/sec, far inside the core from $V_{max}$.

It has already been mentioned that Smith (1968) and Leslie
and Smith (1970) used a momentum integral method. Frank (1977) in composite studies of hurricane pointed out that the vertical velocity at a radius of one degree from the centre and at a height of 950 mb was about 400 mb/day (≈4.5 cm/sec). Gray and Shea (1973) computed the vertical motion kinematically utilising divergence from composite studies above 900 mb. The highest ascending vertical motion was of the order of 80 cm/sec above 650 mb. It was concentrated in a narrow zone around the radius of maximum wind. Just above 900 mb, it was of the order of a few cm/sec. This pattern supports the view first mentioned by Wexler (1947) and later emphasised by Hughes (1952), Simpson (1952) and others, that a large fraction of ascending motion often occurs through a relatively narrow ring near the storm's centre. A study by Riehl and Malkus (1961) indicates that the hurricane does not have a uniform vertical motion pattern; only a small portion of the inner core is covered by a strong updraft. From the results of Malkus and others (1961), it was estimated that only 4% of the inner rain band area between 10-100 km radii had hot towers with 10 m/sec updraft near the tower top. Anthes (1982) pointed out that the vertical velocity at the cloud base is considerably larger than the mean vertical velocity. If updraft of a cloud base covers 5% of the area and there is no subsidence between the clouds, then the cloud base vertical velocity for a mean value $W$ of 2.7 cm/sec would be 54 cm/sec. In reality air is subsiding between clouds, so that cloud base vertical velocity would be greater.

The inflow angles at the surface for four cases are shown in Fig. 3.7. In all cases the inflow angle is small at large radii.
it gradually increases with decreasing radius and becomes maximum outside the region of maximum velocity, then it decreases rapidly at different rates. In the central core, it decreases slowly or remains nearly constant. In case C, it is too large throughout the entire storm area. In the outer region, the variation is more in A than in B or D. In the core region, the rate of decrease with radius is more in B than in A; while in D, it is nearly constant. The maximum inflow angle in A and B is 34°; but in D it is 31°. The variation in different cases may be attributed to the variation of the eddy coefficient of viscosity near the surface. The maximum inflow angle in Rosenthal's (1962) model was about 33°. Frictional convergence enhances the convective heating and this increases the convergence through modification of the pressure field. Consequently, one might expect that frictional convergence is relatively large in the inner part of the convective area except near the centre. Inflow angle in B, appears to be consistent with natural storm.

3.6. Conclusions

It is evident from our results that the magnitude and distribution of induced meridional circulation depends upon the structure of turbulence. In the core region, if $K_2$ is constant both radially and axially, the upward velocity $W$ does not decrease from its maximum value towards the centre. This is not consistent with the existence and favourable for creation of an eye of a tropical storm. If $K_2$ increases from surface with height but does not vary with radius, both the radial velocity and inflow angle become too high and in the core region $W$ does not decrease
to the centre. On the otherhand, if the eddy coefficient of vis-
cosity is assumed to vary with the superimposed flow above the
boundary layer the solutions compare favourably well with observ-
vations including inflow angle and boundary layer thickness. One
of the important results of this study is that it can simulate
the outflow from the core region in the upper part of boundary
layer. This is essential for creation of an eye. Though the ver-
tical velocity is a very small quantity in comparison to other
components, its magnitude and distribution is very significant
for the growth and maintenance of tropical storm. It seems that
this model can simulate it well. This model can be used to study
the boundary layer of a tropical storm from outer region to the
centre of the storm without neglecting any term in the equations
of motion. Moreover, there is no necessity of assuming any velo-
city profile, as required in momentum integral method. Represen-
tation of different kinds of boundary layer flow in the interior
and exterior regions of a storm by a single Stoke’s stream fun-
ction as developed in this study seems to be consistent with the
physical process of real storm. Finally, the finite difference
method applied here can be used to solve other non-linear
simultaneous differential equations.

On the sea surface, only the no-slip boundary conditions of
the variables have been tested. Further studies are to be made
incorporating suitable surface stress terms.
FIG. 3.1 TANGENTIAL VELOCITY PROFILE AT THE TOP OF THE BOUNDARY LAYER
FIG. 3.2 RADIAL PROFILES OF MAXIMUM INFLOW VELOCITY - U

LEGEND
A
B
C
D
FIG. 3-9 VERTICAL PROFILES OF U AT DIFFERENT RADIAL DISTANCES CASE-B
FIG. 3. VERTICAL PROFILES OF G, CASE-B
FIG. 3.5 VERTICAL MOTION AT THE TOP OF THE INFLOW LAYER
FIG. 3.6 DEPTH OF INFLO LAYER NEAR THE CORE
FIG. 3.7 SURFACE INFLOW ANGLE AS A FUNCTION OF RADIUS

LEGEND

A
B
C
D